Game Theory with Applications to Finance and Marketing, II

Some Examples

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1. There are I agents, $i = 1, 2, \cdots, I$. Let I denote the set $\{1, 2, \cdots, I\}$ as well. Let A denote the set of feasible social choices. The profile of the I agents' preferences on A is state-dependent, where in state $\theta \in \Theta$, agent i has a preference ordering $R_i(\theta)$ on A (which defines the strict preference $P_i(\theta)$ and indifference $I_i(\theta)$). Here we assume that agents have *complete information*; i.e., the realization θ is the agents' common knowledge.

   A central planner endowed with a social choice rule (SCR) $f : \Theta \to 2^A$ wishes to design a game form or mechanism, denoted by (g, S), to *(fully) implement* f, where $S = S^1 \times S^2 \times \cdots \times S^I$ with S^i denoting the set of feasible (pure) strategies for agent i, and $g : S \to A$ maps each strategy profile chosen by the I agents into a social choice in A. By full implementation of f by (g, S), we mean that for each $\theta \in \Theta$, $f(\theta)$ *is exactly* the set of equilibrium outcomes of the game form (g, S) in state θ, and we denote the latter by $g(E_g(\theta))$.

   Note that a game form differs from a normal-form game in that the agents' preferences on A are left unspecified in the former. Given any $\theta \in \Theta$, however, (g, S, θ) becomes a well-defined normal-form game.

2. **Example 1.** Two women, Amy and Beth, carry one baby to the king, and each of them claims to be the mother of the baby. There are two possible states: the mother is either Amy (state α) or Beth (β). Thus let $\Theta = \{\alpha, \beta\}$. For the king, there are 4 feasible actions: to give the baby to Amy (a); to give it to Beth (b); to cut the baby in half and let each woman take one half (c); or to let both women and the baby die (d). The king knows that Amy prefers a to b to c to d in state α and

she prefers a to c to b to d in state β, and that Beth prefers b to a to c to d in state β and she prefers b to c to a to d in state α. Can you find a game form to fully implement in Nash equilibrium the social choice rule f satisfying $f(\alpha) = a$ and $f(\beta) = b$?

3. Define the lower contour set at a for agent i in state θ by

$$L^i(a, \theta) \equiv \{b \in A : aR^i(\theta)b\}.$$

We say that f is *monotonic* if and only if for all $\theta, \phi \in \Theta$ and $a \in A$ such that $a \in f(\theta) \setminus f(\phi)$, there must exist some i such that the following statement is false:

$$L^i(a, \theta) \subset L^i(a, \phi).$$

In other words, if f is *monotonic* and the latter is true for all i, then $a \in f(\theta)$ implies that $a \in f(\phi)$.

4. **Theorem N1.** (Maskin 1977; Maskin 1999) If there exists some game form (g, S) that implements f in Nash equilibrium, then f must be monotonic.[1]

Proof. Suppose that $a \in f(\theta) \setminus f(\phi)$ and for all i, $L^i(a, \theta) \subset L^i(a, \phi)$, and yet, game form (g, S) implements f. We shall demonstrate a contradiction. By assumption, there exists some Nash equilibrium s^* for the game (g, S, θ) such that $g(s^*) = a \in f(\theta)$. That is, for agent i, given his rival agents would play s^*_{-i} in state θ,

$$a = g(s^*_i, s^*_{-i})R^i(\theta)g(s_i, s^*_{-i}), \ \forall s_i \in S^i,$$

$$\Rightarrow g(s_i, s^*_{-i}) \in L^i(a, \theta) \subset L^i(a, \phi), \ \forall s_i \in S^i,$$

but then s^*_i continues to be agent i's best response against s^*_{-i} in state ϕ! As this is true for all agents i, we conclude that s^* would also arise as a pure-strategy Nash equilibrium in state ϕ. But then $a \in g(E_g(\phi)) \setminus f(\phi)$, showing that (g, S) does not fully implement f in Nash equilibrium.

[1] Maskin, E., 1977, Nash Equilibrium and Welfare Optimality, MIT working paper. Maskin, E., 1999, Nash Equilibrium and Welfare Optimality, *Review of Economic Studies*, 66, 23-38.

5. An SCR f satisfies (weak) no veto power if for all $i \in \{1, 2, \cdots, I\}$, for all $\theta \in \Theta$, and for all $a \in A$,

$$L^j(a, \theta) = A, \forall j \neq i \Rightarrow a \in f(\theta).$$

In words, if a is top ranked by all agents $j \neq i$ in state θ, then $a \in f(\theta)$ whether agent i likes a or not. (Agent i has no veto power!)

6. **Theorem N2.** (Maskin 1977; Repullo 1987[2]) Suppose that $I \geq 3$, and that f is monotonic and satisfies no veto power. Then f is Nash implementable.

Proof. The proof is by construction of a canonical game form (g, S) which fully implements f. Define for all i, $S^i = \Theta \times A \times \mathbf{Z}_+$, where $\mathbf{Z}_+$ denotes the set of positive integers, and define $g : S \to A$ as follows:

(a) If s is such that there exists $i \in \{1, 2, \cdots, I\}$ such that $s_i = (\eta, a_i, k_i)$ and for all $j \neq i$, $s_j = (\theta, a, k)$ with $a \in f(\theta)$, then

$$g(s) = \begin{cases} a_i, & \text{if } a_i \in L^i(a, \theta); \\ \\ a, & \text{otherwise.} \end{cases}$$

(b) If s is such that (a) does not apply, then $g(s) = a_i$ where i is an agent announcing the highest k_i, with ties being broken by selecting among the agents announcing the highest k_i the person with the smallest i.

We shall show first that $f(\theta) \subset g(E_g(\theta))$ for all $\theta \in \Theta$, and then that $g(E_g(\theta)) \subset f(\theta)$ for all $\theta \in \Theta$.

- $f(\theta) \subset g(E_g(\theta))$ for all $\theta \in \Theta$.

  Given any $a \in f(\theta)$, define for all i, $s_i = (\theta, a, 1)$. Then s is such that (a) holds, and if agent i alone would like to deviate and to implement another a_i, he must choose some $a_i \in L^i(a, \theta)$,

[2]Repullo, R., 1987, A Simple Proof of Maskin's Theorem on Nash Implementation, *Social Choice and Welfare*, 4, 39-41.

and hence he has no incentive to make unilateral deviations. Thus $a \in g(E_g(\theta))$, and this being true for all $\theta \in \Theta$ and for all $a \in f(\theta)$, we conclude that $f(\theta) \subset g(E_g(\theta))$ for all $\theta \in \Theta$.

