Game Theory with Applications to Finance and Marketing, I

Solutions to Homework 5

1. (**Bayesian Equilibrium**) Two workers can each choose to or not to make an effort for their joint project. The project generates one unit of utility to each worker if at least one worker chooses to make the effort. Making effort incurs a disutility c_i to worker i, where c_i is worker i's private information, and worker j believes that c_i is uniformly distributed over $[0,2]$. Ex-ante it is common knowledge that c_1 and c_2 are independent random variables. Find a symmetric pure-strategy Bayesian equilibrium.[1]

Solution. Suppose that such a pure-strategy BE exists, and that given worker j's equilibrium strategy, a type c_i chooses to make the effort in equilibrium and obtain the equilibrium payoff

$$1 - c_i \geq \pi_{i,d},$$

where $\pi_{i,d}$ stands for the deviation payoff that the type c_i would obtain if he chose to make no effort; note that $\pi_{i,d}$ is independent of worker i's type c_i. We claim that a type c_i' must also choose to make the effort in equilibrium, if $c_i' < c_i$. To see this, simply note that

$$1 - c_i' > 1 - c_i \geq \pi_{i,d}.$$

On the other hand, if a type c_i'' chooses to make no effort in equilibrium; i.e., if

$$1 - c_i'' < \pi_{i,d},$$

then a type c_i''' must also choose to make no effort if $c_i''' > c_i''$. It follows that given worker j's equilibrium strategy, there must exist some c_i^* such

[1]**Hint**: There should be a cut-off level of c_i, say c_i^*, such that a type-c_i chooses to make an effort if and only if $c_i \leq c_i^*$.

that a type c_i makes the effort if and only if $c_i \leq c_i^*$. The argument applies to worker j as well.

The above argument suggests that the type c_i^* must feel indifferent about making and not making the effort, given worker j's strategy. That is, we must have

$$1 - c_i^* = \int_0^{c_j^*} 1 \cdot \frac{1}{2} dc_j + \int_{c_j^*}^2 0 \cdot \frac{1}{2} dc_j$$

$$\Rightarrow 1 - c_i^* = \frac{c_j^*}{2}.$$

It follows that

$$1 - c_1^* = \frac{c_2^*}{2} = \frac{1 - \frac{c_1^*}{2}}{2} \Rightarrow c_1^* = \frac{2}{3} = c_2^*.$$

Thus this game has a symmetric pure-strategy BE in which, for $i = 1, 2$, worker i makes an effort if and only if $c_i \leq \frac{2}{3}$.

Note that to fulfill productive efficiency we should have exactly one worker making the effort in equilibrium. Thus the equilibrium inefficiency takes place in two manners. First, with probability

$$\int_0^{\frac{2}{3}} \frac{1}{2} dc_1 \times \int_0^{\frac{2}{3}} \frac{1}{2} dc_2 = \frac{1}{9}$$

both workers make the effort; one worker's effort is totally redundant in this event. Second, with probability

$$\int_{\frac{2}{3}}^2 \frac{1}{2} dc_1 \times \int_{\frac{2}{3}}^2 \frac{1}{2} dc_2 = \frac{4}{9}$$

none of the workers make the effort, when each worker i has $c_i < 1+1 = 2$.

2. In the following two signaling games, player 1 is equally likely to be of type t_1 and type t_2, and can send signal m_1 or m_2 or m_3, and player 2 can respond by taking action a_1 or a_2 or a_3. The three tables indicate their payoffs following each of the 3 signals sent by player 1.

   - There is a separating PBE for the following game, where m_3 is not an equilibrium signal. Find this PBE. Is this PBE an intuitive equilibrium?

m_1	a_1	a_2	a_3
t_1	$(1, 0)$	$(4, 3)$	$(2, 4)$
t_2	$(10, 5)$	$(4, 4)$	$(4, 1)$

m_2	a_1	a_2	a_3
t_1	$(2, 2)$	$(6, 0)$	$(8, 1)$
t_2	$(2, 2)$	$(2, 3)$	$(6, 2)$

m_3	a_1	a_2	a_3
t_1	$(6, 1)$	$(4, -2)$	$(1, 2)$
t_2	$(6, 2)$	$(2, 3)$	$(0, -1)$

   - There is a pooling PBE for the game below, where player 1's equilibrium signal is not m_1. Find this PBE. Is this PBE an intuitive equilibrium?

