Investments
The Answer Sheet for Closed-book Section
Name: __________ ID: __________

Instructor: Chyi-Mei Chen
(TEL) (02) 3366-1086
(Email) cchen@ccms.ntu.edu.tw

Instructions. This page is the answer sheet for the closed-book section. Make sure that you have written down your name and ID before getting started. Submit this page around 6:50pm, at the end of the closed-book section.

Put your solutions into the following table:

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Answer the following true-false questions.

- (No. 1) Derivative securities such as options and futures contracts provide payoffs that determine the prices of other assets such as bond or stock prices. (page 8)

- (No. 2) In Europe, where universal banking had been prohibited, large banks had long maintained both commercial and investment banking divisions. (pages 29)

- (No. 3) Eurodollars are dollar-denominated deposits at foreign banks or foreign branches of European banks. (page 35)

- (No. 4) American Depository Receipts, or ADRs, are certificates traded in foreign markets that represent ownership in shares of a U.S. company. (pages 47)

- (No. 5) Primary offerings can be sold in a public offering rather than a private placement. Public offerings can be far cheaper than private placements. (page 63)

- (No. 6) While its DirectPlus is simply an electronic order-routing system that transmits orders to the specialist’s post, the NYSE also has instituted a fully automated trade-execution system called SuperDot. (pages 73)

- (No. 7) Like mutual funds, hedge funds are commonly structured as private partnerships and thus subject to less regulation, and hence their managers can pursue investment strategies involving heavy use of derivatives, short sales, and leverage. (page 99)

- (No. 8) Exchange-traded funds (ETFs), first introduced in 2003, are offshoots of mutual funds that allow investors to trade index portfolios just as they do shares of stock. (page 108)
• (No. 9) The primary lesson from the history of short-term interest rates in the U.S. is that even a high rate of inflation cannot offset the nominal gains provided by Treasury bills. (page 134)

• (No. 10) VaR is written into regulation of banks, and closely watched by risk managers. Practitioners commonly estimate the 10% VaR, meaning that 90% of returns will exceed the VaR, and 10% will be worse. (pages 146)

• (No. 11) An active strategy describes a portfolio decision that avoids any direct or indirect security analysis. It generates an investment opportunity set that is represented by the CML. (page 187)

• (No. 12) Money market funds hold T-bills and short-term obligations such as CP and CDs, which are perfectly risk-free assets in nominal terms only. (pages 190)

• (No. 13) In a large investment company, the top management optimizes the security selection of each asset-class portfolio, and at the same time, continually updates the asset allocation of the organization. (page 227)

• (No. 14) Adding additional risky assets to a portfolio does not make the rate of return more predictable, even if it reduces dollar risk. (pages 233)

• (No. 15) The Markowitz procedure has two merits. First, the model does not require a huge number of estimates to fill the covariance matrix. Second, past returns are reliable guides to expected future returns and hence can be used to construct the efficient frontier of risky assets. (page 254)

• (No. 16) A tracking portfolio for portfolio P is a portfolio designed to match the unsystematic component of P’s return. (page 377)

• (No. 17) Resistance levels and support levels are respectively price levels below which it is difficult for stock prices to fall, and above which it is unlikely for them to rise. (page 359)
• (No. 18) The efficient market hypothesis (EMH) implies that stock prices should follow a random walk. The EMH has been widely accepted on Wall Street. (pages 354, 366)

• (No. 19) Limits to arbitrage activity enhance the ability of rational investors to exploit pricing errors induced by behavioral investors. (page 410)

• (No. 20) Technical analysis uses volume data and sentiment indicators, and is hence consistent with several behavioral models of investor activity. (page 250)
Part II, Computations. This is an open-book section. You can refer to any printed materials at hand or at our course website (which you can look up using a notebook or ipad), but you are not allowed to communicate with any other human being while working on this section. Don’t work on this part until the TA announces that you can start. Solutions must be supported by explicit computations; an answer without associated computations will not earn you any credit. Put your solutions in the green-stripe answer sheets.