- $g(E_g(\theta)) \subset f(\theta)$ for all $\theta \in \Theta$

  Let $s \in E_g(\theta)$, and we shall show that $g(s) \in f(\theta)$. Suppose θ is the true state. We take cases.

  - Suppose that s is such that $s_i = (\eta, a, k)$ $\forall i \in \{1, 2, \cdots, I\}$, with $a \in f(\eta)$, so that $g(s) = a$.

    For all $i \in \{1, 2, \cdots, I\}$, if agent i wishes to deviate unilaterally from s, then according to (a) above, agent i must announce some $s_i' = (\phi, a_i, k_i)$ with $a_i \in L^i(a, \eta)$. Since s is a Nash equilibrium in the true state θ, agent i weakly prefers the equilibrium outcome $a = g(s)$ to a_i in the true state θ, and this implies that

    $$a_i \in L^i(a, \eta) \Rightarrow a_i \in L^i(a, \theta),$$

    and this being true for all $i \in \{1, 2, \cdots, I\}$, we conclude that $a \in f(\theta)$ since f is monotonic.

  - Suppose that s is such that $s_i = (\eta, a, k)$ $\forall i \in \{1, 2, \cdots, I\}$, with $a \notin f(\eta)$.

    In this case, by (b), any agent i can deviate and announce $s_i' = (\phi, a_i, k')$, where $k' > k$, so that the outcome a_i rather than $g(s)$ would be implemented. Since s is a Nash equilibrium in the true state θ, it must be that

    $$g(s)R^i(\theta)a_i, \ \forall a_i \in A,$$

    or equivalently,

    $$L^i(g(s), \theta) = A,$$

    and with this being true for each single agent i, we conclude that $g(s) \in f(\theta)$ by the fact that f satisfies no veto power.

– Suppose that s is such that there exist $i \neq j$, $s_i \neq s_j$.

In this case, thanks to the fact that $I \geq 3$, some agent $h \notin \{i, j\}$ can implement any $a_h \in A$ by announcing an integer k_h exceeding k_n for all $n \neq h$. Since s is a Nash equilibrium in the true state θ, it must be that

$$L^h(g(s), \theta) = A, \ \forall h \notin \{i, j\}.$$

Moreover, it is impossible that $s_h = s_i$ and $s_h = s_j$, simply because $s_i \neq s_j$. Suppose that $s_h \neq s_i$. Then we can repeat the above argument and conclude that

$$L^j(g(s), \theta) = A.$$

It follows that $g(s)$ is top ranked in state θ by all agents $n \neq i$, so that $g(s) \in f(\theta)$ by the fact that f satisfies no veto power.

7. In this example, we suppose that $I = 3$ and $\Theta = \{t, t'\}$. The SCR f concerns how to divide 1 dollar for the three agents in a given state θ. Suppose that f is such that $f(t) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and $f(t') = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$. The first two agents prefer more money to less in both states. Agent 3's utility function has a single peak at $\frac{1}{2}$ in state t, and a single peak at $\frac{1}{3}$ in state t'.

   Consider the following mechanism (g, S), which asks agents to simultaneously report $\theta \in \Theta$, with agent i's report written as s_i. The following two matrices summarize (g, S) with agent 1 choosing a row, agent 2 choosing a column, and agent 3 choosing between the two matrices; the outcome, or the way that the dollar is to be divided among the three agents is shown as an entry in a matrix.

	$s_2 = t$	$s_2 = t'$
$s_1 = t$	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	$\frac{1}{3}, 0, \frac{1}{2}$
$s_1 = t'$	$0, \frac{1}{3}, \frac{1}{2}$	$0, 0, \frac{1}{2}$

$s_3 = t$

	$s_2 = t$	$s_2 = t'$	
$s_1 = t$	$0, 0, \frac{1}{3}$	$0, \frac{1}{3}, \frac{1}{3}$	$s_3 = t'$
$s_1 = t'$	$\frac{1}{3}, 0, \frac{1}{3}$	$\frac{1}{4}, \frac{1}{4}, \frac{1}{2}$	

(i) Show that f is not monotonic, and is hence not Nash implementable.

(ii) Show that in each state $\theta \in \Theta$, the equilibrium outcome of (g, S) that results from iterated elimination of weakly dominated strategies is exactly $f(\theta)$.

Solution. For part (i) note that the division of the dollar, $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, moves up in each agent's preference ranking when the state changes from t to t', and yet $f(t) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) \neq f(t')$, showing that f is not monotonic.

For part (ii), first note that each agent has four feasible pure strategies: to always tell the truth, to always tell a lie, to always report $\theta = t$, and to always report $\theta = t'$. Now, observe that from agent 3's perspective, his payoff under (g, S) does not depend on s_3 whenever $s_1 = s_2$, and whenever $s_2 \neq s_3$, truth-telling generates the maximum payoff for agent 3. We conclude that lying about θ always or on one occasion is a weakly dominated strategy for agent 3. Upon deleting all the weakly dominated strategies for agent 3, we see that it is agents 2 and 3's common best choice to match agent 3's report about θ; un-matching would result in zero payoffs for the two agents. Thus we can perform a new round of deletion of weakly dominated strategies for agents 2 and 3, which leads to every agent telling the truth. Thus iterated deletion of weakly dominated strategies results in a unique outcome where all three agents are truthfully reporting θ, and the desired $f(\theta)$ is implemented.

