Game Theory with Applications to Finance and Marketing, I

Homework 0, to be discussed by TA

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1. Players 1 and 2 are bargaining over 1 dollar. The game proceeds in $2N$ periods. In period i, where i is odd, if no concensus has been reached before, then player 1 can make an offer $(x_i, 1-x_i)$ to player 2, where x_i is player 1's share, and player 2 can either accept or reject that offer, and if the offer is accepted, then the dollar is so divided; or else, the game moves on to the $i+1$st period. In period j, where j is even, if no concensus has been reached before, then player 2 can make an offer $(x_j, 1-x_j)$ to player 1, where x_j is again player 1's share, and player 1 can either accept or reject that offer, and if the offer is accepted, then the dollar is so divided; or else, the game moves on to the $j+1$st period.

   Both players would have zero payoff if no concensus has been reached at the end of period $2N$. If instead concensus is reached in period k, then player 1's payoff is $\delta_1^{k-1}x_k$ and player 2's payoff is $\delta_2^{k-1}(1-x_k)$.

   Now, suppose that $N=4$, $\delta_1=\frac{5}{8}$ and $\delta_2=\frac{4}{5}$. Then player 1's equilibrium payoff is ___A___ and player 2's equilibrium payoff is ___B___.

   Solution. It can be shown that, using backward induction,

   $$x_8=0,\ x_7=\frac{1}{5},\ x_6=\frac{1}{8},\ x_5=\frac{3}{10},$$

   $$x_4=\frac{3}{16},\ x_3=\frac{7}{20},\ x_2=\frac{7}{32},\ x_1=\frac{3}{8}.$$

   Thus we have $A=\frac{3}{8}$ and $B=\frac{5}{8}$.

2. Suppose that firms 1 and 2 must engage in Cournot competition at date 1. The date-1 inverse demand is $p=4-q_1-q_2$. Assume that initially

both firms have date-1 marginal cost equal to 2, but firm 1 alone has the chance to decide at date 0 whether to spend F and reduce its date-1 marginal cost by 1. At date 1, only firm 1 knows whether F was spent at date 0.

It can be shown that firm 1 would adopt a mixed strategy at date 0 if and only if F lies in the open interval (a, b), where $a+b = \underline{\quad C \quad}$, and in that case two prices $p^H > p^L$ may appear at date 1, with $p^H - p^L = \underline{\quad D \quad}$. If it is equally likely in equilibrium that firm 1 may or may not spend F, then we must have $F = \underline{\quad E \quad}$.

Solution. Suppose that F is spent with probability $1 - \pi$. Then firm 2 would seek to

$$\max_{q_2} q_2(4 - \overline{q}_1 - q_2 - 2),$$

where

$$\overline{q}_1 \equiv \pi q_1(2) + (1 - \pi)q_1(1),$$

and where $q_1(c_1)$ is firm 1's supply quantity when firm 1's marginal cost is $c_1 \in \{1, 2\}$. Thus we have

$$q_2 = \frac{2 - \overline{q}_1}{2}. \tag{1}$$

On the other hand, after spending F, firm 1 would seek to

$$\max_{q_1} q_1(4 - q_1 - q_2 - 1),$$

so that we have

$$q_1(1) = \frac{3 - q_2}{2}. \tag{2}$$

Similarly, without spending F, firm 1 would seek to

$$\max_{q_1} q_1(4 - q_1 - q_2 - 2),$$

so that we have

$$q_1(2) = \frac{2 - q_2}{2}. \tag{3}$$

It follows from (2) and (3) that

$$\overline{q}_1 = \pi q_1(2) + (1-\pi)q_1(1) = \frac{3-\pi-q_2}{2},$$

which, together with (1), implies that

$$\overline{q}_1 = \frac{4-2\pi}{3},\ q_2 = \frac{1+\pi}{3},$$

and hence by (2) and (3), we have

$$q_1(1) = \frac{8-\pi}{6},\ q_1(2) = \frac{5-\pi}{6}.$$

It follows that

$$p^H = 4 - q_1(2) - q_2 = \frac{17-\pi}{6},$$

and

$$p^L = 4 - q_1(1) - q_2 = \frac{14-\pi}{6}.$$

We conclude that, independent of π, we have

$$p^H - p^L = q_1(1) - q_1(2) = \frac{1}{2},$$

which is part D.