1. Suppose that in a two-period perfect-markets economy, two risky assets (assets 1 and 2) together with a riskless asset (asset 0) are traded at date 0, which pay one-time cash flows at date 1. Thus in terms of our notation in Lecture 4, we have \( N + 1 = 3 \) traded assets at date 0. Suppose that for assets 1 and 2, we have the following data:

\[
\mathbf{e} = \begin{bmatrix} 0.18 \\ \mu_2 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} \sigma_1^2 & -0.015 \\ -0.015 & \sigma_2^2 \end{bmatrix}.
\]

Suppose that asset 0 is in zero net supply and assets 1 and 2 are in positive supply. Moreover, we maintain all the other assumptions made in the CAPM model, including that markets are perfect and every investor cares about only the first and second moments of his terminal wealth, and hence in equilibrium he will hold a mean-variance efficient portfolio. Finally, we assume that the Sharpe-Lintner CAPM holds in the date-0 equilibrium.

You are given the following facts.

- The minimum variance portfolio which consists of assets 1 and 2 only, is

\[
\mathbf{w}_{\text{mvp}} = \begin{bmatrix} 5/26 \\ 21/26 \end{bmatrix}.
\]

- \( \beta_1 = 4 \beta_2 \), where for \( j = 1, 2 \),

\[
\beta_j = \frac{\text{cov}(\tilde{r}_j, \tilde{r}_m)}{\text{var}(\tilde{r}_m)}.
\]
There are many investors at date 0, and among them, Mr. A’s initial wealth is $W_0 = 1,000,000$, and he borrows 250,000. The expected rate of return on his equilibrium portfolio is $E[\tilde{r}_A] = \frac{119}{820}$.

Mr. B’s initial wealth is $W_0 = 1,000,000$, and he lends 500,000. The expected rate of return on his equilibrium portfolio is $E[\tilde{r}_B] = \frac{121}{1025}$.

(i) Compute the standard deviation of $\tilde{r}_A$.

(ii) Compute the expected rate of return on $w_{mvp}$.

(iii) Compute $\sigma_1$ and $\sigma_2$.

**Solution.** According to the information about Mr. A and Mr. B, we must have

\[
\begin{aligned}
-\frac{1}{4}r_f + \frac{5}{4}\mu_m &= \frac{119}{820}, \\
\frac{1}{2}(r_f + \mu_m) &= \frac{121}{1025}.
\end{aligned}
\]

Thus we have

\[r_f = 0.1, \quad \mu_m = \frac{279}{2050}\]

It follows that

\[4 = \frac{\beta_1}{\beta_2} = \frac{0.18 - 0.1}{\mu_2 - 0.1} \Rightarrow \mu_2 = 0.12.
\]

It follows from

\[[w_m]'e = \frac{279}{2052}\]

that

\[w_m = \begin{bmatrix}
\frac{11}{34} \\
\frac{30}{34}
\end{bmatrix}.
\]
Now, we have

\[ 4 = \frac{\beta_1}{\beta_2} = \frac{11}{41} \cdot \sigma_1^2 + \frac{30}{41} \cdot (-0.015) \]

and

\[ \text{cov}(\hat{r}_1, \frac{5}{26} \hat{r}_1 + \frac{21}{26} \hat{r}_2) = \text{cov}(\hat{r}_2, \frac{5}{26} \hat{r}_1 + \frac{21}{26} \hat{r}_2), \]

which together imply the following system of equations:

\[
\begin{align*}
1,000\sigma_1^2 &= 48 + 4,200\sigma_2^2, \\
11,000\sigma_1^2 - 450 &= -660 + 120,000\sigma_2^2.
\end{align*}
\]

Solving, we have

\[ \sigma_1 = 0.3, \quad \sigma_2 = 0.1. \]

This finishes part (iii).