Remark. This exercise is taken from Sjöström (1994),[3] where it

[3]Sjöström, T., 1994, Implementation in Undominated Nash Equilibria without Integer Games, *Games and Economic Behavior*, 6, 502-511.

is shown that when there are three or more agents, essentially any social choice function (the author does not consider general social choice correspondences) in well behaved convex economic environments can be implemented in undominated Nash equilibria. An agent i is asked to report his own and his two adjacent neighbors' types, $(R^i_{i-1}, R^i_i, R^i_{i+1})$, where R^i_j is agent i's report about agent j's type. Then agent i is punished with zero consumption (the worst possible outcome for i in the economic environments) if $R^i_j \neq R^j_j$ for some $j \in \{i-1, i+1\}$, and in the opposite case, agent i is allocated with the consumption bundle $M^i(B(s^{-i}), R^i_i)$, which is agent i's favorite consumption bundle in the set $B(s^{-i})$ (which depends only on other agents' joint reports s^{-i}) when agent i's preference type is R^i_i. Since $B(s^{-i})$ is independent of agent i's report R^i_i about his own type, and since telling the truth about his own type would allow agent i to choose his favorite consumption bundle in the set $B(s^{-i})$, agent i has no reason to lie about his own type. This being true for agents $i-1$ and $i+1$ as well, agent i would honestly report the types of his two adjacent neighbors in order to avoid the worst possible outcome. This mechanism is feasible, of course, only when the family of choice sets $\{B(s^{-i})\}$ is rich enough to make any untruthful strategies weakly dominated for agent i, a condition that the author refers to as the *separation property.*

8. (**Example 2.**) This exercise concerns the optimal licensing of a new technology by a monopolist. Consider firms 1 and 2 that will engage in Cournot competition at date 1 with unit costs $\tilde{c}_1$ and $\tilde{c}_2$, which are independently and uniformly distributed over $[0, 1]$. The date-1 inverse demand function is, in the relevant range,

$$p = 3 - q_1 - q_2,$$

where q_i is firm i's date-1 sales volume. The realizations of $\tilde{c}_1$ and $\tilde{c}_2$ become public information right before the date-1 Cournot competition gets started, but firm i alone has privately learned about the realization of $\tilde{c}_i$ at date 0.

(I) At date 0, firm 0 wishes to sell a new production technology to exactly one of the above two firms by holding either a first-price or a second-price auction. With the new technology, the winning firm alone will have its unit cost drop to zero when the two firms engage in the date-1 Cournot competition.

(I-i) Consider the date-1 subgame where it is the two firms' common knowledge that their unit costs are (c_1, c_2), and where the two firms are about to engage in Cournot competition. Let $\pi(c_i, c_j)$ denote firm i's date-1 profit given that its unit cost is c_i and its rival's unit cost is c_j. Then $\pi(c_i, c_j) =$ ___No. 11___. Define

$$v(c_i, c_j) \equiv \pi(0, c_j) - \pi(c_i, 0).$$

Then $v(x, y) =$ ___No. 12___.

(I-ii) Suppose that firm 0 decides to hold a first-price auction. Then in a symmetric equilibrium for the auction game firm i given its original unit cost $c_i = x$ would bid $\beta^I(x) =$ ___No. 13___. Firm 0's expected revenue is equal to ___No. 14___.

(I-iii) Now, suppose instead that firm 0 decides to hold a second-price auction. Then in a symmetric equilibrium for the auction game firm i given its original unit cost $c_i = x$ would bid $\beta^{II}(x) =$ ___No. 15___. Firm 0's expected revenue is equal to ___No. 16___.

(II) Now, suppose that at date 0 firm 0 decides to announce a price $P \geq 0$, and allows the two firms to simultaneously decide whether to purchase the new technology. (Thus the sales volume can be either zero, or one, or two.)

(II-i) Suppose that given P, there exists a symmetric equilibrium where both firms choose to purchase the new technology regardless of $\tilde{c}_1$ and $\tilde{c}_2$. Then $P =$ ___No. 17___.

(II-ii) Suppose instead that given P, there exists a symmetric equilibrium where both firms choose to not purchase the new technology regardless of $\tilde{c}_1$ and $\tilde{c}_2$. Then it is necessary and sufficient that $P \geq \overline{P}$, where $\overline{P} =$ ___No. 18___.

(II-iii) Now, suppose that $P \in (0, \overline{P})$. In this case, there exists a symmetric equilibrium associated with some $c^*(P) \in (0, 1)$, such that for $i \in \{1, 2\}$, firm i would purchase the technology if and only if $c_i \geq c^*(P)$. In equilibrium, expecting its rival to follow the equilibrium strategy specified above, firm i given c_i would get the payoff

$$\int_0^{c^*(P)} \pi(c_i, c_j) dc_j + \int_{c^*(P)}^1 \pi(c_i, 0) dc_j$$

if firm i decides to not purchase the technology; and firm i would get the payoff

$$-P + \int_0^{c^*(P)} \pi(0, c_j) dc_j + \int_{c^*(P)}^1 \pi(0, 0) dc_j$$

if firm i decides to purchase the technology. It follows that P and $c^*(P)$ must satisfy

$$P = H(c^*(P)),$$

where

$$H(x) = \int_0^x [\pi(0, c_j) - \pi(x, c_j)] dc_j + \int_x^1 [\pi(0, 0) - \pi(x, 0)] dc_j.$$

Equivalently, firm 0 would choose $c \in (0, 1)$ to maximize

$$f(c) \equiv 2(1 - c)H(c),$$

and $f(c)$ attains its interior maximum at some $\hat{c}$, where for some integer n, $\frac{n}{100} < \hat{c} < \frac{n+1}{100}$, with $n =$ ___No. 19___. Is $\max_{c\in[0,1]} f(c)$ greater or less than firm 0's expected revenue from holding the second-price auction? (Answer 'greater' or 'less!') ___No. 20___.

Solution. Consider part (I-i). The date-1 Cournot equilibrium is such that

$$q_1 = \frac{3 + c_2 - 2c_1}{3}, \ q_2 = \frac{3 + c_1 - 2c_2}{3}, \ P = \frac{3 + c_1 + c_2}{3},$$

implying that

$$\pi(c_i, c_j) = (\frac{3 + c_j - 2c_i}{3})^2.$$

By definition, we have

$$v(x, y) = \pi(0, y) - \pi(x, 0) = (\frac{3 + y}{3})^2 - (\frac{3 - 2x}{3})^2 = \frac{1}{9}(6 + y - 2x)(y + 2x).$$

It is easy to verify that

$$v(0, 0) = 0,$$

and for all $(x, y) \in [0,1] \times [0,1]$, we have

$$\frac{\partial}{\partial x}v(x,y) = \frac{4}{9}(3-2x) > 0,\ \frac{\partial}{\partial y}v(x,y) = \frac{2(3+y)}{9} > 0.$$

Moreover, we have

$$v(x,x) = 2x - \frac{x^2}{3}, \Rightarrow \frac{dv(x,x)}{dx} > 0,\ \forall x \in (0,1).$$

