Evaluating Natural Resource Investments: A Dynamic Option Simulation Approach

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Abstract

A Dynamic Option Simulation (DOS) approach is proposed for evaluating a natural resource investment in this paper. The DOS combines simulation and dynamic programming techniques and can value natural resource investments with properties of multi-variables, early exercise, several embedded options, and finite reserves. To construct a practical pricing model, several stochastic variables are considered. A copper mine investment example is presented, in which the mine holder is allowed to temporarily close, reopen, and abandon the mine at specific times before the expiration day. Furthermore, the mine holder has options to accelerate the mining speed. By applying the DOS approach, the value of a natural resource investment and the values of options embedded in the investment can be efficiently and accurately derived.

Keywords: simulation, dynamic programming, early exercise, embedded option
JEL Classification: C15, C61, G13

1. Introduction

Ever since the famous option pricing models introduced by Black and Scholes (1973) and Merton (1973), the valuation of financial derivatives has leapt into a new era. The option pricing model can provide a fair price to both counterparties of trading, and it can thus increase the liquidity and growth of financial markets. The contributions of option pricing models are not only in the valuation for financial instruments, but they can also be used to price the so-called "managerial flexibilities" in an investment project. Applying an option pricing model into the valuation of an investment has now become a mainstream technique, which is called the Real Option approach. People can evaluate an investment project efficiently by using a Real Option approach, and the resources of an enterprise can thus be allocated optimally.

An entrepreneur traditionally evaluates investment projects according to the Net Present Value (NPV) criterion. People will reject a good project and make a wrong decision according to NPV rule. This is because the NPV rule cannot take into account the values of the managerial flexibilities embedded in an investment project. The under-valuation problems mentioned above can be solved by applying the Real Option approach, since the Real Option approach can efficiently quantify the values of managerial flexibilities. People gradually pay attention to the Real Option approach both for academic and practical fields. It is worth further exploring how the Real Option approach has been used to accurately evaluate an investment project.

A Real Option is defined as the right, but not the obligation, to take an investment action on a real asset at a pre-determined price (the cost called exercise price), for a predetermined period of time. A real Option approach to capital investment has the advantage to capture the value of managerial flexibilities which a traditional NPV cannot properly address. This value is manifest as a collection of call or put options embedded in capital investment opportunities. These options typically include: option to defer, time-to-build option, option to alter operating scale (expand or contract), option to abandon, option to switch, growth option, and multiple interacting options. A deferral option is an American call option found in most projects where one has the right to delay the start of a project. Its exercise price is the money invested in getting the project started. The option to abandon a project for a fixed price (even when that price decreases through time) is formally an American put; so is the option to contract (scale back) a project by selling a fraction of it for a fixed price. The option to expand a project by paying more to scale up the operations is an American call. The option to extend the life of a project by paying an exercise price is also an American call. Switching options are portfolios of American call and put options that allow their owner to switch at a fixed cost between two modes of operation.[1]

For venture capital investment in high-risk technology development, traditional discount cash flow techniques cannot capture the full value of investment. Myers (1987), Kester (1984), and Dixit and Pindyck (1995) suggest the use of option-based techniques to value the managerial flexibility implicit in investment opportunities. Concerning the valuation of natural resource investment, Brennan and Schwartz (1985, hereafter B&S) first applied an option model for evaluating a copper mine. Castillo-Ramirez (1999, hereafter C-R), followed B&S (1985) by modifying the Raymar and Zwecher (1997) model for evaluating a copper mine.

For the task of multiple Real Options valuation, it may be difficult to solve the problem using analytic solutions by partial differential equations. Three types of numerical techniques have been developed for option valuation: (1) approximate the underlying stochastic process directly by Monte Carlo simulation as first introduced by Boyle (1977); (2) use various lattice (tree) approaches such as Cox, Ross, and Rubinstein's (1979) binomial tree method; (3) discretize a partial differential equation by Finite Difference methods, such as Brennan and Schwartz (1978). Both the lattice (tree) model and Finite Difference method will become inefficient when the state variable number is large enough. This is because the memory space and computation time will grow exponentially as the state variable increases. Generally, as the number of state variables is more than three, these two numerical methods are incapable of valuing an option. On the contrary, the simulation approach seems to be the best choice for a complex option valuation problem.