It follows that firm 1 would expect to get

$$q_1(2)(p^H - 2) = \frac{(5-\pi)^2}{36}$$

if firm 1 decides to not spend F at date 0; and firm 1 would expect to get

$$q_1(1)(p^L - 1) - F = \frac{(8-\pi)^2}{36} - F$$

if firm 1 decides to spend F at date 0. Since firm 1 must feel indifferent about to or not to spend F in a mixed strategy equilibrium, we have

$$\frac{(5-\pi)^2}{36} = \frac{(8-\pi)^2}{36} - F \Rightarrow \pi = \frac{13}{2} - 6F,$$

and since

$$0 < \pi < 1,$$

we have

$$\frac{11}{12} = a < F < b = \frac{13}{12},$$

so that $a + b = 2$, which is part C.

In particular, if $\pi = \frac{1}{2}$, then we have $F = 1$, which is part E.

3. Given a metric space (X, d), recall the following definition:

   - Given any $x \in X$ and $e > 0$, the set $B(x, e) = \{y \in X : d(x, y) < e\}$ is an *open ball.*
   - A is an *open set* in X if and only if A is a (empty or non-empty) union of open balls.
   - A is a *closed set* in X if and only if A^c is an open set in X.
   - The intersection of all closed sets that contain A is called the *closure* of A, and denoted by $\overline{A}$. Thus $\overline{A}$ is the *smallest* closed set containing A.
   - The union of all open sets contained by A is called the *interior* of A, and denoted by $\text{Int}(A)$. Thus $\text{Int}(A)$ is the *largest* open subset of A.
   - Given $A \subset X$ and $x \in X$, we say x is a *limit point* of A if and only if *every* open ball $B(x, e)$ with $e > 0$ contains an element of A which is not x.
   - Given $A \subset X$ and $x \in X$, we say x is an *interior point* of A if and only if we can find *one* open ball B such that $x \in B \subset A$.
   - Given $A \subset X$ and $x \in X$, we say x is an *isolated point* of A if and only if we can find *one* open ball $B(x, e)$ (with $e > 0$) such that $B(x, e) \bigcap A = \{x\}$.

   Now, prove the following assertions.

   (a) If $x \in A$, then x is an interior point of A if and only if x is *not* a limit point of A^c.

Proof. Note that if x is an interior point of A so that $B(x, e) \subset A$ for some $e > 0$, then $B(x, e)$ cannot intersect A^c at some point $y \neq x$, and hence x is not a limit point of A^c. Conversely, if x is *not* an interior point of A so that for each and every $e > 0$, $B(x, e)$ would contain some element of A^c, then that element of A^c certainly differs from x (recall that $x \in A$), so that x is indeed a limit point of A^c.

(b) A is closed if and only if A contains all its limit points.

Proof. Suppose that A is closed and x is a limit point of A. If $x \in A^c$, where note that A^c is open, then by Fact 8 x is an interior point of A^c, so that x cannot be a limit point of A, which is a contradiction. We conclude that a closed set must contain all its limit points.

Now, suppose that A contains all its limit points. If $x \in A^c$ and x is not an interior point of A^c (so that A is not a closed set), then x is a limit point of A, and hence $x \in A$, which is a contradiction. Thus a set containing all its limit points must be closed.

(c) If $A \subset X$ and A is a finite set, then A has no limit points and is automatically a closed set. Moreover, every element of A is an isolated point.

Proof. Suppose that $A = \{x_j \in X : \ j = 1, 2, \cdots, n\}$, where $x_i \neq x_j$ whenever $j \neq i$. Define

$$e \equiv \min\{d(x_i, x_j) : i \neq j, \ i, j = 1, 2, \cdots, n\}.$$

Then $x_k \in B(x_i, \frac{e}{2}) \Rightarrow x_k = x_i$, proving that all x_i's are isolated points of A, and that none of them are limit points.

If $y \in A^c$ and y is a limit point of A, then for all n, $B(y, \frac{1}{n})$ must contain some $x_n \in A$ with $x_n \neq y$, proving that A cannot be a finite set. Thus A^c does not contain any of A's limit points either. Since the set of limit points for A is empty, A is closed.

(d) Suppose that $X = \Re$ and $d(x, y) = |x - y|$. Suppose that the sequence $\{x_n; n \in \mathbf{Z}_+\}$ converges to x_0. Let A contains exactly each and every x_n. Then x_0 is a limit point of A if and only if A is *not* a finite set.

