

Can an Optimal Insurance Contract Violate the Principle of Indemnity?

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Abstract

The principle of indemnity is well-accepted in the field of insurance and is also commonly assumed in the literature of optimal contract design. However, should this principle be imposed on all insurance contracts without exceptions? Does there exist an insurance contract that is optimal for both the insurer and the insured even as the principle of indemnity is violated? The answer is yes. This paper demonstrates that in the insurance market there do exist contracts in which optimality is accompanied by a violation of the indemnity principle if the insurer and insured have deviant beliefs in loss distribution. We further use retrospective rating as an example to explain why an insurance contract, violating the principle of indemnity, commonly exists in commercial insurance, especially where moral hazard and adverse selection is less severe.

Keywords: optimal insurance contract, deviant beliefs, retrospective rating, moral hazard, adverse selection

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1 Introduction

The principle of indemnity, i.e., the insured should not receive an indemnity greater than the loss, is widely adopted in the industry. In this paper, we intend to show that violation of the principle of indemnity does not prevent an insurance policy from being an optimal contract. One example is retrospective rating, which is commonly used in commercial insurance, such as workers' compensation and general liability insurance. Under such a plan, the insurance company charges a premium based on the insured's loss experience during the current policy period. Typically, if the insured's claims during the policy period are low (high), then the insurance premium paid for that period is low (high). In practice, the insured may pay a higher premium in the beginning of the policy year and receive a rebate if the insured's claims experience during the policy year is good. Although many larger firms adopt retrospective rating in business, this may violate the principle of indemnity. Consider a case in which the insured's claims experience during the policy year is good; the insured receives not only payments from the insurer to cover his losses but also a partial rebate of the insurance premium. Depending on the policy design, for a small loss claimed, the insured may obtain full coverage along with the partial rebate of the premium. In such a case, the insured enjoys a net profit; consequently the principle of indemnity is violated. Now, two questions need to be answered: What are the mechanisms that check and balance the temptation to make a profit from moral hazard? Could this kind of contract still be optimal for both the insurer and insured?

Since Raviv [12] studied the optimal insurance contract between an insurer and an insured, many papers (Huberman, Mayers, and Smith [7]; Gollier [4, 5]; and Kaplow [8]) have followed this thread and have investigated how an optimal insurance policy varies under different circumstances. Although the literature has provided the rationale for many contracts, such as deductible, coinsurance, policy limit, franchise deductible, etc., few papers have ever studied why contracts that violate the principle of indemnity can still be found in the market. This paper intends to fill

this gap.

Our paper is different from Raviv [12] in two ways. First of all, Raviv [12] imposed a two-sided constraint for the indemnity, i.e., the indemnity should be neither negative nor larger than the losses. Gollier [4] released the nonnegative constraint. Our model removes the constraint that the indemnity should be less than or equal to the loss. In other words, the principle of indemnity is not imposed. Secondly, we assume, as Marshall [10] did, that the insurer and the insured may have deviant beliefs in the loss distribution.

We show that even if it violates the principle of indemnity, such as retrospective rating, a contract could still be optimal when the insurer and insured have deviant beliefs in loss distribution. Under certain conditions, when the losses are small, the optimal indemnity could be larger than the losses, and it, indeed, makes the contract look like a retrospective rating. We further discuss several practical concerns, such as moral hazard and adverse selection, and demonstrate that a retrospective rating may exist in insurance where moral hazard and adverse selection are less severe.¹

We review the problem in Section 2 with a model revised from Raviv [12]. Section 3 discusses some practical considerations in implementing retrospective ratings, including moral hazard and adverse selection. Section 4 concludes this paper.

¹In this paper, we do not intend to provide a unique reason to explain all the exceptions to the principle of indemnity, for example, life insurance. Instead, we try to offer a rationale, which can explain at least some cases with a violation of the indemnity principle.

