Investments
Solutions to the Midterm Exam, May 2, 2012.

Part I, True-False Questions. You have 20 minutes to finish this closed-book section.

1. ( F ) By the end of 2008 all the major stand-alone investment banks in Europe had been absorbed into commercial banks or had reorganized themselves into bank holding companies. (page 52)

2. ( F ) The financial crisis of 2008 showed the importance of systematic risks. Policies that limit these risks include transparency, capital adequacy, frequent settlement of gains and losses, incentives to discourage excessive risk taking, and so on. (pages 52)

3. ( T ) Securities trading may take place in dealer markets, via electronic communication networks, or in specialist markets. In the latter, specialists maintain a limit order book. (page 114)

4. ( T ) Exchange-traded funds (ETFs), first introduced in 1993, are offshoots of mutual funds that allow investors to trade index portfolios just as they do shares of stock. (page 132)

5. ( T ) Investments in risky portfolios do not become safer in the long run. On the contrary, the longer a risky investment is held, the greater the risk. (page 182)

6. ( T ) An asset that is perfectly negatively correlated with a portfolio can serve as a perfect hedge. The perfect hedge asset can reduce the portfolio variance to zero. (page 252)

7. ( F ) The CAPM fails empirical tests. On average, low-beta securities have negative alphas and high-beta securities have positive alphas. (page 326)

8. ( T ) A passive investment strategy aims only at establishing a well-diversified portfolio of securities without attempting to find under- or overvalued stocks. One common strategy for passive management is to create an index fund. (page 379)
9. (F) Proponents of behavioral finance argue that an investor tends to exhibit the following behavioral biases: forecasting errors, overconfidence, conservatism, and sample size neglect and representativeness. (page 412)

10. (F) Empiricists have found pronounced positive long-term serial correlation in the performance of the (aggregate) U.S. stock market. (page 386)

Part II, Computations. This is an open-book section. Don’t work on this part until the TA announces that you can start. Solutions must be supported by explicit computations; an answer without associated computations will not earn you any credit.

1. Consider a two-period economy with perfect financial markets, where two risky assets (asset 1 and asset 2) and one riskless asset (asset 0) are traded at date 0, and cash flows are generated at date 1. The riskless asset is in zero net supply (a lending and borrowing opportunity), but the 2 risky assets are in strictly positive net supply. The Sharpe-Lintner CAPM holds in the date-0 equilibrium. There are only three investors in the date-0 financial markets, and we refer to them as A, B, and C. They only want to consume at date 1, and hence each of them will invest all the initial wealth in the date-0 traded assets. Investor \( i \in \{A, B, C\} \) is endowed with initial wealth \( W_i^0 > 0 \), together with a mean variance utility function \( U_i(E[\tilde{W}_i], \text{var}[\tilde{W}_i]) \), where \( U_i(\cdot, \cdot) \) is increasing in its first argument and decreasing in its second argument, and \( \tilde{W}_i \) is investor \( i \)'s date-1 (random) terminal wealth. In the following, for \( j = 1, 2, \tilde{r}_j, \tilde{r}_m \) and \( r_f \) stand for respectively the rate of return on asset \( j \), the rate of return on the market portfolio, and the riskless rate of interest. The following information is relevant.

- The expected rates of return on the two risky assets are

\[
E[\tilde{r}_1] = \frac{6}{25}, \quad E[\tilde{r}_2] = \frac{2}{25}.
\]

- Moreover, assume that \( \text{cov}[\tilde{r}_1, \tilde{r}_2] = -\frac{1}{25} \) and \( \text{var}[\tilde{r}_1] = \frac{1}{25} \).
• Investor A’s date-0 initial wealth is $W_{A0} = \$9,000,000$, and she lends $\$3,000,000$ and spends $\$4,000,000$ on asset 1 in equilibrium.

• Investor B’s date-0 initial wealth is $W_{B0} = \$10,000,000$, and she lends $\$5,000,000$ in equilibrium. Her equilibrium portfolio earns an expected rate of return equal to $\frac{448}{3000}$.