Boyle (1977) first proposed a simulation approach for pricing options. The simulation approach is very flexible and can be used to price complex European-style options. Before 1993, there were few published works on the use of simulation techniques to value American options. Tilley (1993) was the first to develop such a technique and since then, many related articles have followed, such as Boyle, Broadie, and Glasserman (1997), Grant, Vora, and Weeks (1996), and Glasserman (1997). Barraquand and Martineau (1995) presented a new simulation approach to approximate the maximum American option prices numerically. Barraquand and Martineau (1997) extended from their article in 1995. They believe that the value of the second highest underlying asset is meaningful, and will give some information on whether to exercise immediately or hold to expiration. In short, simulation approaches are particularly useful, powerful, and efficient when there are multiple stochastic factors that

[1] For a general overview of the Real Option, Trigeorgis (1996) and Schwartz and Trigeorgis (2001) provided an in-depth review and examples on different Real Options.

determine the option's value. Longstaff and Schwartz (2001) developed a simple least-squares Monte Carol simulation (hereafter LSM) method to price American options. LSM can easily handle the option pricing problem with several variables and path-dependent exotic features.

A natural resource investment valuation can be thought of as a multi-stage Real Option pricing problem. There are usually several options involved, generally, each of the two options is dependent and these options can be exercised before expiration day. Also, the value of a natural resource investment has relations with its last period conditions and the resource reserves, this means the options embedded in a natural resource investment are path-dependent. To construct a practical pricing model, this study assumes several stochastic variables into the model.

According to the settings above, evaluating a natural resource investment is just like pricing a multi-factor path-dependent American option. It's impossible to derive a closed-form solution for a natural resource investment under the complex conditions mentioned above. It is also inefficient by applying a tree model or a Finite Difference method to price natural resource investments. To effectively price a natural resource investment, this study develops a Dynamic Option Simulation (DOS) approach, which combines the LSM proposed by Longstaff and Schwartz (2001) and dynamic programming techniques. LSM will be applied to deal with the problems of multi-factor, early exercise, and several embedded options. A dynamic programming technique can solve the finite reserves issues in a natural resource investment pricing problem.

A copper mine investment example is presented, in which the mine holder is allowed to temporarily close, reopen, and abandon the mine at specific times before the expiration day. The mine holder is allowed to accelerate the mining speed which is defined as an Acceleration option in this study. By applying the DOS approach, the value of a natural resource investment and the values of options embedded in the investment can be efficiently and accurately derived

The other sections are organized as follows. Section 2 demonstrates the model. Section 3 presents the numerical analyses, the values of natural resource investment, and the options embedded in this investment project will be derived. Section 4 concludes.

2. The model

In this section the stochastic processes of state variables are first proposed. The LSM and DOS will then be discussed. Finally, the options embedded in a natural resource investment, or as we can call them the managerial flexibilities, will be further analyzed.

2.1 The stochastic variable processes

It is vary impractical and will cause serious biases by assuming a fixed interest rate environment for the valuation of an investment project with a maturity of more than ten years. To construct a practical Real Option model, this study integrates four stochastic variables into a simulation approach. The stochastic variable includes: copper prices, convenience yield, production costs, and the interest rate. Under the risk-neutral measure, the four stochastic variable processes are as follows

$$dS = S(r-\delta)dt + S\sigma_S dz_S \tag{1}$$

$$d\delta = [\kappa_\delta(\alpha_2 - \delta) - \lambda_\delta]dt + \sigma_\delta dz_\delta \tag{2}$$

$$da = a(\alpha_a - \lambda_a)dt + a\sigma_a dz_a \tag{3}$$

$$dr = \kappa_r(\mu_r - r)dt + \sigma_r\sqrt{r}dz_r\ . \tag{4}$$

Equation (1) is the copper price stochastic process which follows a geometric Brownian motion. Equation (2) is a convenience yield stochastic process, and it follows an Ornstein-Uhlenbeck (O-U) process; for the sake of convenience, the yield usually possesses a mean-reverting characteristic. Equation (3) is a production cost stochastic process. Equation (4) is an interest rate stochastic process, and it follows a CIR (Cox, Ingersoll, and Ross, 1985) term structure model. The meanings of the parameters in Equations (1) to (4) are as follows. Term S is the copper price, δ is the convenience yield, σ_S is the volatility of copper prices, κ_δ is the convergence speed of convenience yield, α_δ is the long-term mean of convenience yield, σ_δ is the volatility of the convenience yield, a is copper mine production costs, α_a is the growth rate of production costs, σ_a is the volatility of production costs, r is the risk-free interest rate, κ_r is the convergence speed of the risk-free interest rate, μ_r is the long-term mean of the risk-free interest rate, σ_r is the volatility of the risk-free interest rate, and dz_i is a standard Wiener process, $i = S, \delta, a, r$. Each standard Wiener process, dz_i and dz_j , are correlated, and the correlation coefficient is ρ_{ij} , where $i, j = S, \delta, a, r$. Terms λ_δ and λ_a are the market prices of the risk of convenience yield and production costs, separately.