2 Model

2.1 Optimal Insurance Contract Under Deviant Beliefs

Assume that there are two representative agents, one insured and one insurance company, in the market. As Marshall [10] did, we assume that the insured and the insurer have different beliefs in risk. More specifically, we assume that the insured and the insurer believe that the probability density functions of loss x are, respectively, $f(x)$ and $g(x)$ and they are common knowledge to both agents. Assume that $f(x)$ and $g(x)$ are positive, continuous, and differentiable for $0 \leq x \leq L$, where L is the maximum possible loss. Assume that there exists an insurance contract between the insured and the insurer. The insurer pays indemnity $I(x)$ to the insured when loss x happens. Assume that $I(x) \geq 0$, and $I'(x) \geq 0, \forall x$.² To obtain the insurance policy from the insurance company, the insured needs to pay premium P to the insurer.

Let w be the initial wealth of a potential insurance buyer. If he/she purchases a policy $I(x)$ with premium P to cover the possible loss, his/her final wealth, W , will be $w - x + I(x) - P$ in case loss x happens. The insured is assumed to maximize his/her expected utility³ in a way that can be described as:

$$EU = \int_0^L u(w - x - P + I(x))f(x)dx \quad (1)$$

where u is the utility function. Here we assume that $u' > 0$, $u'' < 0$ for all wealth levels. In addition, the insured is assumed to be either constant or decreasing absolute risk averse.

The insurance company operates with administrative cost $c(I(x))$. Without losing any generality, $c(I(x))$ is assumed to be equal to $mI(x)$. The marginal cost of indemnity, m , is a constant.

²Unlike Gollier [4], we focus on non-negative indemnity cases. Furthermore, if the marginal indemnity is negative, that means the insurer would pay less when larger losses happen. This would cause some difficulties in administering an insurance policy.

³The necessary condition for the insured to purchase the insurance is $EU \geq \int_0^L u(w - x)f(x)dx$, and is assumed to hold.

Furthermore, the insurer is assumed to be risk neutral.

Following the approach in Raviv [12], the optimal policy can be developed by Equation (2).

$$\begin{aligned} \max_{I(x), P} EU &= \int_0^L u(w - x - P + I(x))f(x)dx \\ \text{s.t. } P &= \int_0^L (1 + m)I(x)g(x)dx \\ I(x) &\geq 0, I'(x) \geq 0. \end{aligned} \tag{2}$$

Proposition 1 *The optimal coverage function is $I^*(x) = 0$, for $x \leq D$; $I^*(x) > 0$, and*

$$I^{*'}(x) = \frac{R_u(W) + \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}}{R_u(W)}, \tag{3}$$

for $D < x \leq L$, where R_u is the absolute risk averse index of the insured.

Proof: See Appendix A.

Arrow [1] showed that the optimal insurance contains a deductible and full coverage above the deductible. Comparing Proposition 1 with Arrow's work [1], we notice that the difference in the likelihood-ratio for the insurer and the insured plays a significant role in the optimal marginal coverage function. Obviously, Arrow's result is a special case where the insurer and the insured have the same belief about risk, and thus $\frac{f'(x)}{f(x)} = \frac{g'(x)}{g(x)}$ for all x .

2.2 Violation of the Principle of Indemnity

In this subsection we try to find the conditions under which the indemnity can be larger than the loss amount.

Proposition 2 *$I^*(x) > x$, if*

$$\int_D^x \frac{\frac{f'(t)}{f(t)} - \frac{g'(t)}{g(t)}}{R_u(w - t - P + I^*(t))} dt > D. \tag{4}$$

Proof: Since $I^*(x) = \int_D^x I^{*'}(t)dt = x - D + \int_D^x \frac{\frac{f'(t)}{f(t)} - \frac{g'(t)}{g(t)}}{R_u(w - t - P + I^*(t))} dt$, the condition for $I^*(x) > x$ is the sum of the last two terms is positive.