m_1	a_1	a_2	a_3
t_1	$(8, 0)$	$(4, 3)$	$(2, 4)$
t_2	$(10, 5)$	$(4, 4)$	$(4, 1)$

m_2	a_1	a_2	a_3
t_1	$(2, 2)$	$(6, 0)$	$(8, 1)$
t_2	$(2, 2)$	$(2, 3)$	$(6, 2)$

m_3	a_1	a_2	a_3
t_1	$(6, 1)$	$(4, -2)$	$(2, 2)$
t_2	$(6, 2)$	$(2, 3)$	$(0, -1)$

Solution. According to the hint, in the separating PBE either the type-t_j player 1 sends m_j, $j = 1, 2$, or the type-t_j player 1 sends m_{3-j}, $j = 1, 2$.

Suppose that the former is the case. Then upon seeing m_2, player 2 belives that player 1 is of type t_2, and hence player 2 must respond by choosing a_2, yielding a payoff of 2 for the type-t_2 player 1, who can however ensure himself a payoff of at least 4 by sending signal m_1 instead. This is a contradiction.

Thus suppose that the latter is the case. Then upon seeing m_2, player 2 belives that player 1 is of type t_1, and hence player 2 must respond by choosing a_1, yielding a payoff of 2 for the type-t_1 player 1. Upon seeing m_1, player 2 belives that player 1 is of type t_2, and hence player 2 must respond by choosing a_1, yielding a payoff of 10 for the type-t_2 player 1. It is clear that this type of player 1 would never deviate unilaterally by sending m_2 or m_3. What about the type-t_1 player 1? By sending m_1, he will be regarded as type t_2 for sure, and player 2 will choose a_1 accordingly, which yields only a payoff of 1 for him. If he sends m_3, player 2 must respond by choosing a_3 in order to sustain the separating PBE, and for a_3 to be player 2's best response, player 2 must believe that the signal-sender is of type t_1 with a probability exceeding $\frac{3}{4}$. Hence we have verified that these beliefs and strategies indeed constitute a separating PBE.

Finally, to see that the separating PBE is intuitive, note that only m_3 is an off-equilibrium signal, and hence seeing m_3 is the only zero-probability event relevant in Cho-Kreps refinement. Intuition suggests that the type-t_2 player 1 should not have sent m_3, after player 2 receives m_3, because no matter what action player 2 will take after seeing m_3, the type-t_2 player 1 would get a payoff strictly less than 10, where 10 is his equilibrium payoff. But if player 2 believes for sure that it was the type-t_1 player 1 that has sent m_3, then player 2's best response would be a_3, which yields a deviation payoff of 1 for the type-t_1 player 1, so that this type would never deviate in the first place. To sum up, the separating PBE is intuitive.

3. In the following dynamic game with incomplete information, player 1 has two equally likely types, denoted by t_1 and t_2, and given his type, the informed player 1 must choose either strategic game A or strategic game B. After observing player 1's choice, the informed player 1 and the uninformed player 2 must simultaneously take actions in the chosen strategic game. In each strategic game, player 1 can choose either U or D, and player 2 can choose either L or R. The resulting payoff x for the type-t_1 player 1, y for the type-t_2 player 1, and z for player 2, are

written as a row vector (x, y, z). The following two tables summarize the players' type-and-action-contingent payoffs. For example, if player 1 chooses game A and then action U, and if player 2 chooses action L in game A, then $x = 2$, $y = 1$, and $z = 3$.

Strategic Game A

	L	R
U	$(2, 1, 3)$	$(1, 2, 5)$
D	$(1, 2, 0)$	$(0, 12, 10)$

Strategic Game B

	L	R
U	$(\frac{3}{2}, 21, 3)$	$(\frac{3}{4}, 2, 1)$
D	$(0, 0, 0)$	$(0, 10, 4)$

We shall only consider PBEs in which players use pure strategies in each and every subgame. For supporting beliefs, let us define $\mu_A \equiv \text{prob}(t_1|A)$ and $\mu_B \equiv \text{prob}(t_1|B)$, where A and B stand for "strategic game A" and "strategic game B" respectively.
(i) Find all separating and pooling PBEs of this game.[2]
(ii) For each PBE obtained in part (i), determine whether it is a Cho-Kreps intuitive equilibrium or not.[3]

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[2]**Hint**: For each PBE, you must write down explicitly player 1's and player 2's strategies, together with μ_A and μ_B. In particular, for player 1's strategy, you must state clearly

$$\begin{pmatrix} t_1 \to \text{(A,U) or (A,D) or (B,U) or (B,D)} \\ \\ t_2 \to \text{(A,U) or (A,D) or (B,U) or (B,D)} \end{pmatrix},$$

and for player 2's strategy, you must state clearly

$$\begin{pmatrix} A \to \text{L or R} \\ \\ B \to \text{L or R} \end{pmatrix}.$$

[3]**Hint**: For part (i), show that this game has two pooling but no separating equilibria; and for part (ii), show that both pooling PBEs are intuitive.