2.2 The LSM

The key point of LSM is that it can efficiently identify the conditional expected holding value of contingent claims at times before expiration by a simple regression from the maturity day backward to the starting time. Once the holding values at each time spot of the different paths are identified, we can compare the values of early exercise and holding value at any time on each path, and an optimal early exercise strategy can thus be made.

The objective of the LSM is to provide a pathwise approximation to the optimal stopping rule that maximizes the contingent claim's value. Let $C(\omega, s; t, T)$ denote the path of cash flow generated by the option, conditional on the option not being exercised at, or prior to, time t and on the option holder following the optimal stopping strategy for all $s, (t \le s \le T)$. Term ω represents a simple path and we approximate the American-style option by discretization. It is assumed that the American option can only be exercised at K discrete times, where $0 < t_1 \le t_2 ... \le t_K = T$, such that when K is sufficiently large, the LSM can be used to approximate the theoretical American option value.

At any time t_K during times 0 and T, the investor knows the immediate exercise value, however, he does not know the holding value. The holding value depends on the cash flows $C(\omega, s; t, T)$ by taking the expectation of its value with respect to the risk-neutral pricing measure *Q*. The holding value $F(\omega; t_K)$ can now be expressed as:

$$F(\omega; t_K) = E_Q[\sum_{j=K+1}^{N} \exp(-\int_{t_K}^{t_j} r(\omega, s)ds)\, C(\omega, t_j; t_{K,} T) \mid \Im_{t_K}] \tag{5}$$

where $r(\omega, s)$ is the risk-free discount rate, and the expectation is taken conditional on the information set $\Im_{t_K}$ at time t_K. When the holding value $F(\omega; t_K)$ is identified, the problem of optimal early exercise is just to compare the immediate exercise value and with this conditional expectation holding value.

In order to identify the conditional expectation function at $t_{K-1}, t_{K-2}, ..., t_1$, we have to work backwards from maturity day t_K, since there is only one certain boundary condition at t_K. At time t_{K-1} we assume $F(\omega; t_{K-1})$ can be represented as a linear combination of a countable set of $\Im_{t_{K-1}}$-measurable basis functions. There are many basis functions that can be used as Longstaff and Schwartz (2001) have mentioned, and after our experiments, this study used the simplest basis function - the power of state variable - since it is easy to implement and can also give accurate results. The conditional expectation function is:

$$E[Y_t \mid S_t] = \beta_{0t} + \beta_{1t} S_t + \beta_{2t} S_t^2 + \beta_{3t} S_t^3, \tag{6}$$

where Y_t denote the corresponding discounted cash flows received at time t, and S_t denote the stock prices at time t for each paths. As Table 1 shows, pricing an American put option by LSM with simple basis functions can be very accurate compared with the tree method.

[Insert Table 1 about here]

2.3 The DOS

Since the mine reserves will influence the mine value directly, it is worth noting how to develop a practical model that can integrate the information of mine reserves. This study develops a DOS approach that combines the LSM and dynamic programming techniques and can efficiently value natural resource investments with properties of multi-variables, early exercise, finite reserves, and several embedded options.

2.3.1 DOS cash flow analyses

This study assumes the copper mine is developed, Q_t means that the copper reserves at time t, the initial time reserve Q_1, is known and finite. The normal mining speed is set to be q_1 (pounds/year), as Q_1 is known, and the mining years are certain, n=Q_1/q_1. However, the mine holder always has an option to accelerate the mining speed, as he thinks the copper prices are appealing and high enough. This paper also assumes that the mine holder has an Acceleration option to increase the mining speed to q_2, and $q_2 > q_1$.

The mine holder has three options at each decision point: {temporary closure, open, abortion}. If the mine holder decides to abort the mine, a fixed cost C_3 will incur. Furthermore, once the mine is aborted, it cannot return back to the other condition. This means the abortion decision is irreversible.