• Investor C’s equilibrium portfolio earns an expected rate of return equal to $\frac{672}{3000}$.

(i) Compute $\text{var}[\hat{r}_2]$.
(ii) Compute $r_f$.
(iii) Compute investor C’s initial wealth $W_{C0}$.

**Solution.** Recall that every investor must hold a portfolio consisting of the riskless asset and the market portfolio only; see Theorem 5 of Lecture 4. From investor A’s data we know that the market portfolio must be

\[
\mathbf{w}_m = \begin{bmatrix}
0 \\
\frac{4,000,000}{9,000,000 - 3,000,000} \\
\frac{9,000,000 - 3,000,000 - 4,000,000}{9,000,000 - 3,000,000}
\end{bmatrix} = \begin{bmatrix}
0 \\
\frac{2}{3} \\
\frac{1}{3}
\end{bmatrix}.
\]

It follows that the expected rate of return on the market portfolio is

\[
E[\hat{r}_m] = \frac{2}{3} \times \frac{6}{25} + \frac{1}{3} \times \frac{2}{25} = \frac{14}{75}.
\]

Investor B’s equilibrium portfolio must have an expected rate of return equal to

\[
\frac{5,000,000}{10,000,000}r_f + \frac{10,000,000 - 5,000,000}{10,000,000}E[\hat{r}_m],
\]

which by assumption is equal to $\frac{448}{3000}$. Thus we have

\[
r_f = 2 \times \frac{448}{3000} - E[\hat{r}_m] = \frac{112}{1000}.
\]

This finishes part (ii).

Since the market for the riskless asset must clear in equilibrium, and since there are only 3 investors in this economy, we know that investor C
must borrow $8,000,000 in equilibrium. Thus investor C’s equilibrium portfolio must have an expected rate of return equal to

$$W_{C0} + \frac{8,000,000}{W_{C0}} E[\tilde{r}_m] + \frac{-8,000,000}{W_{C0}} r_f,$$

which by assumption is equal to \(\frac{672}{9000}\). From here we obtain

$$W_{C0} = 16,000,000.$$

This finishes part (iii).

Now, it is easy to compute \(\text{cov}[^\tilde{r}_1, ^\tilde{r}_m]\). We have

$$\text{cov}[^\tilde{r}_1, ^\tilde{r}_m] = \frac{2}{3} \text{var}[^\tilde{r}_1] + \frac{1}{3} \text{cov}[^\tilde{r}_1, ^\tilde{r}_2]$$

$$= \frac{2}{3} \times \frac{1}{25} + \frac{1}{3} \times \left[-\frac{1}{50}\right] = \frac{1}{50}.$$ 

It follows from the CAPM equation that

$$\text{cov}[^\tilde{r}_2, ^\tilde{r}_m] = \left[\frac{E[^\tilde{r}_1] - r_f}{E[^\tilde{r}_2] - r_f}\right]^{-1} \text{cov}[^\tilde{r}_1, ^\tilde{r}_m]$$

$$= \frac{2}{25} - \frac{112}{1000} \times \frac{1}{50} = -\frac{1}{200}.$$

It follows that

$$-\frac{1}{200} = \frac{1}{3} \text{var}[^\tilde{r}_2] + \frac{2}{3} \text{cov}[^\tilde{r}_1, ^\tilde{r}_2]$$

$$= \frac{1}{3} \text{var}[^\tilde{r}_2] + \frac{2}{3} \times \left[-\frac{1}{50}\right] \Rightarrow \text{var}[^\tilde{r}_2] = \frac{1}{40}.$$

This finishes part (i).