Solution. For part (i), we can show that this game has two pooling but no separating equilibria. For part (ii), we can show that both pooling PBEs are intuitive. Now we give details.

At first, there is a pooling PBE where player 1's strategy is

$$\begin{pmatrix} t_1 \to \text{(A,U)} \\ \\ t_2 \to \text{(A,D)} \end{pmatrix},$$

and player 2's strategy is

$$\begin{pmatrix} A \to \text{R} \\ \\ B \to \text{R} \end{pmatrix},$$

and where

$$\mu_A = \frac{1}{2},\ \mu_B \leq \frac{2}{3}.$$

This PBE is intuitive, because by sending the off-the-equilibrium signal B, (1) type-t_1 may get $\frac{3}{2}$ (if player 2 is willing to choose L), which is greater than t_1's equilibrium payoff, which is 1; and (2) type-t_2 may get 21 (if player 2 is willing to choose L), which is greater than t_1's equilibrium payoff, which is 12. Thus any $\mu_B \in [0, 1]$ is consistent with the intuitive criterion.

There is another pooling PBE where player 1's strategy is

$$\begin{pmatrix} t_1 \to \text{(B,U)} \\ \\ t_2 \to \text{(B,U)} \end{pmatrix},$$

and player 2's strategy is[4]

[4]Following the equilibrium signal B, player 2's pure strategy must be L: if R were to be taken, then the type-t_2 A would rather send signal A and then play D, which would yield a payoff 12, higher than the maximal payoff 10 that he could get in the supposed equilibrium.

$$\begin{pmatrix} A \to \text{R} \\ \\ B \to \text{L} \end{pmatrix},$$

and where

$$\mu_A \in [0,1],\ \mu_B = \frac{1}{2}.$$

This PBE is also intuitive, because (1) by sending the off-the-equilibrium signal A, type-t_2's maximal possible payoff is 12 (if player 2 is willing to choose R), which is still less than t_2's equilibrium payoff, which is 21; and (2) any μ_A that rules out t_2 must be such that $\mu_A = 1$, and given $\mu_A = 1$, player 2's best response is R, so that by sending the off-the-equilibrium signal A, type-t_1's payoff would become 1 (as long as player 2 adopts the intuitive belief $\mu_A = 1$ and chooses R), which is still less than t_1's equilibrium payoff, which is $\frac{3}{2}$, showing that t_1 would not want to deviate from his equilibrium strategy.

There is no separating PBE for this game. To see this, suppose first that there were a separating PBE where the type-t_2 player 1 sends signal A. In this PBE, player 2 would correctly expect the type-t_2 player 1 to play D after sending signal A, and hence player 2's best response would be R, yielding 12 for the type-t_2 player 1. However, the type-t_2 player 1 could have deviated and sent signal B, which would convince player 2 that U would then follow, and hence player 2 would choose L after seeing B, yielding a deviation payoff of 21 for the type-t_2 player 1, which is a contradiction.

Next, suppose that there were a separating PBE where the type-t_1 player 1 sends signal A. In this PBE, player 2 would correctly expect the type-t_1 player 1 to play U after sending signal A, and hence player 2's best response would be R, yielding 1 for the type-t_1 player 1. On the other hand, the type-t_2 player 1 is expected to send signal B. Thus seeing B, player 2 expects to have 2 possible pure-strategy NEs, (U,L) and (D,R). To sustain the current PBE, however, player 2 must believe in (U,L) only: if player 2 believed in (D,R), so that the type-t_2 player

1 must play D after sending signal B, then the type-t_2 player 1's equilibrium payoff would be 10, but he could have deviated and sent signal A and then played D to reach the outcome (A,(D,R)), which would yield $12 > 10$ for the type-t_2 player 1. Thus we conclude that in this supposed separating PBE, player 2 would expect player 1 to play U after seeing B, and hence player 2's best reponse upon seeing B is L. But then the type-t_1 player 1 could have sent B and then played U to reach the outcome (B,(U,L)), which would yield $\frac{3}{2} > 1$ for the type-t_1 player 1, which is also a contradiction. Hence we conclude that this game has no separating PBEs.