Suppose the last period condition of the mine is {open}, but the mine holder decides to temporarily close the mine. The mine holder should pay a fixed closure fee C_1 and a maintenance fee C_4. As the copper price rises to a higher level, the mine holder can switch the mine condition from {temporary closure} into {open}. If the last period condition of the mine is {closed}, but the mine holder decides to reopen the mine, the mine holder should pay a fixed reopening fee C_2. This study assumes the legal production maturity N to be 15 years, N>n. The mine holder loses the ownership of the mine and stops mining.

At time t, the value of the copper mine before real estate tax is H_t, and t_2 means the real estate tax rate. The value of the copper mine after the real estate tax would be $H_t(1-t_2)$, where $H_t(1-t_2)$ is the function of S_t, a_t, C_1, C_2, C_3, C_4, t_1, t_2, Q_t, and j_{t-1}. Term j_{t-1} is an indicator function, j_{t-1}=1 means the copper mine is {open} at the last period time t-1, and j_{t-1}=0 means the copper mine is {closed} at the last period time t-1.

If the mine is {open} at the last period (t-1), then the cash flow generated at this period (t) is indicated as Equation (7) shows,

$$CF_t(j_t = 1) = q_1(S_t - a_t)(1-t_1) - t_2 H_t - c_2(1 - j_{t-1}). \tag{7}$$

As the mine is {open} at time t-1, then j_{t-1}=1; otherwise, if the mine is {closed}, j_{t-1}=0. The value H_t can be derived by estimating the expected present value of the cash flows generated after time t under an optimal decision making.[2]

Suppose the mine is {closed} at last period time t-1. The cash flow generated at this period (t) is then indicated as Equation (8) shows,

$$CF_t(j_t = 0) = -t_2 H_t - c_1 j_{t-1} - c_4. \tag{8}$$

2.3.2 DOS steps

The steps for valuing a copper mine by applying DOS are as follows.

【Step 1】

One first generates the future values of the four state variables according to Equations (1) to (4). This study assumes that the mine holder makes the decision of changing the mine condition once a year. After a simulation trial, the mine value at each period can be derived by Equation (9).[3]

$$BRETCF_t(j_t = 1) = q_1(S_t - a_t)(1-t_2). \tag{9}$$

At initial time t=1, the initial copper reserve is known to be Q_1, and the mining maturity will be n=Q_1/q_1 years under a normal mining speed q_1. If the mine holder makes a decision each year, then there will be m (=n+1) possible reserve conditions that exist. At time t=2, only two possible reserve conditions exist. (Condition 1) If the mine is {closed} at time t=1, then the mine holder can still mine for n years. (Condition 2) If the mine is {open} at time

[2] The setting is referred to as C-R (1999).
[3] This study refers the setting according to C-R (1999). BRETCF = before real estate tax cash flow.

t=1, then the mine holder can only mine for n-1 years. There will be two reserve conditions that happen at time t=2: {n-1, n}. According to the same rule, three possible reserve conditions exist at time t=3: {n-2, n-1, n}. If the mine reserve is known to be 15 years (n=15) and the concession to be 45 years, then there will be 16 possible reserve conditions at time t=15: {0, 1, …, 15}.

[Insert Table 2 about here]

【Step 2】

Compute the mine value at maturity according to the boundary conditions. Assume the concession is N years, at the last year time of t=N. If the mine condition was {open} at the last period time N-1, then the mine value would be

$$V(t = N, m) = \max\{-C_3,\ BRETCF_N(1-t_2),\ CF(j_N = 0)\},\ m = 0,1,...,n. \tag{10}$$

At the last period, the mine holder can make a best decision from the three options: {temporary closure, reopen, abort}. It is worth noting that the {reopen} or {open} option can only used as the mine still has reserves. Thus, the mine reserve will influence the value of the copper mine directly.

At the last year time of t=N, if the mine condition was {closed} at the last period time of N-1, then the mine value would be

$$W(t = N, m) = \max\{-C_3,\ (BRETCF_N - C_2)(1-t_2),\ CF(j_N = 0)\},\ m = 0,1,...,n. \tag{11}$$

The $CF(j_N = 0)$ in Equations (10) and (11) is different. Since in Equation (10), the last period is {open}, if the mine holder decides to temporarily close the mine, then $CF(j_N = 0)$ should include a fixed closure fee C_1 and a maintenance fee C_4, and then $CF(j_N = 0)$ = $-(BRETCF_N)t_2 - C_1 - C_4$. In Equation (11), since the last period is {closed}, if the mine holder decides to reopen the mine, then $CF(j_N = 0)$ = $-(BRETCF_N)t_2 - C_4$.