For part (ii), we have

\[ E[\hat{r}_{mvp}] = \frac{5}{26} \times (0.18) + \frac{21}{26} \times (0.12) = \frac{171}{1300}. \]

Finally, for part (i), we have

\[ \sqrt{\text{var}(\hat{r}_A)} = \frac{5}{4} \sigma_m = \frac{5}{4} \sqrt{\left(\frac{11}{41}\right)^2(0.09) + \left(\frac{30}{41}\right)^2(0.01) + 2\left(\frac{11}{41}\right)(\frac{30}{41})(-0.015)} \]

\[ = \frac{5}{4} \cdot \frac{1}{41} \sqrt{\frac{1089}{100} + \frac{900}{100} + \frac{66 \cdot (-15)}{100}} \]

\[ = \frac{5}{4} \cdot \frac{1}{41} \cdot \frac{1}{10} \sqrt{1089 + 900 - 990} \]

\[ = \frac{5}{4} \cdot \frac{1}{41} \cdot \frac{3}{10} \sqrt{111} \]

\[ = \frac{3\sqrt{111}}{328}. \]

This finishes part (i).
2. There are $N$ people in a two-period production economy with futures trading. At date 0, person $i$ seeks to maximize the expectation of the following utility function

$$U(w_i) - C(a_i) = [w_i - \frac{b}{2}w_i^2] - C(a_i),$$

where $b > 0$ is a very small positive constant, $a_i \geq 0$ the individual’s effort, and $w_i$ her date-1 wealth (which equals her date-1 consumption). Person $i$ has the following production function

$$\pi_i = f(a_i) + \tilde{\epsilon}_i,$$

where $\tilde{\epsilon}_i$ has zero mean.

There are $M$ traded futures at date 0, where contract $j$ pays a random payoff $\tilde{z}_j - p_j$ at date 1, with $p_j$ being the date-0 futures price. Denote $E[\tilde{\epsilon}_i\tilde{z}] = k_i, k = \sum_i k_i, E[(\tilde{z} - p)(\tilde{z} - p)'] = V, and E[\tilde{z} - p] = e.$

At date 0, person $i$’s problem is to choose effort $a_i$ and the positions $x_i$ in the $M$ futures contracts, so that his expected utility

$$E[U(\pi_i + (z - p)'x_i)] - C(a_i)$$

can be maximized. A Walrasian equilibrium is a tuple

$$(a_{N \times 1}^*, p_{M \times 1}^*, x_1^*, x_2^*, \ldots, x_N^*)$$

such that given $p^*$, $a_i^*$ and $x_i^*$ maximize person $i$’s expected utility, and the $M$ futures markets clear at the prices $p^*$.

In this exercise, we shall assume that

$$b = 1, \; M = N = 2, \; f(a) = a, \; C(a) = \frac{a^2}{2}.$$
Moreover, we assume that the random vector

\[
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
z_1 \\
z_2
\end{bmatrix}
\]

has zero mean (i.e., \(E[\epsilon_1] = E[\epsilon_2] = E[z_1] = E[z_2] = 0\)), and its covariance matrix is

\[
\begin{bmatrix}
1 & 0 & \frac{1}{2} & -\frac{1}{2} \\
0 & 1 & -\frac{1}{3} & \frac{1}{3} \\
\frac{1}{2} & -\frac{1}{3} & 1 & 0 \\
-\frac{1}{2} & \frac{1}{3} & 0 & 1
\end{bmatrix}.
\]

(i) Compute the equilibrium futures prices \(p_1\) and \(p_2\).

(ii) Compute \(V^{-1}\) and \(e\).

(iii) Compute \(a_1^*\) and \(x_1^*\).

(iv) Compute \(a_2^*\) and \(x_2^*\).