Consider part (I-ii). Expecting that firm j given c_j would bid $\beta(c_j)$, where $\beta(\cdot)$ is strictly increasing and differentiable over $[0,1]$, firm i given c_i would get the following payoff by bidding $\beta(z)$:

$$\begin{aligned} &\int_0^z [\pi(0,c_j) - \beta(z)]dc_j + \pi(c_i,0)[1-z] \\ = &\int_0^z \{[\pi(0,c_j) - \pi(c_i,0)] - \beta(z)\}dc_j + \pi(c_i,0) \\ = &\int_0^z [v(c_i,c_j) - \beta(z)]dc_j + \pi(c_i,0). \end{aligned}$$

Thus other than the constant term $\pi(c_i, 0)$, bidder i's (firm i's) payoff function is essentially the same as described in section 6.4 of Krishna (2002). Note that winning the auction generates two benefits for a firm: *it reduces the firm's unit cost to zero, and it prevents the other firm from having the chance to reduce the unit cost to zero.*

Following Proposition 6.3 in Krishna (2002),[4] we have

$$\beta^I(x) = \int_0^x v(y,y)dL(y|x),$$

where, for all $y \in [0, x]$, $L(y|x) = \frac{y}{x}$. Hence we have

$$\beta^I(x) = \frac{1}{x}\int_0^x v(y,y)dy = x - \frac{x^2}{9}.$$

It is easy to see that

$$\frac{d\beta^I(x)}{dx} > 0,\ \forall x \in (0,1).$$

[4]Krishna, V., 2002, Auction Theory, Amsterdam: Elsevier.

Thus firm 0's expected revenue is equal to, by letting $\tilde{x} = \tilde{c}_1$ and $\tilde{y} = \tilde{c}_2$,[5]

$$\begin{aligned}
&E[\max(\beta^I(\tilde{x}), \beta^I(\tilde{y}))] \\
&= \text{prob.}(\tilde{x} \geq \tilde{y})E[\beta^I(\tilde{x})|\tilde{x} \geq \tilde{y}] + \text{prob.}(\tilde{x} \leq \tilde{y})E[\beta^I(\tilde{y})|\tilde{x} \leq \tilde{y}] \\
&= \int_{x=0}^{x=1} \int_{y=0}^{y=x} \beta^I(x)dydx + \int_{y=0}^{y=1} \int_{x=0}^{x=y} \beta^I(y)dxdy \\
&= 2 \cdot \int_{x=0}^{x=1} \int_{y=0}^{y=x} \beta^I(x)dydx = 2 \cdot \int_{x=0}^{x=1} x\beta^I(x)dx \\
&= 2 \cdot [\frac{x^3}{3} - \frac{x^4}{36}]_0^1 = \frac{11}{18}.
\end{aligned}$$

Consider part (I-iii). By Proposition 6.1 in Krishna (2002), we have

$$\beta^{II}(x) = v(x, x) = 2x - \frac{x^2}{3}.$$

Thus firm 0's expected revenue is equal to, by letting $\tilde{x} = \tilde{c}_1$ and $\tilde{y} = \tilde{c}_2$,[6]

$$\begin{aligned}
E[\min(\beta^{II}(\tilde{x}), \beta^{II}(\tilde{y}))] &= 2 \int_{y=0}^{y=1} \int_{x=0}^{x=y} \beta^{II}(x)dxdy \\
&= 2 \int_{y=0}^{y=1} (y^2 - \frac{y^3}{9})dy = \frac{11}{18}.
\end{aligned}$$

Consider part (II-i). For firm i with $c_i = 0$ to be willing to pay P, given that firm j always pays P, we have

$$\pi(0, 0) - P \geq \pi(0, 0) \Rightarrow P = 0.$$

Consider part (II-ii). For firm i with $c_i = 1$ to be unwilling to pay P, given that firm j always refuses to pay P, we must have

$$-P + \int_0^1 \pi(0, c_j)dc_j \leq \int_0^1 \pi(1, c_j)dc_j$$

[5] Alternatively, we can define $\theta \equiv \max(\tilde{x}, \tilde{y})$ and verify that the distribution function of θ is $F(\theta) = \theta^2$ for $\theta \in [0, 1]$. Firm 0's expected revenue is then equal to $E[\beta^I(\theta)] = \int_0^1 [\theta - \frac{\theta^2}{9}]d\theta^2$, which equals $\frac{11}{18}$.

[6] Alternatively, we can define $\lambda \equiv \min(\tilde{x}, \tilde{y})$ and verify that the distribution function of λ is $G(\lambda) = 1 - (1 - \lambda)^2$ for $\lambda \in [0, 1]$. Firm 0's expected revenue is then equal to $E[\beta^{II}(\lambda)] = \int_0^1 [2\lambda - \frac{\lambda^2}{3}]dG(\lambda) = \int_0^1 [2\lambda - \frac{\lambda^2}{3}]2(1 - \lambda)d\lambda$, which equals $\frac{11}{18}$.

$$\Leftrightarrow P \geq \overline{P} \equiv \frac{1}{9}\int_0^1 [(3+y)^2 - (1+y)^2]dy = \frac{10}{9}.$$

Consider part (II-iii). Since firm 0 would get zero expected revenue by setting $P = 0$ or $P \geq \overline{P}$, the best firm 0 can do given that there will be a symmetric equilibrium following its announcement of P is to induce each firm to purchase the new technology with a positive probability that is less than one. Recall that

$$\pi(x,y) = \frac{(3+y-2x)^2}{9},$$

so that

$$H(x) = \int_0^x [\frac{(3+y)^2 - (3+y-2x)^2}{9}]dy + \int_x^1 [\frac{(3+0)^2 - (3+0-2x)^2}{9}]dy$$

$$= \frac{1}{9}\{\int_0^x [4x(3+y) - 4x^2]dy + \int_x^1 [12x - 4x^2]dy\}$$

$$= \frac{1}{9}\{(12x-4x^2)x + 2x\cdot(x^2-0^2) + (12x-4x^2)(1-x)\} = \frac{12x - 4x^2 + 2x^3}{9}.$$