2. This problem is adapted from Example 12 of Lecture 4. In this two-period economy, the date-0 markets for the N risky assets are perfect. The N risky assets are either in positive net supply or in zero net supply. The riskless asset (asset 0), however, is in positive net supply. Regarding the return on the riskless asset, the lending rate \(r_L\) differs from the borrowing rate \(r_B\). Recall that \(\tilde{r}_{mvp}\) denotes the rate of return on the minimum variance portfolio composed of risky assets only.
Here, the key difference from Example 12 of Lecture 4 is that now we assume
\[ r_B > E[\tilde{r}_{mvp}] = r_L > 0. \]

(i) Suppose that we need at least \( K \) distinct portfolios to span the portfolio frontier, where \( K \) is a positive integer. Compute \( K \). Draw the portfolio frontier on the \( \sigma - \mu \) space.
(ii) Suppose that we need at least \( k \) distinct portfolios to span the efficient frontier, where \( k \) is a positive integer. Compute \( k \). Draw the efficient frontier on the \( \sigma - \mu \) space.

**Solution.** The portfolio frontier and efficient frontier will be demonstrated in class. Recall that in this case an efficient portfolio consists of putting one's initial wealth in the riskless asset and then holding an arbitrage portfolio consisting of risky assets only. Recall that the optimal arbitrage portfolio can always be spanned by the two risky portfolios \( \frac{V^{c+1}}{V^{c+1} + 1} \) and \( \frac{V^{c+1}}{V^{c+1} + 1} \). Hence the portfolio frontier can be spanned by \( K = 4 \) funds (which include the above two risky portfolios, the lending opportunity, and the borrowing opportunity), and the efficient frontier can be spanned by \( k = 3 \) funds (which include the above two risky portfolios and the lending opportunity).

3. This last problem tests your knowledge about stochastic dominance.

A risk-neutral monopolistic firm, \( M \), is faced with the following inverse demand function (in the relevant range)
\[ p = 1 - q, \]
where \( p \) is the demand price for its product, and \( q \) is its choice of sales volume. \( M \)'s unit cost is originally \( c = \frac{1}{2} \). \( M \) chooses \( q \) to maximize its expected profit.

Right now, \( M \) is considering introducing a machine, which costs \( F > 0 \), and which can replace the unit cost \( \frac{1}{2} \) by a random unit cost \( \tilde{C} \), which is uniformly distributed on the unit interval \([0, 1]\). \( M \) can first observe the outcome of \( \tilde{C} \) and then choose \( q \).

(i) What is \( M \)'s equilibrium expected profit in the absence of the machine?
(ii) Show that there exists a threshold level \( F^* \) such that \( M \) is willing
to introduce such a machine if and only if \( F < F^* \). Compute \( F^* \).

**Solution.** Without the machine, M’s expected profit is

\[
(p - c)q = (1 - q - c)q = (\frac{1}{2} - q)q,
\]

which is maximized at the output level

\[
q^*(c) = \frac{1 - c}{2} = \frac{1}{4},
\]

leading to a payoff of \( \frac{1}{16} \) for M. So, M will be willing to introduce the machine if and only if by doing so M’s payoff can exceed \( \frac{1}{16} \).

In the presence of this machine, M’s optimal choice of sales volume is still

\[
q^*(\tilde{C}) = \frac{1 - \tilde{C}}{2},
\]

implying that M’s expected profit is

\[
E[(\frac{1 - \tilde{C}}{2})^2] = \frac{1}{4}[1 - 2E[\tilde{C}] + \int_0^1 C^2 dC] = \frac{1}{12}.
\]

It follows that M will be willing to buy the machine if and only if

\[
F < F^* = \frac{1}{12} - \frac{1}{16} = \frac{1}{48}.
\]

Given any realized unit cost \( c_0 \), M’s payoff before deducting \( F \) is

\[
q^*(c_0)[1 - q^*(c_0) - c_0] = \frac{(1 - c_0)^2}{4},
\]

which is convex in \( c_0 \). In other words, although M is risk-neutral with respect to its profit, M is actually risk-seeking with respect to its unit cost \( c_0 \). Now observe that the original unit cost \( c = \frac{1}{2} \geq SSD \hat{C} \). Thus M prefers \( \hat{C} \) to \( c = \frac{1}{2} \) when \( F = 0 \). The above part (ii) shows that M still prefers \( \hat{C} \) to \( c = \frac{1}{2} \) as long as \( F \) is not to high; i.e. \( F \leq F^* = \frac{1}{48} \).