Recall that in the game of beer and qiche discussed in Lecture 4, the uninformed player's payoff depends on his own action and the informed player's type, but not on the informed player's action. In the signalling games discussed in Example 3 of Lecture 4, the uninformed player's payoff depends on everything—his own action, the informed's action and the informed's type all affect the uninformed's payoff. Those games are said to have *common values*, because the two players' payoffs both depend on the informed's type. Here, we have a game with *private values*, where the informed's type per se does not affect the uninformed's payoff. The uninformed player still cares about the informed's type, because different types of the informed player may take different actions, and those action choices affect the uninformed's payoff. This distinction is relevant in certain signaling games where the informed's signals are "contracts" that the informed designs and offers to the uninformed player; see for example Maskin and Tirole's two articles in *Econometrica*, The Principal-Agent Relationship with an Informed Principal: The Case of Private Values (1990) and The Principal-Agent Relationship with an Informed Principal, II: Common Values (1992).

4. Consider the following stock trading model with one traded common stock and three classes of traders: one insider (or informed speculator), one noise trader, and several Bertrand-competitive market makers. Everyone is risk-neutral without time preferences. Stock trading takes place at date 0, and the true value of the stock, denoted v, will become public information at date 1. The extensive game proceeds as follows.

- At the beginning of date 0, the insider alone learns about the realization of v, when everyone else only knows that v is equally likely to be $-2, -1, 1$ or 2.
- Then simultaneously, the insider and the noise trader must each submit one market order. The insider's market order is denoted by X, and the noise trader's market order is denoted by u, and we assume that u is equally likely to be 1 or -1; that is, the noise trader is equally likely to buy one share or sell one share. By submitting a market order a trader commits to accepting order execution at the market-clearing price subsequently announced by the stock-trading platform.
- At the same time when the insider and the noise trader submit their market orders, the market makers must each submit one *pricing schedule*, denoted by $P(\cdot)$. By submitting a schedule $P_i(\cdot)$, a market maker i commits to absorbing any market order $z \in \Re$ at the share price $P_i(z)$ that he specifies via $P_i(\cdot)$.
- Then, the stock-trading platform receives X, u and the market makers' pricing schedules. The platform insists on matching X and u first, and in case $z = X + u \neq 0$, then the platform will pick one market maker i whose $P_i(z)$ appears to be the lowest when $z > 0$ or whose $P_i(z)$ appears to be the highest in case $z < 0$. In case $z = 0$, then the platform will just pick $P(0) = E[v]$.
- Then, the date-0 stock trading session ends, and the game moves on to date 1. Then the realization of v becomes publicly known, and each stock-trading participant gets his realized gain or loss from the date-0 stock-trading.

The above is a signaling game, where v is the informed insider's type, and X is the signal he sends. This is referred to as a signaling game *with noise*, because market makers (i.e., the uninformed players) do not observe X directly; rather, what they learn from the stock-trading platform is $z = X + u$ only (not u and X separately), where we recall that u is a zero-mean random variable.

We shall look for pure strategy perfect Bayesian equilibria in which the market makers submit the same $P(\cdot)$. Let us call them symmetric

PBEs. A symmetric PBE is formally a pair $\{P(z), X(v)\}$ such that (i) given $P(\cdot)$, $X(v) \in \arg\max_y E[y(v - P(y + u))|v]$; and (ii) given any $z = X + u$, either $P(z)$ would ensure that no trade would occur between the selected market maker and the traders submitting market orders, or in the opposite case, the selected market maker must break even by absorbing $z = X(v) + u$; that is, $P(z) = E[v|X(v) + u = z]$.

Show that for each $a \in (\frac{2}{3}, 1)$, $\{P_a(\cdot), X_a(\cdot)\}$ is one symmetric PBE, where $X_a(\cdot)$ is such that

$$X_a(2) = -X_a(-2) = 1 + a, \quad X_a(1) = -X_a(-1) = 1 - a,$$

and $P_a(\cdot)$ is such that

$$\forall z \in \{-2 - a, -2 + a, -a, a, 2 - a, 2 + a\}, \ P_a(z) = -P_a(-z),$$

$$P_a(a) = \frac{1}{2}, \ P_a(2 + a) = 2, \ P_a(2 - a) = 1,$$

$$\forall z > 0, \ z \neq 2 + a, 2 - a, a, \ P_a(z) = 2,$$

and

$$\forall z < 0, \ z \neq -2 + a, -2 - a, -a, \ P_a(z) = -2.$$

Solution. Given $X_a(\cdot)$, define the order imbalance observed by the market makers by $Z(u, v)$, with