Thus firm 0 seeks to

$$\max_{c\in[0,1]} f(c) \equiv 2H(c)[1-c] = \frac{2}{9}(1-c)(12c - 4c^2 + 2c^3),$$

where we have

$$f'(c) = \frac{2}{9}[-12c+4c^2-2c^3+(1-c)(12-8c+6c^2)] = \frac{2}{9}[12-32c+18c^2-8c^3],$$

and

$$f''(c) = \frac{2}{9}[-12 + 8c - 6c^2 - 12 + 8c - 6c^2 + (1-c)(12c-8)]$$

$$= \frac{2}{9}[-24c^2 + 36c - 32] < 0, \ \forall c \in [0,1].$$

Hence $f(\cdot)$ is strictly concave on $[0,1]$, and one can verify that

$$f'(0.47) = 0.02347 > 0 > -0.02167 = f'(0.48),$$

so that $f(\cdot)$ reaches its unique maximum at some $\hat{c} \in [0,1]$, with $f'(\hat{c}) = 0$ and

$$\frac{47}{100} < \hat{c} < \frac{48}{100}.$$

Moreover, we have

$$f(\hat{c}) \leq 0.58468 < \frac{11}{18}.$$

Thus by restricting the sales volume to one unit, exclusive selling via a first-price or second-price auction generates a higher expected revenue for firm 0 than the current selling format.

Intuitively, by denying the losing firm's access to the new production technology, exclusive selling ensures that the winning firm has the highest cost advantage relative to the losing firm in the subsequent Cournot competition, thereby inducing the firms to bid high in the auction. The sales volume in this exercise is restricted to be zero or one or two, which, like the uniform distribution, is rather special. In general, the number of firms in the industry, the distribution of $(\tilde{c}_1, \tilde{c}_2)$, and how the new technology may alter the firms' costs and demands all matter. This exercise is adapted from an old working paper of mine.

9. (**Example 3.**) **The Bolton-Scharfstein (1990) CSV model**.[7]

   Firm B needs F dollars to operate in the product market at respectively date 0 and date 1. Profits are generated at respectively date 1 and date 2. Firm B has no cash initially, and it has to borrow from an investor who has all bargaining power against firm B. Profits are only observable to firm B and verifying profits is prohibitively costly for the investor. The revelation principle implies that the contract-design problem between B and the investor can be modeled as a direct revelation game with no loss of generality. In the direct revelation game, the repayment of the financial contract only depends on the firm's report of profit. Assume that at each date t ($t = 1, 2$) the profit of B can be either π_1 (with probability θ) or π_2, with $\pi_2 > \pi_1$, $\overline{\pi} \equiv E(\pi) = \theta\pi_1 + (1-\theta)\pi_2 > F$, and $\pi_1 < F$. Also, assume all parties are risk neutral with no time preference.

[7]Bolton, P., and D. Scharfstein, 1990, A Theory of Predation Based on Agency Problems in Financial Contracting, *American Economic Review*, 80, 93-106.

(i) Show that if firm B operates for only one period, the investor will refuse to lend F. (**Hint**: If the investor does, B will *always* report $\pi_1 < F$.)

Because of (i), we now suppose that B operates for two periods and that $\pi_2 - \pi_1 < F$. The financing is assumed to proceed as follows. At date 0, the investor lends F to B. Then at date 1, B reports its date-1 profit $\pi_i \in \{\pi_1, \pi_2\}$. If B reports its date-1 profit to be π_i, then it has the obligation of paying the investor R_i at date 1. After this repayment is made, with probability β_i the debt is renewed. In case the debt is renewed at date 1, then the investor gives F to B at date 1, and at date 2, B reports its profit π_j. The second period repayment is denoted by R_{ij} if at date 1 firm B has reported π_i and at date 2 it reports π_j.

The game proceeds as follows. The investor first decides to or not to lend at date 0. If lending is the decision, then the investor offers a financial contract $(R_1, R_2, \beta_1, \beta_2, R_{11}, R_{12}, R_{21}, R_{22})$ to B, and B can either accept or reject.[8] Such a contract specifies only variables that can be subsequently observed by both contracting parties and can be verified by the court of law (so that the latter can enforce it). When specifying these contract variables, the investor must make sure that B will accept (accepting generates for B a utility higher than otherwise), which is called B's individual rationality condition (IR condition). The investor must also make sure that B will truthfully report its profits (truthtelling is better than lying), which is called B's incentive compatibility condition (IC condition). Finally, the repayment R_i and R_{ij} must really be affordable by B when the true profts are respectively π_i and π_j at dates 1 and 2. This is called the limited liability condition (LL condition).

Any contract satisfying these three conditions is said to be *feasible*. The investor wants to find a feasible contract that maximizes her own expected utility. The solution is called an *optimal contract* (because such a contract is Pareto optimal within the set of feasible contracts). Thus, when deciding to lend at date 0, the investor's optimal contract

[8]This implies that the borrower may be a small firm, which lacks bargaining power when negotiating the loan contract with a large bank.

problem is

$$\max_{\beta_i, R_i, R^i} -F + \theta[R_1 + \beta_1(R^1 - F)] + (1-\theta)[R_2 + \beta_2(R^2 - F)],$$

subject to
(IC at date 1) $\pi_2 - R_2 + \beta_2(E(\pi) - R^2) \geq \pi_2 - R_1 + \beta_1(E(\pi) - R^1)$;
(LL at date 1) $\pi_i \geq R_i$,
(LL at date 2) $\pi_i - R_i + \pi_1 \geq R^i, i = 1, 2$;
(IR at date 0) $\theta[\pi_1 - R_1 + \beta_1(E(\pi) - R^1)] + (1-\theta)[\pi_2 - R_2 + \beta_2(E(\pi) - R^2)] \geq 0$.
Note that in the above, we have used the fact that R_{ij} must be independent of π_j in order to satisfy B's second period IC condition, and we have written R_{ij} as R^i. This is also why we did not impose B's IC condition at date 2.

(ii) Show that under optimal contract, firm B always tells the truth when reporting the second-period profit. (**Hint**: Like the reasoning in part (i), if the repayment were dependent on the second-period report, B would always report π_1 in the second period, violating B's IC constraint.)