Proof. If x_0 is a limit point of A then for all $m \in \mathbf{Z}_+$ the open ball $B(x_0, \frac{1}{m})$ must contain some $x_m \in A$ with $x_m \neq x_0$. A cannot be a finite set in this case.

If x_0 is *not* a limit point of A then there exists $e > 0$ such that $B(x_0, e)$ does not contain any $y \in A$ with $y \neq x_0$, but then x_0 cannot be the limit of the sequence $\{x_n\}$ unless there exists $m \in \mathbf{Z}_+$ such that $x_k = x_0$ for all $k \geq m$, proving that A is a finite set.

(e) Suppose that $X = \Re$ and $d(x, y) = |x - y|$. Suppose that A is a non-empty subset of X and u is an upper bound for A. Then $\sup A$ exists, and there exists a sequence in A that converges to $\sup A$. In particular, if A does not contain $\sup A$, then there exists an increasing sequence in A that converges to $\sup A$.

Proof. There exists $x_0 \in A$ because $A \neq \emptyset$. Define $d_1 \equiv x_0$ and $u_1 \equiv u$. Suppose that we have defined u_j and d_j for all $j = 1, 2, \cdots, k$, then define u_{k+1} and d_{k+1} as follows:

If $\frac{u_k + d_k}{2}$ is an upper bound for A, then define $u_{k+1} \equiv \frac{u_k + d_k}{2}$ and $d_{j+1} \equiv d_k$; otherwise, define $u_{k+1} \equiv u_k$ and d_{k+1} as some element of A such that $d_{k+1} \geq \frac{u_k + d_k}{2}$. Thus $\{d_k\}$ is an increasing sequence[1] contained in A, and $\{u_k\}$ is a decreasing sequence of upper bounds for A.

Now, we have an increasing sequence $\{d_k\}$ bounded above by u_1 and a decreasing sequence $\{u_k\}$ bounded below by x_0. By Axiom of Continuity,[2] the two sequences are convergent sequences, and

[1]Here, increasing means weakly increasing or non-decreasing. I would say strictly increasing if the sequence is not just weakly increasing.

[2]This axiom says that a decreasing real-valued sequence which is bounded below has a limit, and so does an increasing real-valued sequence which is bounded above.

they share the same limit, which is easy to show to be $\sup A$.[3] If $\sup A \in A$, then $\sup A$ is the limit of the constant sequence $\{\sup A\}$; otherwise, $\sup A$ is the limit of the increasing sequence $\{d_k\}$.

4. We now state a generalized version of the Kuhn-Tucker Theorem.

   Consider the following maximization problem:

$$\max_{\mathbf{x} \in \Re^n} f(\mathbf{x})$$

subject to

$$g_i(\mathbf{x}) \leq 0, \ \forall i = 1, 2, \cdots, m;$$

$$h_j(\mathbf{x}) = 0, \ \forall j = 1, 2, \cdots, l.$$

This maximization problem involves m inequality constraints and l equality constraints. The associated Slater condition is as follows: there exists $\hat{\mathbf{x}} \in \Re^n$ such that

$$g_i(\hat{\mathbf{x}}) < 0, \ \forall i = 1, 2, \cdots, m;$$

$$h_j(\hat{\mathbf{x}}) = 0, \ \forall j = 1, 2, \cdots, l.$$

When the Slater condition holds, the following Kuhn-Tucker necessary conditions must hold: at an optimal solution $\mathbf{x}^*$, where $\{Dh_j(\mathbf{x}^*); j = 1, 2, \cdots, l\}$ consists of a set of linearly independent vectors, there must exist $(\mu_1, \mu_2, \cdots, \mu_m)^T \in \Re^m_+$ and $(\lambda_1, \lambda_2, \cdots, \lambda_l)^T \in \Re^l$ such that

$$(\textbf{Stationarity}) \ Df(\mathbf{x}^*) = \sum_{i=1}^{m} \mu_i Dg_i(\mathbf{x}^*) + \sum_{j=1}^{l} \lambda_j Dh_j(\mathbf{x}^*);$$

$$h_j(\mathbf{x}^*) = 0, \ \forall j = 1, 2, \cdots, l;$$

$$(\textbf{Complementary Slackness}) \ \mu_i g_i(\mathbf{x}^*) = 0, \ \forall i = 1, 2, \cdots, m.$$

Conversely, when the Slater condition holds, f is concave, g_i's are convex, and h_j's are affine, if $\mathbf{x}^*$, $(\mu_1, \mu_2, \cdots, \mu_m)^T \in \Re^m_+$, and $(\lambda_1, \lambda_2, \cdots, \lambda_l)^T \in$

[3] Show that $\lim d_k \leq \lim u_k$, and if the inequality were strict, then a contradiction would arise. Moreover, show that $\lim u_k$ is one upper bound for A, and u' cannot be an upper bound for A if $u' < \lim u_k$.