**Solution.** From the covariance matrix, we have

\[
k_1 = \begin{bmatrix}
\frac{1}{2} \\
-\frac{1}{2}
\end{bmatrix}, \quad k_2 = \begin{bmatrix}
-\frac{1}{3} \\
\frac{1}{3}
\end{bmatrix}, \quad k = \begin{bmatrix}
\frac{1}{6} \\
-\frac{1}{6}
\end{bmatrix}.
\]
It follows from the two investors’ first-order conditions that

\[
x_1^* = V^{-1}\{e[1 - a_1^*] + \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}\},
\]

\[
x_2^* = V^{-1}\{e[1 - a_2^*] + \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}\},
\]

\[
a_1^* = 1 - a_1^* - e'x_1^*,
\]

\[
a_2^* = 1 - a_2^* - e'x_2^*.
\]

From the last two first-order conditions, we have, using the markets-clearing condition,

\[a_1^* + a_2^* = 1;\]

and from the first two first-order conditions, we have, using again the markets-clearing condition, that

\[x_1^* + x_2^* = 0 \Rightarrow e = -p = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} \\ -\frac{1}{6} \end{bmatrix}.\]

This finishes part (i).

It follows that

\[V = E[zz'] - E[z]p' - pE[z'] + pp';\]

\[= \begin{bmatrix} 37/36 & -1/36 \\ -1/36 & 37/36 \end{bmatrix}.\]

It follows that

\[V^{-1} = \frac{1}{\left(\frac{37}{36}\right)^2 - \left(-\frac{1}{36}\right)^2} \begin{bmatrix} 37/36 & 1/36 \\ 1/36 & 37/36 \end{bmatrix} = \frac{1}{(37/36)(37/36)} \begin{bmatrix} 37/36 & 1/36 \\ 1/36 & 37/36 \end{bmatrix}\]

\[= \frac{36}{38} \begin{bmatrix} 37/36 & 1/36 \\ 1/36 & 37/36 \end{bmatrix} = \begin{bmatrix} 37/38 & 1/38 \\ 1/38 & 37/38 \end{bmatrix}.\]
This finishes part (ii).

Now, combining the first-order conditions for \( a_1^* \) and for \( x_1^* \), we have

\[
2a_1^* = 1 - e'x_1^*
\]

\[
= 1 - e'V^{-1}\{e[1 - a_1^*] + \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix}\},
\]

which, using

\[
e = \begin{bmatrix} 1/6 \\ -1/6 \end{bmatrix}
\]

and

\[
V^{-1} = \begin{bmatrix} 37/38 & 1/38 \\ 1/38 & 37/38 \end{bmatrix}
\]

implies that

\[
a_1^* = \frac{21}{37}.
\]

It follows from \( a_1^* + a_2^* = 1 \) that

\[
a_2^* = \frac{16}{37}.
\]

Now, using the first-order conditions for \( x_1^* \) and for \( x_2^* \), we have

\[
x_1^* = \begin{bmatrix} 37/38 & 1/38 \\ 1/38 & 37/38 \end{bmatrix} \begin{bmatrix} 16/37 \\ -1/6 \end{bmatrix} + \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} -15/37 \\ 15/37 \end{bmatrix}.
\]

Similary, one can obtain

\[
x_2^* = \begin{bmatrix} 37/38 & 1/38 \\ 1/38 & 37/38 \end{bmatrix} \begin{bmatrix} 21/37 \\ -1/6 \end{bmatrix} + \begin{bmatrix} 1/3 \\ -1/3 \end{bmatrix} = \begin{bmatrix} 15/37 \\ -15/37 \end{bmatrix}.
\]

This finishes parts (iii) and (iv).
3. This last problem tests your knowledge about stochastic dominance.

At date 0, a monopolistic firm, which is protected by limited liability, has one shareholder S and one debtholder D. The firm is currently run by S, and is faced with the following inverse demand function

\[ p = \begin{cases} 
1 - q, & \text{if } 0 \leq q \leq 1; \\
0, & \text{if } q > 1.
\end{cases} \]

At date 0, S must make two decisions. First, there are two production technologies facing the firm, called A and B, and S must decide which technology to adopt. If technology A is chosen, then the firm’s unit cost is \( c_A = \frac{1}{2} \). If technology B is chosen, then the firm’s unit cost is \( \tilde{c}_B \), which has two equally likely realizations \( \frac{2}{3} \) and \( \frac{1}{3} \). Second, after choosing the technology, S must choose an output level \( q \). The firm has no fixed costs, and it generates a random date-1 cash flow

\[ q(1 - q - \tilde{c}_j) \]

if S has chosen technology \( j \) and output level \( q \) at date 0. We assume that the decision-maker S is risk-neutral.