As the reserves will influence the value of a copper mine, the model should consider all possible reserve conditions at the last period. By Equations (10) and (11), the mine holder can evaluate the real mine value and make a best decision.

【Step 3】

By integrating the LSM and dynamic programming techniques, and the values of mine at maturity derived from Step 2, a mine holder can value at each period for every possible reserve condition backward from the maturity time. At time period 1 to N-1, $0 \le t \le N-1$, the mine value is $V(t,m)$ if the last period is {open}, and is $W(t,m)$ if the last period is {closed}. Compute backward from maturity t=N to initial time t=1. The DOS can easily derive $V(t=1, m=n)$ and $W(t=1, m=n)$, and they are the answers for the real value of natural resource investments. Terms $V(t,m)$ and $W(t,m)$ are denoted as

$$V(t,m) = \max\{-C_3, BRETCF_t(1-t_2) + E_t[V(t+1, m-1)], CF(j_N = 0) + E_t[W(t+1,m)]\} \quad (12)$$

$$W(t,m) = \max\{-C_3, BRETCF_t(1-t_2) + E_t[V(t+1, m-1)] - C_2, CF(j_N = 0) + E_t[W(t+1,m)]\} \quad (13)$$

where $E_t[V(t+1, m-1)]$ means the expected value of the mine at time t+1 from time t, the mine is {open} at time t, and the mine reserve condition is m-1. Term $E_t[W(t+1,m)]$ means the expected value of the mine at time t+1 from time t, the mine is {closed} at time t, and the mine reserve condition is m. At each period in this model, LSM will be applied to compute the holding value of the mine. Under the four state variables' framework, this paper uses Equation (14) to derive the holding value.[4]

$$\begin{aligned} E[Y_t \mid S_t, \delta_t, a_t, r_t] = {} & \beta_0 + \beta_{1t} S_t + \beta_{2t} S_t^2 + \beta_{3t} S_t^3 + \beta_{4t} \delta_t + \beta_{5t} \delta_t^2 + \beta_{6t} \delta_t^3 \\ & + \beta_{7t} a_t + \beta_{8t} a_t^2 + \beta_{9t} a_t^3 + \beta_{10t} r_t + \beta_{11t} r_t^2 + \beta_{12t} r_t^3 \\ & + \beta_{13t} S_t \delta_t + \beta_{14t} S_t a_t + \beta_{15t} S_t r_t + \beta_{16t} \delta_t a_t + \beta_{17t} \delta_t r_t + \beta_{18t} a_t r_t \end{aligned} \quad (14)$$

Assume that the reserve condition is {m} at time N-1. The possible reserve conditions at time N can only be {m} or {m-1}. In other words, the value of the mine at time N-1 with reserve condition {m} can be derived from the value of the mine at time N with reserve conditions of {m} and {m-1}.

2.4 The embedded options

A mine holder always has an option to accelerate the mining speed, when he thinks the copper prices are high enough. This study defines this managerial flexibility as an Acceleration option. It is assumed that the mine holder can increase the mining speed to q_2, and $q_2 > q_1$. Contrary to the previous settings, a mine holder will have four options at the times when he wants to make a decision; they are {temporary closure, open at q_1, open at q_2, abortion}. Considering the Acceleration option, the possible reserve conditions will be as Table 3 shows.

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[4] Considering the non-linearity of option prices and under a four-state variables model, this study takes a third degree power polynomial function for each variable as the basis function. To capture the correlations of each two variables, 6 additional cross terms of each variable are considered put into the basis functions.

[Insert Table 3 about here]

3. Numerical analyses

This section firstly analyzes the valuation of a copper mine under the conditions that the reserve is infinite and the mine holder can only keep mining (without managerial flexibilities). The valuations of a copper mine with embedded options are then presented. The embedded options of a copper mine can be extracted. Finally, interest rate effects to the values of a copper mine are also discussed.

The frequency of making a decision is once a year; that is, the mine holder can make the decision of switching the mine condition at a specific time during a year. The mine holder can temporary close the mine which is previously {open}, if he thinks the copper is too low, and closure is better than being open. Furthermore, if the copper price drops low enough, then the mine holder can even abort the mine. The base parameters used in this paper are shown in Table 4.