$$Z(-1, 2) = Z(1, -1) = a, \quad Z(1, -2) = Z(-1, 1) = -a.$$

Thus, for example, when seeing an order imbalance a, the market makers think that $(u, v) = (-1, 2)$ and $(u, v) = (1, -1)$ are equally likely, and hence they set $P_a(a) = \frac{2+(-1)}{2} = \frac{1}{2}$. Similarly, they set $P_a(-a) = \frac{-2+1}{2} = -\frac{1}{2}$.

On the other hand, the order imbalance $2 + a$, if it appears, is fully revealing: it must be that $(u, v) = (1, 2)$, so that $P_a(2 + a) = 2$. The order imbalance $2 - a$, likewise, can appear only when $(u, v) = (1, 1)$, and hence $P_a(2 - a) = 1$. Finally it is easy to check that

$$\forall z \in \{-2 - a, -2 + a, -a, a, 2 - a, 2 + a\}, \ P_a(z) = -P_a(-z).$$

Now consider the off-the-equilibrium order imbalances. Let us specify the following supporting posterior beliefs for the market makers:

$$\forall z > 0,\ z \neq 2+a, 2-a, a,\ \text{prob.}(v = 2|z) = 1,$$

and

$$\forall z < 0,\ z \neq -2+a, -2-a, -a,\ \text{prob.}(v = -2|z) = 1.$$

Apparently, with these posterior beliefs, $P_a(z)$ is as asserted when z is not contained in $\{-2-a, -2+a, -a, a, 2-a, 2+a\}$.

It remains to check that $X_a(\cdot)$ is the insider's best response given $P_a(\cdot)$. Given a, define the type-v insider's payoff in equilibrium a from submitting market order x as

$$B_a(v, x) \equiv x(v - E[P_a(x+u)]),\ \forall x \in \Re,\ v = -2, -1, 1, 2.$$

Given the above supporting beliefs, if x is such that $x + u$ is not contained in the set $\{-2-a, -2+a, -a, a, 2-a, 2+a\}$, then x can never be optimal. Finally, it is easy to show that when $a \in (\frac{2}{3}, 1)$,

$$B_a(2, 1+a) \geq B_a(2, 1-a),\quad B_a(1, 1-a) \geq B_a(1, 1+a).$$

Thus $X_a(\cdot)$ is indeed optimal. This finishes the proof.

5. Firm A has a single owner-manager Mr. A, who needs to raise \$100 for a positive-NPV investment project at date 0. There are two possible date-0 states, called G and B, and the date-0 state is Mr. A's private information. In state G, the assets in place of firm A are worth \$150 and the new project's NPV equals \$20. In state B, the assets in place are worth only \$$x$ and the new project's NPV is accordingly \$$y$. The public investors (also referred to as the outsiders) believe that the state may be G with prob. a. Mr. A and public investors are all risk-neutral without time preferences.

   The game proceeds as follows. Mr. A first decides to or not to issue new equity to raise \$100 (two feasible signals!). Then, upon seeing Mr. A's decision, the public investors form posterior beliefs about the date-0 state, and they engage in Bertrand competition to determine the fraction α of equity that Mr. A must sell in order to raise \$100.

(i) Suppose that $x = 50$ and $y = 10$. Find all the pure-strategy PBE's of this signaling game.

(ii) Suppose that $x = 60$ and $y = -25$. Assume that the firm, after raising \$100 from new investors, can either undertake the new investment project or put \$100 in a riskless money market account. The risk-free interest rate is zero. In this case, a pooling equilibrium where both types of the firm choose to issue new equity exists if and only if the prior probability a for the good state is such that $a \geq a^*$. Compute a^*.

(iii) Suppose that $x = 60$ and $y = -25$. Unlike in part (ii), assume instead that the firm, after raising \$100 from new investors, must spend it on the new investment project, regardless of the state. In this case, a pooling equilibrium where both types of the firm choose to issue new equity exists if and only if the prior probability a for the good state is such that $a \geq a^{**}$. Compute a^{**}.

(iv) Suppose that $x = 60$ and $y = -25$. Suppose that $a = a^{**}$. Then in the pooling equilibrium obtained in part (iii), Mr. A ends up possessing a fraction $1 - \alpha$ of firm A's equity. Compute α.