At the beginning of date 0, the firm has already borrowed some money from D. The firm promises to repay \( F \geq 0 \) to debtholder D at date 1, so that by holding the debt, D will receive the following random date-1 payoff:

\[ \min\{F, \max[q(1 - q) - q\tilde{c}_j, 0]\}, \]

if S decides to choose technology \( j \) and output level \( q \).\(^1\)

The timing of relevant events is as follows.

\(^1\)Note that the total cost \( q\tilde{c}_j \) represents a liability which is senior to the debt held by D. For example, \( q\tilde{c}_j \) may represent a paycheck to a worker, or, when the firm is a retailer, \( \tilde{c}_j \) may represent the wholesale price of the product.
At date 0, given $F$, S must first choose a technology $j \in \{A, B\}$. Then, given the chosen technology, S must choose $q$ to maximize his expected date-1 payoff, which is

$$E\{\max[q(1 - q - \hat{c}_j) - F, 0]\}.$$

Then, at date 1, the firm’s revenue $q(1 - q)$ and cost $q\hat{c}_j$ (both in cash) are both realized, and S and D get respectively their date-1 payoffs after paying the cost $q\hat{c}_j$.

(i) Suppose that $F = 0$. Which technology would S choose at date 0? What is the equilibrium output level at date 0?

(ii) Ignore part (i). Suppose instead that $F = \frac{1}{36}$. Which technology would S choose at date 0? What is the equilibrium output level at date 0? Suppose that at date 0 financial markets are perfect and all investors are risk-neutral without time preferences, and the efficient market hypothesis holds. Then what is the firm’s date-0 equity value, and what is the date-0 value of debt held by D?

**Solution.** Consider part (i). First suppose that technology A has been chosen. Then S seeks to

$$\max_q \max[q(1 - q - c_A), 0] = \max[q(\frac{1}{2} - q), 0] \equiv G(q(\frac{1}{2} - q)).$$

Since the function $G(\cdot)$ is non-decreasing, S should equivalently seek to

$$\max_q q(\frac{1}{2} - q),$$

implying that the optimal output level for S to go along with technology A is

$$q^* = \frac{1}{4}.$$

This yields for S the expected date-1 payoff of $\frac{1}{16}$. 

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Next, suppose that technology B has been chosen. Then S seeks to
\[ \max_q J(q) \equiv \frac{1}{2} \max[q(1 - q - \frac{1}{3}), 0] + \frac{1}{2} \max[q(1 - q - \frac{2}{3}), 0]. \]
Note that
\[
J(q) = \begin{cases} 
q(1 - q - \frac{1}{2}), & \text{if } 1 - q \geq \frac{2}{3}; \\
\frac{1}{2}q(1 - q - \frac{1}{3}), & \text{if } \frac{1}{3} \leq 1 - q < \frac{2}{3}; \\
0, & \text{if } 1 - q < \frac{1}{3}.
\end{cases}
\]
Direct computations show that
- for \( q \in [0, \frac{1}{3}] \),
  \[ J(q) \leq J\left(\frac{1}{4}\right) = \frac{1}{16}; \]
- for \( q \in (\frac{1}{3}, \frac{2}{3}] \),
  \[ J(q) < \frac{1}{2} \left(1 - \frac{1}{3} - \frac{1}{3}\right) = \frac{1}{18}; \]
and
- for \( q > \frac{2}{3} \),
  \[ J(q) = 0. \]
This means that \( q^* = \frac{1}{4} \) is again S’s optimal output choice to go along with technology B, and S’s expected date-1 payoff will equal \( \frac{1}{16} \) independently of the chosen technology. Thus, S feels indifferent about the two technologies, and either technology can be chosen at date 0.