[Insert Table 4 about here]

3.1 The value of a copper mine and the values of embedded options

This section assumes that the mine holder can only keep mining (without managerial flexibilities) for 15 years. The values of a copper mine and the values of embedded options are presented at Table 5.

[Insert Table 5 about here]

From Table 5, the differences between B&S (1985), C-R (1999), and DOS are significant as the copper prices are below 0.7 $/pound. According to the cost parameters, the production cost is 0.5 $/pound, if the initial copper price is 0.3 $/pound. The mine holder should keep mining for 15 years without the options to temporarily close, reopen, or abort. It is more reasonable that the mine holder will lose money and the mine value will become negative.[5]

The DOS model combines the LSM and dynamic programming techniques, which are different from the model of C-R (1999). DOS can efficiently integrate the reserve conditions into the values of copper. On the contrary, the C-R (1999) model seems to have the disadvantages of undervaluing a copper mine under some conditions. From Figure 1,

[5] C-R (1999) pointed out this in his footnote 15.

we can easily recognize that the C-R (1999) model presents a non-linear relation between mine values and copper prices, which should not happen under a mine without any options. The DOS model outperforms the C-R (1999) model from this point.

[Insert Figure 1 about here]

From column 3 of Table 4, as the copper price is significantly lower than the production cost (ex. S=0.3 $/pound, and a=0.5 $/pound) the embedded options are valuable (11.46 Million $). This is because as the mine is {open} at the previous time, to reduce loses, the mine holder will use the {temporary closure} option. As the copper price rises above a certain level, the mine holder can switch the mine condition into {open} by using the {open} option. The possibility of options being used is great as the copper price is at a low level. That is why the option is so valuable under such a condition (when the copper price is 0.3 $/pound, the value of the embedded option is about 8.36 times[6] the value of the copper mine!). On the contrary, if the initial mine is {open} and the copper price is larger than the production cost, then the mine holder will tend to remain {open}, and so the embedded option is less valuable (only 0.85 Million$).

3.2 The value of a mine under several variables and the Acceleration options values

This section analyzes the mine value under several variables and also discusses the values of Acceleration options under different conditions.

[Insert Table 6 about here]

From Table 6, the mine value is less than that of one variable in Table 5, and the reason could be the high long-term convenience yield. We can also tell from Table 6 that the Acceleration option values are more valuable as the copper price is higher. The result is very intuitive, since when the copper price is higher, the mine holder will tend to increase the mining speed, and so the Acceleration option will be triggered with greater possibility.

3.3 Interest rate effects

This section analyzes the effects of the interest rate to the value of a copper mine under different conditions. Firstly, this study discusses the effects of interest patterns, such as upward sloping, flat, and downward sloping. Furthermore, convergence speed and

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[6] According to Table 4, as the initial copper price is 0.3$/pound and the mine is previously {open}, the mine value is 1.37 Million$, whereas the mine value is -10.09 Million$ under the condition without options and the mine holder should keep mining. The embedded option value extracted would be 1.37-(-10.09)= 11.46 Million$. The embedded value is roughly (11.46/1.37=)8.36 times the value of the copper mine.

volatility effects to the value of the copper mine are also discussed.

[Insert Table 7 about here]

From Table 7 we can tell that, as the future interest rate is upward sloping, the mine value will increase. The differences of the mine value between different interest rate patterns are significant. In other words, when pricing a long-term investment project, a stochastic interest rate model is necessary, and we should cautiously estimate the interest rate parameters; otherwise, the investment value will be significantly biased.

4. Conclusions

A natural resource investment valuation is a complex multi-variable American Real Option pricing problem. This study combines LSM and dynamic programming techniques and develops a DOS model, which can practically solve an investment project with multi-variables, early exercise, finite reserves, and several embedded options. From the numerical results, this paper shows that the DOS can efficiently and accurately value a copper mine under different conditions. Furthermore, we can evaluate the importance of each state variable parameter and the values of options embedded in an investment.

We learn from this paper that the embedded options could be very valuable, and these results cannot be derived by applying the traditional NPV approach. The results of this study also demonstrate the powers of the Real Option and DOS approaches. It goes without saying that the DOS developed herein could also be applied into other types of Real Option investment pricing problems.