Although Proposition 2 provides us with the condition where the optimal contract violates the principle of indemnity, we need to check whether the condition can really exist in both theory and practice. For simplicity, let us assume that the insured is constant absolute risk aversion with the absolute risk aversion index equal to K , $f(0) = g(0)$, and $m = 0$.⁴ Let us further consider a case where $f(x) \geq g(x)$, $\forall x \in [0, x_1]$, $x_1 \in [0, L]$. Thus, from Proposition 2, $I(x) \geq x$, $\forall x \in [0, x_1]$, since $I(x) = \int_0^x I'(t)dt = x + \frac{1}{K} \ln \frac{f(x)}{g(x)}$. If $f(x) \geq g(x)$, $\forall x \in [0, x_1]$, it can be shown that $F(x) \geq G(x)$, $\forall x \in [0, x_1]$, where $F(x)$ and $G(x)$ are cumulative density functions of $f(x)$ and $g(x)$, respectively. $F(x) \geq G(x)$, $\forall x \in [0, x_1]$ means the insured is more optimistic than the insurer. In other words, the insured may think of himself as one in a good risk group, whereas the insurer has totally opposite ideas. Although those beliefs are common knowledge to each other, they cannot agree on the objective perspective in the risk classification of the insured. Thus, the insurer and the insured may settle with a retrospective rating such that the insured can get a premium rebate if the loss experience is good.

3 Retrospective Rating

In this section, we use retrospective rating as an example to explain why a contract that violates the principle of indemnity could still be an optimal policy if there exists deviant beliefs in the underlined loss distribution. The assumptions of the theoretical model in the previous section are also useful for discussing practical conditions of implementing the retrospective rating in reality. In the theoretical model, we assume that there is no moral hazard or adverse selection in the market. Indeed, these two assumptions are crucial for the implementation of a retrospective rating.

⁴According to Raviv [12], the deductible equals zero while there is no variable cost of the insurer. Even with deviant belief of losses, the condition of zero deductible remains the same. We make this assumption in order to simplify our example, so that the violation of the indemnity principle can be seen easily.

3.1 Moral Hazard

It is well accepted that the principle of indemnity can prevent moral hazard. If the indemnity transferred from the insurer to the insured can be larger than the loss suffered, the insured is provided an immoral incentive to be careless or even deliberately make the loss happen, collect the claim, and enjoy the net profits. The model presented in Section 2 could predict that for a case of small loss, the insured may situate in a moral hazard. On the other hand, we can see that the retrospective rating is widely used in the industry. This implies that there must be a counter mechanism that balances the moral hazard.

Let us take workers' compensation insurance, for example. The employer, i.e., the insured, claims for the coverage only when his/her employees are injured. If the employer tries to profit from the insurance, this means that he/she needs to persuade a third party to exaggerate the injury to cheat the insurance company and split the compensation. This generally never ever happens, since the employee must to suffer physical injury and legal responsibility.⁵ This is the passive thread of the mechanism. There are also active threads. For example, the insurer reviews the claim record right before the end of the coverage period and calculates the retrospective premium. In other words, if the insured shows a good track record during the coverage, a portion of the premiums paid is refunded. However, if a claim exceeds a certain point, an assessment for additional premiums would be adjusted. Therefore, the insured tends to keep the premium as low as possible by carrying out some kind of risk-management programs.

3.2 Adverse Selection

Traditionally, under the model of adverse selection, such as documented in Rothschild and Stiglitz [15], the insured is assumed to have full information on loss distribution whereas the insurer is assumed to have none. In this work, we follow the model depicted in Marshall's work [10].

⁵Moral hazard may still exist between employees and employer.

Marshall assumed that the insured and the insurer have different beliefs on the loss distribution, but they are common knowledge to both agents. With the difference between $f(x)$ and $g(x)$ (it is also true in reality), we can study other interesting aspects of an optimal contract.