$\Re^l$ satisfy the above Kuhn-Tucker necessary conditions, then $\mathbf{x}^*$ must be an optimal solution.[4]

Now, consider the following exercises.

(a) Consider the following maximization problem: for some constant $a \in \Re_+$,

$$\max_{x \in \Re} f(x; a) \equiv 2a - (x - a)^2$$

subject to

$$g(x) = (x - 3)^2 - 1 \leq 0.$$

Given a, let $x^*(a)$ be the unique optimal solution and $\pi(a)$ the Lagrange multiplier associated with the constraint $g \leq 0$.

(i) Show that in the above maximization problem the Slater condition is satisfied, and that f and g are respectively concave and convex.

(ii) Show that there exist $\underline{a}, \overline{a} \in \Re_+$ with $\underline{a} < \overline{a}$ such that $\pi(a) > 0$ if and only if either $a < \underline{a}$ or $a > \overline{a}$. Find $\underline{a}$ and $\overline{a}$.

(iii) Compute $x^*(\underline{a}-1)$, $\pi(\underline{a}-1)$, $Df(x^*(\underline{a}-1))$ and $Dg(x^*(\underline{a}-1))$.

(iv) Compute $x^*(\overline{a}+1)$, and $\pi(\overline{a}+1)$, $Df(x^*(\overline{a}+1))$ and $Dg(\overline{a}+1))$.

(b) Suppose that $f : \Re \to \Re$ is twice continuously differentiable and strictly concave. Let x^* attain the maximum value of f. Show that if $f'(x) > 0$ then $x^* > x$, and if $f'(x) < 0$ then $x^* < x$.

[4] Visit for example
https://en.wikipedia.org/wiki/Karush%E2%80%93Kuhn%E2%80%93Tucker_conditions
to see a list of regularity conditions (or constraint qualification conditions) including the Slater condition. After Harold W. Kuhn and Albert W. Tucker published the Kuhn-Tucker conditions in 1951, scholars discovered that these necessary conditions had been stated by William Karush in his master's thesis in 1939. Hence nowadays people also refer to the Kuhn-Tucker conditions as Karush-Kuhn-Tucker conditions.

(c) Consider the following maximization problem:

$$\max_{(x,y)\in\Re^2} x,$$

subject to

$$g(x,y) = (x+1)^2 + (y-2)^2 - 4 \leq 0;$$

$$h(x,y) = y - 3 = 0.$$

Let μ_g and μ_h denote the Lagrange multipliers associated with respectively $g \leq 0$ and $h = 0$. Let (x^*, y^*) denote *the* optimal solution.

(i) Are constraint qualification conditions stated in section 48 of Lecture 0 satisfied?

(ii) Find μ_g, μ_h, and (x^*, y^*).

(d) Consider the following maximization problem:

$$\text{Problem (P):} \max_{T_1,T_2,q_1,q_2} \frac{1}{2}[T_1 - cq_1] + \frac{1}{2}[T_2 - cq_2]$$

subject to

$$\theta_1 V(q_1) - T_1 \geq 0, \tag{4}$$

$$\theta_2 V(q_2) - T_2 \geq 0, \tag{5}$$

$$\theta_1 V(q_1) - T_1 \geq \theta_1 V(q_2) - T_2, \tag{6}$$

$$\theta_2 V(q_2) - T_2 \geq \theta_2 V(q_1) - T_1. \tag{7}$$

The interpretation is as follows. A large bank is facing a small borrowing firm, which is equally likely to be of type θ_1 or type θ_2. By borrowing q dollars from the bank today (date 0), a type-θ_j borrowing firm will generate a cash flow $\theta_j V(q)$ tomorrow (date 1). The bank cannot tell the borrowing firm's type, and hence it offers a menu of choices to the borrowing firm, and asks the firm to pick one. The menu says that, for $j \in \{1, 2\}$, if the firm would borrow q_j dollars today, then it has to repay the bank T_j dollars tomorrow.