(iii) Show that the optimal contract $(R_1^*, \beta_1^*, R_2^*, \beta_2^*)$ is $(\pi_1, 0, E(\pi), 1)$ if

$$\theta F + (1-\theta)E(\pi) > \pi_1,$$

and $(\pi_1, 1, \pi_1, 1)$ if otherwise. (**Hint**: Show that the above IC condition has to be binding at optimum. Thus,

$$\pi_2 - R_2 + \beta_2(E(\pi) - R^2) = \pi_2 - R_1 + \beta_1(E(\pi) - R^1).$$

Replace this equality into the objective function and note that the objective function becomes strictly increasing in β_2. This implies that the objective function does not depend on R_2 and R^2 separately; rather, it depends on $R_2 + R^2$ only (and similarly for the constraints.) Thus, there is no loss to set $R^{2*} = \pi_1$. Also, the objective function is increasing in both R_1 and R^1. Finally, note that the objective function is decreasing in β_1 if and only if

$$\theta F + (1-\theta)E(\pi) > \pi_1.$$

Depending on whether this inequality holds, the optimal contract can be fully solved.)

(iv) Show that the investor lends F to B at date 0 if and only if

$$F < \frac{(\pi_1 + (1-\theta)E(\pi))}{2-\theta}.$$

Up to now, we have assumed that $\pi_2 - \pi_1 < F$, and so refinancing at date 1 is necessary for firm B to continue its business in the second period.
(v) Show that, if instead,

$$\min(\pi_2 - F, F) \geq \pi_1,$$

then the investor refuses to lend at date 0 even if B can operate for two periods.

Solution. Consider part (i). Apparently, B will *always* report $\pi_1 < F$ in this one-period setting, and expecting this, investors never want to lend to B in the first place.

Consider part (ii). As in part (i), if the second-period repayment were made dependent on the second-period profit report in a non-trivial way, then B will always report π_1 in the second period, violating B's second-period IC constraint. Thus, the optimal contract requires that R_{ij} be independent of π_j.

Consider part (iii). First it can be proved that the above IC condition must be binding at optimum. Thus,

$$\pi_2 - R_2 + \beta_2(E(\pi) - R^2) = \pi_2 - R_1 + \beta_1(E(\pi) - R^1).$$

Replace this equality into the objective function and note that the objective function becomes strictly increasing in β_2. This means that $\beta_2^* = 1$, which in turn implies that the objective function does not depend on R_2 and R^2 separately; rather, it depends on $R_2 + R^2$ only (and the same is also true for the constraints). Thus, there is no loss to set $R^{2*} = \pi_1$. Also, the objective function is increasing in both R_1 and R^1. Thus, $R_1 = R^1 = \pi_1$, according to LL. Finally, note that the objective function is decreasing in β_1 if and only if

$$\theta F + (1-\theta)E(\pi) > \pi_1,$$

and in this case, $\beta_1^* = 0$. On the other hand, when

$$\theta F + (1-\theta)E(\pi) \leq \pi_1,$$

it is optimal to set $\beta_1^* = 1$.

Next, consider part (iv). We need to show that the investor lends F to B at date 0 if and only if

$$F < \frac{(\pi_1 + (1-\theta)E(\pi))}{2-\theta}.$$

This follows from the fact that the investor's expected profit is

$$\pi_1 - F + (1-\theta)(E(\pi) - F),$$

which cannot be negative.

Finally, assume in part (v) that

$$\min(\pi_2 - F, F) \geq \pi_1.$$

We need to show that the investor would refuse to lend to B at date 0 even if B can operate for two periods. To see that this is so, note that if the above inequality holds, B will always report π_1, with no concern about whether he will get refinancing. This happens because, by the above assumption, once B gets to operate for one period, B will collect enough money to cover the second-period F. Recognizing this fact, investors will not lend to B in the first place. This is one version of the *free cash flow* problem discussed in Jensen (1986, *American Economic Review*). There, Michael Jensen points out that there may be substantial benefits resulting from a voluntary reduction of a firm's internal funds (by buying shares back, paying dividends, or repaying existing debts).

10. (**Example 4.**) Here we consider the optimal three-period contract for the Bolton-Scharfstein (1990) model.

Consider the Bolton-Scharfstein 1990 model, but with the following modifications. At first, assume that firm B can invest the same 1-period investment project for 3 times (at respectively date 0, date 1, and date 2). Second, if firm B does not undertake the project at date t, then the project becomes unavailable at date $s > t$. Third, assume that the investor can adopt only pure-strategy loan-approving policies; that is, the β's must take value of 0 or 1.

To highlight the value of long-term relationships, let us assume that

$$0 < \pi_1 < \frac{\pi_1(1+\theta-\theta^2)+(1-\theta)^2\pi_2}{2-\theta} < F < \overline{\pi} = \theta\pi_1 + (1-\theta)\pi_2 < \pi_2;$$

that is, the investor would refuse to finance at all if the investment project can only be undertaken twice.

To facilitate our analysis, we shall assume from now on that

$$\theta = \frac{1}{2},$$

so that the above inequalities can be written as

$$(\Lambda)\ 0 < \pi_1 < \frac{5\pi_1 + \pi_2}{6} < F < \overline{\pi} = \frac{\pi_1 + \pi_2}{2} < \pi_2.$$

The investor must design a long-term contract at date 0, which maximizes the investor's payoff by specifying the date-t repayments and the date-t refinancing probabilities contingent on the history of earnings reports up to date t, such that (IR) the investor can at least break even by offering the long-term contract at date 0; (IC) at each date t and in each contingency of date t firm B always finds truth-telling to be optimal; and (LL) the date-t repayments satisfy the date-t limited liability constraints.

To derive the optimal contract, we first make a few observations.

- At first, the optimal contract should specify $\beta_1 = 0$ following a date-1 profit report of π_1.

To see this, suppose instead that $\beta_1 = 1$. To induce truth-telling at date 1, we must have $\beta_2 = 1$ also. Thus the contract promises refinancing at date 1 regardless of the realized date-1 profit. By condition (Λ), the investor cannot break even over the date-1-date-3 period, and by (LL), the investor's date-0 payoff cannot exceed $-F + \pi_1$ plus the optimal payoff over the date-1-date-3 period, and hence the investor would be better off abandoning this contract at date 0!

- Now, given that $\beta_1 = 0$, the contract must specify $\beta_2 = 1$ at date 0.

  Otherwise, the contract would become a one-period contract, and condition (Λ) implies that the investor would become better off abandoning this contract at date 0.