Let us think about the case of workers' compensation again. Usually, insurance companies have their claim record and analysis to classify the risk level of different industries, plants, or even a specific client. However, the claim record reveals only part of the risk type of the insured. The risk type or level is still influenced by other random factors. Thus, if an employer experiences one year of a poor record, he may take it as bad luck and feel confident of himself in the long run. However, he may not be able to fully convince the insurer to see things from the same perspective and share the same belief. Another case would be that, after an employee suffers serious injuries, the employer usually learns a lesson and adopts some risk-management tools to avoid similar losses in the future. In this case, the insured considers himself to be a low risk, but the insurer may classify the insured as a high risk. Rather than the case of adverse selection, the two aforementioned cases are closer to the assumption made in Marshall's model, i.e., the insurer and the insured have different beliefs on the loss distribution but they are common knowledge to each other. This observation suggests that the retrospective rating is more likely to be sustained in the market under the level of risk is perceived asymmetrically as opposed to traditional adverse selection.

Another observation is that, it may be difficult to implement a retrospective rating in an insurance market where the loss in a year provides little information on the loss distribution. Let us compare automobile insurance to workers' compensation. In each automobile insurance contract, only one or two cars are insured under a policy. The automobile accident happens randomly; and usually no cases are claimed during the insurance period. So a driver with a one-year no-accident record, statistically speaking, does not reveal much information on his risk type. As for workers' compensation contracts, hundreds or thousands of employees are involved. By the

law of large numbers, the claim records of workers' compensation in a year give the ensemble, the average performance of the workers under consideration. Thus, we may expect to see retrospective ratings adopted where the realization of losses reveals more information on the loss distribution, such as workers' compensation.

4 Conclusions

This paper shows that even if the principle of indemnity is not enforced, an insurance contract can still be optimal if the insurer and insured have deviant beliefs in loss distribution and in an insurance market, which moral hazard and adverse selection are less severe. The prediction is consistent with the fact that the retrospective rating is commonly used in commercial insurance, especially workers' compensation insurance.

Appendix A

Facing a fixed premium, the insured would like to control coverage function $I(x)$ in order to maximize his/her expected utility subject to the insurer's expected utility.

The Hamiltonian for Equation (2) is

$$H = u(w - x + I(x) - P)f(x) - \lambda(1 + m)I(x)g(x) \quad (5)$$

where λ is constant with respect to x . The necessary conditions for the maximizing problem are discussed as following:

$I^*(x) = 0$, if

$$M = u'(w - x - P)f(x) - \lambda(1 + m)g(x) \leq 0 \quad (6)$$

$0 < I^*(x)$, if

$$u'(w - x + I^*(x) - P)f(x) - \lambda(1 + m)g(x) = 0 \quad (7)$$

M is continuous in x .

From Equation (7), we have

$$\lambda = \frac{u'(w - x + I^*(x) - P)f(x)}{(1 + m)g(x)}, \quad \forall x \quad (8)$$

Differentiating Equation (8) with respect to x , we get the marginal indemnity function as follows.

$$I^{*'}(x) = \frac{R_u(W) + \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}}{R_u(W)}. \quad (9)$$

M in (6) is an increasing function of x . It is obtained from

$$\text{sign } \frac{\partial M}{\partial x} = \text{sign } R_u(w - x - P) + \frac{f'(x)}{f(x)} - \lambda(1 + m) \frac{g'(x)}{u'(w - x - P)f(x)}.$$

Let

$$\lambda = \frac{u'(w - x - P)f(x)}{(1 + m)g(x)}, \quad (10)$$

then

$$\text{sign } \frac{\partial M}{\partial x} = \text{sign } R_u(w - x - P) + \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)} \quad (11)$$

According to the assumption, $I'(x) \geq 0$, $\forall x$, which means

$$R_u(W) + \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)} \geq 0$$

Since we assume the insured is CARA or DARA, the inequality above can ensure that

$$R_u(w - x - P) + \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)} \geq 0$$

That is, M is an increasing function of x .

Let us define D , which satisfies the following equation:

$$M(D) = u'(w - D - P)f(D) - \lambda(1 + m)g(D) = 0 \quad (12)$$

As long as $x \leq D$, the optimal policy pays nothing to the insured. Therefore, we can conclude the following:

$$I^*(x) = 0, \quad \text{if } x \leq D \quad (13)$$

$$0 < I^*(x), \quad \text{if } x > D. \quad (14)$$

Q.E.D.

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