In designing the menu of bank loan contracts, the bank must make sure that the borrowing firm is willing to accept the deal, regardless of its type (so that (1) and (2) must hold); and the bank must also make sure that a type-θ_j borrowing firm would rather accept the deal (q_j, T_j) than accept (q_i, T_i), which is designed for the other type θ_i (and hence (3) and (4) must hold). (We are assuming zero interest rates, so that there is no discounting.)

Finally, note that the objective function in (P) says that the bank considers both types of the borrowing firm equally likely, and that for each dollar lent to the firm, the bank must incur a cost $c > 0$. The bank thus seeks to maximize its expected profit from lending to the firm.

Let us assume from now on that

$$\theta_1 = 3,\ \theta_2 = 4, V(q) = \ln(1+q),\ c = \frac{1}{4}.$$

Let $(q_2^{**}, q_1^{**}, T_2^{**}, T_1^{**})$ denote the optimal solution to the above maximization problem. Show that $q_2^{**} = 15$ and $q_1^{**} = 7$. What are the associated T_2^{**} and T_1^{**}? [5]

5. Let us now practice the concept of common knowledge.

[5] To apply the Kuhn-Tucker Theorem, first define $h(x_j) \equiv e^{x_j} - 1$, and re-write the maximization problem as

$$\text{Problem (P):} \max_{T_1, T_2, x_1, x_2} f(T_1, T_2, x_1, x_2) \equiv T_1 - ch(x_1) + T_2 - ch(x_2)$$

subject to

$$g_1 \equiv T_1 - \theta_1 x_1 \leq 0;$$
$$g_2 \equiv T_2 - \theta_2 x_2 \leq 0;$$
$$g_3 \equiv T_1 - T_2 + \theta_1(x_2 - x_1) \leq 0;$$
$$g_4 \equiv T_2 - T_1 + \theta_2(x_1 - x_2) \leq 0.$$

Let μ_j be the associated Lagrange multiplier for constraint $g_j \leq 0$.

Then, show that f is strictly concave and g_j's are convex, with

$$Df = \begin{bmatrix} 1 \\ 1 \\ -ch'(x_1) \\ -ch'(x_2) \end{bmatrix},\ Dg_1 = \begin{bmatrix} 1 \\ 0 \\ -\theta_1 \\ 0 \end{bmatrix},\ Dg_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -\theta_2 \end{bmatrix},$$

$$Dg_3 = \begin{bmatrix} 1 \\ -1 \\ -\theta_1 \\ \theta_1 \end{bmatrix},\ Dg_4 = \begin{bmatrix} -1 \\ 1 \\ \theta_2 \\ -\theta_2 \end{bmatrix}.$$

It is useful to gain some insights before computing.

- Note that the θ_2 type can always pretend to be the θ_1 type and take the deal (T_1, x_1), which would allow the θ_2-buyer to obtain a payoff

  $$\theta_2 x_1 - T_1 \geq \theta_1 x_1 - T_1 \geq 0,$$

  and the first inequality would be strict if $x_1 > 0$ (or equivalently $q_1 > 0$). Thus $x_1 > 0$ together with $g_4 \leq 0$ would imply $g_2 \leq 0$. That is, if we conjecture that $x_1 > 0$ then removing the second constraint would not alter the optimal solution to (P).
- Following the removal of $g_2 \leq 0$, we can further conjecture that the first constraint $g_1 \leq 0$ must be binding at an optimal solution, for otherwise we could raise T_1 and T_2 by the same tiny positive amount without violating g_1, g_3,and g_4, but this would increase f!
- The removal of $g_2 \leq 0$ and the conjecture that $g_1 = 0$ at optimum now allow us to further conjecture that g_4 must be binding at an optimal solution, for otherwise we could raise T_2 alone by a tiny positive amount without violating the other constaints, but this would increase f!
- Now, following $g_1 = 0 = g_4$ and following the removal of $g_2 \leq 0$, we can re-state $g_3 \leq 0$ as

  $$(\theta_1 - \theta_2)(x_2 - x_1) \leq 0,$$

  but this last inequality would not be binding so long as $x_2 > x_1$.