By assumption, following a date-1 profit report of π_1, the project will never be undertaken again at dates 1 and 2. Thus we only need to consider the date-2 refinancing probabilities following a date-1 profit report of π_2. let β^j denote the probability of date-2 refinancing following a date-1 profit report of π_2 and then a date-2 profit report of π_j. Apparently, the investor would be better off abandoning the contract if the contract specifies $\beta^1 = \beta^2 = 0$. Thus we are left with 3 classes of feasible contracts: those specifying $(\beta^1, \beta^2) = (1, 1)$, those specifying $(\beta^1, \beta^2) = (0, 1)$, and those specifying $(\beta^1, \beta^2) = (1, 0)$. We shall derive the optimal contract in each of these 3 classes, and then compare and find the optimal date-0 contract.

- Let us find the optimal contract with $(\beta^1, \beta^2) = (1, 1)$.

  As in the paper, let R_1 and R_2 be the date-1 repayments following a date-1 profit report of respectively π_1 and π_2. The contractual relationship continues to date 2 if and only if the realized (and reported) date-1 profit is π_2. Thus let R^k be the date-2 repayment following a date-1 profit report of π_2 and a date-2 profit report of π_k. Given $(\beta^1, \beta^2) = (1, 1)$, the date-2 refinancing is guaranteed, and we denote the date-3 repayment following a date-1 profit report of π_2 and a date-2 profit report of π_k by R_3^k, which, because state verification is out of the question, must be independent of the date-3 profit report.

Hence the investor's date-0 contract-design problem given $(\beta^1, \beta^2) = (1,1)$ is

$$\max_{R_1,R_2,R^1,R^2,R_3^1,R_3^2} -F+\theta R_1+(1-\theta)[R_2-F+\theta(R^1-F+R_3^1)+(1-\theta)(R^2-F+R_3^2)]$$

subject to

$$0 \leq R_j \leq \pi_j, \ j=1,2;$$

$$0 \leq R_2 + R^j \leq \pi_2 + \pi_j, \ j=1,2;$$

$$0 \leq R_2 + R^j + R_3^j \leq \pi_2 + \pi_j + \pi_1, \ j=1,2;$$

$$(\text{IC}_2) \begin{cases} (\pi_2 - R_2) + (\pi_2 - R^2) + (\overline{\pi} - R_3^2) \geq (\pi_2 - R_2) + (\pi_2 - R^1) + (\overline{\pi} - R_3^1), \\ (\pi_2 - R_2) + (\pi_1 - R^1) + (\overline{\pi} - R_3^1) \geq (\pi_2 - R_2) + (\pi_1 - R^2) + (\overline{\pi} - R_3^2); \end{cases}$$

$$\Rightarrow R^2 + R_3^2 = R^1 + R_3^1 \equiv R;$$

$$(\text{IC}_1)\ \pi_2-R_2+\theta(\pi_1-R^1+\overline{\pi}-R_3^1)+(1-\theta)(\pi_2-R^2+\overline{\pi}-R_3^2) \geq \pi_2-R_1$$

$$\Leftrightarrow R_2 + R \leq \pi_2 + 2\overline{\pi},$$

where note that (IC_t) is firm B's date-t IC constraint.
It is obvious that at optimum we must have $R_1 = \pi_1$, and

$$R_2 + R \leq \min(2\pi_1 + \pi_2, 2\pi_2 + \pi_1, \pi_2 + 2\overline{\pi}) = 2\pi_1 + \pi_2.$$

Thus the investor's maximization problem can be re-stated as

$$\max_{R_2,R} -F + \theta\pi_1 + (1-\theta)(R_2 + R - 2F),$$

so that one optimal solution is such that

$$R^1 = R^2 = 0, \quad R_2 = \pi_2, \quad R_3^1 = R_3^2 = 2\pi_1.$$

This optimal contract with $(\beta^1, \beta^2) = (1,1)$ yields for the investor the payoff

$$\overline{\pi} - (3-2\theta)F + (2-2\theta)\pi_1,$$

which satisfies the investor's date-0 IR constraint if and only if

$$F \leq \hat{F}_{11} \equiv \frac{(2-\theta)\pi_1 + (1-\theta)\pi_2}{3-2\theta} = \frac{3\pi_1 + \pi_2}{4}.$$

- Now, let us find the optimal contract with $(\beta^1, \beta^2) = (0, 1)$. The investor's date-0 contract-design problem given $(\beta^1, \beta^2) = (0, 1)$ is

$$\max_{R_1, R_2, R^1, R^2, R_3^2} -F + \theta R_1 + (1-\theta)[R_2 - F + \theta R^1 + (1-\theta)(R^2 - F + R_3^2)]$$

subject to

$$0 \le R_j \le \pi_j, \ j = 1, 2;$$

$$0 \le R_2 + R^j \le \pi_2 + \pi_j, \ j = 1, 2;$$

$$0 \le R_2 + R^2 + R_3^2 \le \pi_2 + \pi_2 + \pi_1;$$

$$(\text{IC}_2) \begin{cases} (\pi_2 - R_2) + (\pi_2 - R^2) + (\overline{\pi} - R_3^2) \ge (\pi_2 - R_2) + (\pi_2 - R^1), \\ \\ (\pi_2 - R_2) + (\pi_1 - R^1) \ge (\pi_2 - R_2) + (\pi_1 - R^2) + (\overline{\pi} - R_3^2); \end{cases}$$

$$\Rightarrow R^2 + R_3^2 = R^1 + \overline{\pi};$$

$$(\text{IC}_1) \ \pi_2 - R_2 + \theta(\pi_1 - R^1) + (1-\theta)(\pi_2 - R^2 + \overline{\pi} - R_3^2) \ge \pi_1 - R_1$$

$$\Leftrightarrow R_2 + R^1 \le \pi_2 + \overline{\pi}.$$

It is obvious that at optimum we must have $R_1 = \pi_1$, and

$$R_2 + R^1 \le \min(\pi_1 + \pi_2, 2\pi_2 + \pi_1, \pi_2 + \overline{\pi}) = \pi_1 + \pi_2.$$

Thus the investor's maximization problem can be re-stated as

$$\max_{R_2, R^1} -F + \theta\pi_1 + (1-\theta)[R_2 + R^1 - F + (1-\theta)(\overline{\pi} - F)],$$

so that one optimal solution is such that

$$R^1 = R_3^2 = \pi_1, \quad R_2 = \pi_2, \quad R^2 = \overline{\pi}.$$

This optimal contract with $(\beta^1, \beta^2) = (0, 1)$ yields for the investor the payoff

$$\pi_1(1 + \theta - 2\theta^2 + \theta^3) + \pi_2(2 - 4\theta + 3\theta^2 - \theta^3) - F(3 - 3\theta + \theta^2),$$

which satisfies the investor's date-0 IR constraint if and only if

$$F \le \hat{F}_{01} \equiv \frac{\pi_1(1 + \theta - 2\theta^2 + \theta^3) + \pi_2(2 - 4\theta + 3\theta^2 - \theta^3)}{3 - 3\theta + \theta^2} = \frac{9\pi_1 + 5\pi_2}{14}.$$