Now, consider part (ii). First suppose that technology A has been chosen. Then S seeks to
\[ \max_q \max[q(1 - q - c_A) - F, 0] = \max[q(\frac{1}{2} - q) - F, 0] \equiv H(q(\frac{1}{2} - q)). \]
Note that \( H(\cdot) \) is non-decreasing, and hence S should seek to
\[ \max_q q(\frac{1}{2} - q), \]
implying that the optimal output level for S to go along with technology A is
\[ q_A = q^* = \frac{1}{4}. \]
This yields for S the following expected date-1 payoff
\[ \Pi_A \equiv \frac{1}{4} \left( \frac{1}{2} - \frac{1}{4} \right) - F = \frac{1}{16} - \frac{1}{36} = \frac{5}{144}. \]

Next, suppose that technology B has been chosen. Then S seeks to
\[ \max_q \frac{1}{2} \max \left[ q \left( 1 - q - \frac{1}{3} \right) - F, 0 \right] + \frac{1}{2} \max \left[ q \left( 1 - q - \frac{2}{3} \right) - F, 0 \right]. \]
We claim that for all \( q \geq 0 \),
\[ q \left( 1 - q - \frac{2}{3} \right) - F \leq 0. \]
This can indeed be verified easily:
\[ q \left( 1 - q - \frac{2}{3} \right) - F \leq \frac{1}{6} \left( 1 - \frac{1}{6} - \frac{2}{3} \right) - F = \frac{1}{36} - \frac{1}{36} = 0. \]
Thus the preceding objective function can be re-written as
\[ \max_q \frac{1}{2} \max \left[ q \left( 1 - q - \frac{1}{3} \right) - F, 0 \right], \]
and it is clear that S should equivalently seek to
\[ \max_q q \left( 1 - q - \frac{1}{3} \right), \]
which implies that the optimal output level for S to go along with technology B is
\[ q_B = \frac{1}{3}. \]
This yields for S the following expected date-1 payoff
\[ \Pi_B \equiv \frac{1}{2} \max \left[ \frac{1}{3} \left( 1 - \frac{1}{3} - \frac{1}{3} \right) - F, 0 \right] = \frac{1}{2} \left( 1 - \frac{1}{36} \right) = \frac{1}{24}. \]
Since $\Pi_B = \frac{1}{24} = \frac{6}{144} > \frac{5}{144} = \Pi_A$, we conclude that S will choose technology B and the output level $q_B = \frac{1}{3}$ at date 0. The date-0 equity value is thus $\Pi_B = \frac{1}{24}$ by the efficient market hypothesis.

Finally, what is the date-0 debt value? If we assume that the production cost is a paycheck to a worker, then this paycheck represents a debt senior to the debt held by D. When $\tilde{c}_B = \frac{2}{3}$, the firm must pay the worker $q_B \cdot \frac{2}{3} = \frac{2}{5}$, and yet the firm’s total revenue at date 1 is also $\frac{1}{3}(1 - \frac{1}{3}) = \frac{2}{9}$. Thus debtholder D gets nothing in this event. It follows that D’s date-1 expected payoff is simply $\frac{1}{2} \cdot F = \frac{1}{172}$, which, under the efficient market hypothesis, is exactly the date-0 value of the debt held by D.

**Remark.** This exercise shows that a financially leveraged firm has a risk-seeking tendency, if the firm is run by shareholders who are protected by limited liability. This has been emphasized in Example 6 of Lecture 2, where we pointed out that the shareholders’ payoff as a function of firm earnings is convex, and our theory of stochastic dominance suggests that the shareholders become risk-seeking when running the firm, even though they are risk-neutral with respect to cash flows.

In this exercise we have assumed that the firm is faced with a random unit cost of production. One interpretation is that, even if the hourly wage for the worker is pre-determined, the firm cannot be sure how many hours the worker will have to spend in order to produce one unit of the product—it would depend on the worker’s (stochastic) physical condition.