Thus if we conjecture that $x_2 > x_1 > 0$ at optimum then we would also conjecture that

$$\mu_2 = \mu_3 = 0,\ T_1 = \theta_1 x_1,\ T_2 = T_1 + \theta_2(x_2 - x_1) = \theta_2 x_2 - (\theta_2 - \theta_1)x_1.$$

Now, by the fact that f is concave and g_1, g_2, g_3, g_4 are all convex, the sufficiency of Kuhn-Tucker Theorem applies, and hence we only need to find $\mu_1, \mu_4 \geq 0$ such that

$$Df = \mu_1 Dg_1 + \mu_4 Dg_4 \Rightarrow \begin{bmatrix} 1 \\ 1 \\ -ch'(x_1) \\ -ch'(x_2) \end{bmatrix} = \mu_1 \begin{bmatrix} 1 \\ 0 \\ -\theta_1 \\ 0 \end{bmatrix} + \mu_4 \begin{bmatrix} -1 \\ 1 \\ \theta_2 \\ -\theta_2 \end{bmatrix},$$

Suppose that there are two players (i.e., $I = \{1, 2\}$) and 5 possible uncertain states of the world, and we denote by $\Omega = \{\omega_i; i = 1, 2, \cdots, 5\}$ the set of possible true states. Let H_i denote player i's information, which is formally a partition of Ω. For example, assume that H_1 is depicted as

$$\boxed{\omega_1 | \omega_2, \omega_3 | \omega_4, \omega_5}$$

and H_2 is depicted as

$$\boxed{\omega_1, \omega_2 | \omega_3 | \omega_4 | \omega_5}$$

In this example, if the true state is ω_1, then player 1's information would tell him that the true state is exactly ω_1, but player 2's information only shows that the true state is not ω_3, ω_4, or ω_5. If instead the true state is ω_2, then player 1 only knows that the true state is not ω_1, ω_4, or ω_5; and player 2 only knows that the true state is not ω_3, ω_4, or ω_5. In other words, when the true state is ω_j, the element in H_i that contains ω_j is the set of ω's that player i considers possible to be the true state.

Consider three random variables $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$, whose realizations in true state ω are denoted respectively by $x(\omega)$, $y(\omega)$, and $z(\omega)$. Suppose that

	ω_1	ω_2	ω_3	ω_4	ω_5
$x(\cdot)$	5	4	4	3	1
$y(\cdot)$	1	1	0	0	0
$z(\cdot)$	2	2	2	4	5

Assume that the above 3 tables regarding H_1, H_2, and $(\tilde{x}, \tilde{y}, \tilde{z})$ are the two players' common knowledge.[6]

(i) Suppose that the true state is ω_2. Do both players know the realization of $\tilde{x}$? If your answer is yes, do they also know that they both

and if a solution to this system of equations exists and if the solution implies that $x_2 > x_1 > 0$ and $g_2, g_3 < 0$, then we are done. Moreover, because f is strictly concave in (x_1, x_2), the solution would be unique!

[6] To answer the following questions, we obviously must assume that they have *some* common knowledge!

know the realization of $\tilde{x}$? If your answer is again yes, do they know that they both know that they both know the realization of x? Do you think that the realization of $\tilde{x}$ is the two players' common knowledge in state ω_2?

(ii) Suppose that the true state is ω_1. Do both players know the realization of $\tilde{y}$? If your answer is yes, do they also know that they both know the realization of $\tilde{y}$? If your answer is again yes, do they know that they both know that they both know the realization of y? Do you think that the realization of $\tilde{y}$ is the two players' common knowledge in state ω_1?

(iii) Suppose that the true state is ω_3. Do both players know the realization of $\tilde{z}$? If your answer is yes, do they also know that they both know the realization of $\tilde{z}$? If your answer is again yes, do they know that they both know that they both know the realization of z? Do you think that the realization of $\tilde{z}$ is the two players' common knowledge in state ω_3?

Solution. In part (i), in state ω_2 player 1 knows the realization of $\tilde{x}$ but player 2 does not.

In part (ii), in state ω_1 both players know the realization of $\tilde{y}$, and player 1 knows that player 2 knows the realization of $\tilde{y}$. Player 2, however, is not sure whether player 1 knows the realization of $\tilde{y}$.

In part (iii), in state ω_3 the realization of $\tilde{z}$ is indeed the two players' common knowledge.