- Finally, let us find the optimal contract with $(\beta^1, \beta^2) = (1, 0)$. The investor's date-0 contract-design problem given $(\beta^1, \beta^2) = (1, 0)$ is

$$\max_{R_1, R_2, R^1, R^2, R_3^1} -F + \theta R_1 + (1-\theta)[R_2 - F + \theta(R^1 - F + R_3^1) + (1-\theta)R^2]$$

subject to

$$0 \le R_j \le \pi_j, \ j = 1, 2;$$
$$0 \le R_2 + R^j \le \pi_2 + \pi_j, \ j = 1, 2;$$
$$0 \le R_2 + R^1 + R_3^1 \le \pi_2 + \pi_1 + \pi_1;$$
$$(\mathrm{IC}_2) \begin{cases} (\pi_2 - R_2) + (\pi_2 - R^2) \ge (\pi_2 - R_2) + (\pi_2 - R^1) + (\overline{\pi} - R_3^1), \\ (\pi_2 - R_2) + (\pi_1 - R^1) + (\overline{\pi} - R_3^1) \ge (\pi_2 - R_2) + (\pi_1 - R^2); \end{cases}$$
$$\Rightarrow R^1 + R_3^1 = R^2 + \overline{\pi};$$
$$(\mathrm{IC}_1)\ \pi_2 - R_2 + \theta(\pi_1 - R^1 + \overline{\pi} - R_3^1) + (1-\theta)(\pi_2 - R^2) \ge \pi_1 - R_1$$
$$\Leftrightarrow R_2 + R^2 \le \pi_2 + \overline{\pi}.$$

It is obvious that at optimum we must have $R_1 = \pi_1$, and

$$R_2 + R^2 \le \min(2\pi_1 + \pi_2 - \overline{\pi}, 2\pi_2, \pi_2 + \overline{\pi}) = 2\pi_1 + \pi_2 - \overline{\pi}.$$

Thus the investor's maximization problem can be re-stated as

$$\max_{R_2, R^1} -F + \theta\pi_1 + (1-\theta)[R_2 + R^2 - F + \theta(\overline{\pi} - F)],$$

so that one optimal solution is such that

$$R^1 = R_3^1 = \pi_1, \quad R_2 = \pi_2 - \overline{\pi}, \quad R^2 = 2\pi_1.$$

This optimal contract with $(\beta^1, \beta^2) = (0, 1)$ yields for the investor the payoff

$$\pi_1(2 - 2\theta + 2\theta^2 - \theta^3) + \pi_2(2\theta - 3\theta^2 + \theta^3) - F(2 - \theta^2),$$

which satisfies the investor's date-0 IR constraint if and only if

$$F \le \hat{F}_{10} \equiv \frac{\pi_1(2 - 2\theta + 2\theta^2 - \theta^3) + \pi_2(2\theta - 3\theta^2 + \theta^3)}{2 - \theta^2} = \frac{11\pi_1 + 3\pi_2}{14}.$$

- It is easy to verify that

  $$\frac{5\pi_1 + \pi_2}{6} < \frac{11\pi_1 + 3\pi_2}{14} = \hat{F}_{10} < \frac{3\pi_1 + \pi_2}{4} = \hat{F}_{11} < \frac{9\pi_1 + 5\pi_2}{14} = \hat{F}_{01} < \overline{\pi}.$$

  Moreover, when

  $$\frac{5\pi_1 + \pi_2}{6} < F \leq \frac{11\pi_1 + 3\pi_2}{14} = \hat{F}_{10}$$

  so that the above three optimal contracts all satisfy the investor's IR constraint, the optimal contract with $(\beta^1, \beta^2) = (0, 1)$ dominates both the optimal contract with $(\beta^1, \beta^2) = (1, 1)$ and the optimal contract with $(\beta^1, \beta^2) = (1, 0)$.

Now, we can summarize our findings.

Proposition 1

- *Suppose that condition* (Λ) *holds, and*

  $$\hat{F}_{01} = \frac{9\pi_1 + 5\pi_2}{14} < F < \frac{7\pi_1 + 7\pi_2}{14} = \overline{\pi}.$$

  Then in equilibrium the investor optimally turns down firm B's financing request at date 0.

- *Suppose that condition* (Λ) *holds, and*

  $$\hat{F}_{01} = \frac{9\pi_1 + 5\pi_2}{14} \geq F \geq \frac{5\pi_1 + \pi_2}{6}.$$

  Then in equilibrium the investor optimally offers the following 3-period contract (or one of its equivalent versions) at date 0:

  $$\beta_1 = 0 = \beta^1, \;\; \beta_2 = \beta^2 = 1,$$

  $$R_1 = \pi_1, \;\; R^1 = R_3^2 = \pi_1, \quad R_2 = \pi_2, \quad R^2 = \overline{\pi}.$$

These results are not surprising. Being able to invest the project for 3 times instead of 2 times opens up the possibility that the investor may be able to extract rents at date 1 following a profit realization of π_2, which is achieved by allowing firm B to stay in business for more than 1 more period.

While a contract with $(\beta^1, \beta^2) = (1, 1)$ or $(\beta^1, \beta^2) = (1, 0)$ may also become profitable to the investor given that the project can now be undertaken for 3 times, it is optimal to allow refinancing at date 2 only after a stream of high profit reports.

The problem with the policy $(\beta^1, \beta^2) = (1, 1)$ is that in order to induce truth-telling at date 2, the repayments received after date 1 must be independent of the profit realizations after date 1. This greatly limits the investor's opportunity of extracting rents from firm B.

The problem with the policy $(\beta^1, \beta^2) = (1, 0)$ is that, unlike the policy $(\beta^1, \beta^2) = (0, 1)$, the date-2 refinancing is granted following a low date-2 profit report, which implies a relatively small date-3 verifiable income that can accrue to the investor without state verification. Again, it limits the investor's opportunity of extracting rents from firm B.