Solution. Consider part (i). We first look for separating equilibria. We shall refer to Mr. A of type j as simply "type j."

In a separating PBE where only type G issues new equity, in exchange of the \$100 raised, the outsiders must ask for a share $\alpha = \frac{100}{150+20+100}$ of the ownership. But then type B will deviate: by deviating and issuing, type B would get

$$(1 - \frac{100}{150 + 20 + 100})(50 + 10 + 100) = 100.74,$$

which is greater than 50, the payoff of type B if abanoning the new project. Hence, there is no such separating equilibrium.

Now, consider the separating PBE where only type B issues new equity. Then, the public investors would ask for a share of ownership equal to $\alpha = \frac{100}{10+50+100}$. Type B would indeed want to issue new equity: by issuing, he would get

$$(1 - \frac{100}{10 + 50 + 100})(100 + 50 + 10) = 60,$$

greater than 50. On the other hand, type G insider would not issue new equity if and only if

$$(1 - \frac{100}{10 + 50 + 100})(100 + 150 + 20) = 101.25 < 150,$$

which indeed is true. Thus this separating equilibrium does exist. The supporting beliefs for this PBE are all equilibrium beliefs, and can be pinned down by the Bayes Law.

Next, we look for pooling equilibria. Suppose that in equilibrium neither type issues new equity. But then type B wants to deviate: by issuing, type B cannot do worse than being identified, but even in that case, issuing is preferred to not issuing. Therefore there is no such pooling equilibrium.

Finally, consider the PBE where both types issue new equity. The outsiders would ask for

$$\alpha[a(150 + 20 + 100) + (1 - a)(50 + 10 + 100)] = 100,$$

and hence

$$\alpha = \frac{100}{160 + 110a}.$$

Type G must be willing to issue new equity in equilibrium:

$$(1 - \frac{100}{160 + 110a})(100 + 150 + 20) > 150;$$

and so must type B insider:

$$(1 - \frac{100}{160 + 110a})(100 + 50 + 10) > 50.$$

Thus the pooling equilibrium exists if and only if $a > \frac{13}{22}$.

Note that in this pooling equilibrium the outsiders' beliefs following the off-equilibrium signal "not issuing" is irrelevant. Note also that there does not exist an off-equilibrium signal in a separating equilibrium. Thus in part (i), both pure-strategy PBE's are robust against Cho and Kreps' intuitive criterion.

Consider part (ii). In the assumed pooling equilibrium, we must have

$$\alpha[270a + (160 + 0)(1 - a)] = 100,$$

and hence

$$\alpha = \frac{100}{[270a + 160(1 - a)]}.$$

Implicitly we are assuming here how the firm uses the 100-dollar cash is not verifiable, and since the new project has a negative NPV in the bad state, it is in Mr. A's interest to put the cash in the riskless money market account. For the pooling equilibrium to be viable, we need

$$270(1 - \alpha) \geq 150, \quad (160 + 0)(1 - \alpha) \geq 60,$$

so that we must require that

$$a \geq a^* = \frac{13}{22},$$

as obtained in part (i).

Consider part (iii). In the assumed pooling equilibrium, we must have

$$\alpha[270a + (160 - 25)(1 - a)] = 100,$$

and hence

$$\alpha = \frac{100}{[270a + (160 - 25)(1 - a)]}.$$

Implicitly we are assuming here whether the firm undertakes the new project is verifiable, and even though the new project has a negative NPV in the bad state, Mr. A has to spend the 100-dollar cash on the new project if he wants to pool with his counterpart in the good state. For the pooling equilibrium to be viable, we need

$$270(1 - \alpha) \geq 150, \quad (160 - 25)(1 - \alpha) \geq 60,$$

so that we must require that

$$a \geq a^{**} = \frac{2}{3}.$$

Finally, for part (iv), we obtain

$$\alpha = \frac{100}{[270a^{**} + (160 - 25)(1 - a^{**})]} = \frac{4}{9}.$$

Remark. Unlike in part (i), where a pooling equilibrium is always productively efficient, in part (iii) we have a pooling equilibrium with *over-investments*. Thus in this pooling equilibrium, neither informational efficiency nor productive efficiency is attained. In equilibrium the bad-type firm is willing to undertake a project with negative NPV because it can share with the new investors the proceeds of $100 that it obtains from issuing the new equity. In this case, the new investors lose more than the negative NPV pertaining to the new project.