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Table 1. Pure American put option value comparisons between tree method and LSM

S	σ_S	Tree method (1,000 stages) (1)	LSM (Standard error) (2)	Differences between Tree method and LSM [(2)-(1)]/(1)
38	0.2	3.7512	3.7443 (0.0141)	-0.1834%
38	0.4	7.6763	7.6718 (0.0348)	-0.0585%
40	0.2	2.8895	2.8849 (0.0164)	-0.1578%
40	0.4	6.9225	6.9209 (0.0339)	-0.0219%
42	0.2	2.2170	2.2142 (0.0147)	-0.1267%
42	0.4	6.2515	6.2492 (0.0337)	-0.0370%

1. Strike price K=40, r=0.06, T=2 year.
2. We compute the differences between the tree method and LSM by taking the value derived from the tree method as a base parameter.
3. An option is allowed 50 exercise times per year with LSM.
4. An option price of LSM is derived with 50,000 paths, and the standard error of the option price is derived from 100 option prices of different random numbers seeded.

Table 2. The possible reserves for future years without Acceleration Options

Year (t)	**1**	**2**	**3**	**…**	**15**	**16**	**17**	**…**	**43**	**44**	**45**
	15	15	15	…	15	15	15	…	15	15	15
		14	14	…	…	…	…	…	…	14	14
			13	…	…	…	…	…	…	…	13
m				…	…	…	…	…	…	…	…
				…	…	…	…	…	…	…	…
					1	1	1	…	1	1	1
						0	0	…	0	0	0

Table 3. The possible reserves for future years with Acceleration Options

Year (t)	**1**	**2**	**3**	**…**	**8**	**9**	**10**	**…**	**43**	**44**	**45**
	15	15	15	…	15	15	15	…	15	15	15
		14	14	…	…	…	…	…	…	14	14
		13	13	…	…	…	…	…	…	…	13
m			12	…	…	…	…	…	…	…	…
			11	…	…	…	…	…	…	…	…
					1	1	1	…	1	1	1
						0	0	…	0	0	0

Table 4. Base parameters (annual)

	Notation	Description	Base value
Mining parameters	q_1	Normal mining speed	10,000,000 pounds
	q_2	Fast mining speed	20,000,000 pounds
	Q_0	Initial mine reserves	150,000,000 pounds
Cost and taxes parameters	a_0	Mining cost	0.5 $
	σ_a	Volatility of mining cost	0.2646
	C_1	Cost of temporary closure	200,000 $
	C_2	Cost of reopening	200,000 $
	C_3	Cost of aborting	0
	C_4	Cost of maintenance	500,000 $
	t_1	Income taxes	50%
	t_2	Real estate taxes	2%
Copper price parameter	σ_S	Volatility of copper price	0.2828
Convenience yield parameters	δ_0	Initial convenience yield	0.01
	α_2	Long-term convenience yield	0.248
	κ_δ	Convergence speed of convenience yield	1.156
	σ_δ	Volatility of convenience yield	0.28
	λ_δ	Market price of risk of convenience yield	0.256
Interest rate parameters	r_0	Initial short rate	0.05
	κ_r	Convergence speed of short rate	0.20
	μ_r	Long-term short rate	0.05
	σ_r	Volatility of short rate	0.08

1. The frequency of making a decision is once a year.
2. The correlation of copper prices and convenience is referred to Castillo-Ramirez (1999), which set ρ_{sc}=0.818, and the other correlations are set to be 0. The drift term of mining cost under risk-neutral is $(\alpha_a - \lambda_a)$ =0.
3. The interest rate parameters of CIR are referred to Chen et al. (1992).

Table 5. The values of the copper mine and embedded options

Initial copper price ($/pound)	The value of (1) DOS	Copper mine C-R (1999)	Without options B&S (1985)	Copper mine values (2)	The value of embed options: temporary closure, reopen, and abort the mine (3)=(2)-(1)
0.3	-10.09	-19.74	0.38	1.37	11.46
0.4	-4.07	-7.83	3.12	3.84	7.91
0.5	1.95	2.04	7.22	7.35	5.40
0.6	7.98	7.99	12.01	11.67	3.69
0.7	13.99	13.95	17.19	16.61	2.62
0.8	20.02	19.91	22.61	21.90	1.88
0.9	26.04	25.87	28.18	27.36	1.32
1.0	32.06	31.82	33.85	32.91	0.85

1. The initial condition of the mine is “open”.
2. “The value of copper mine without options” means the mine holder can only keep mining for 15 years, even if the copper price is lower enough, he cannot temporarily close or abort the mine.
3. C-R (1999) means the values of the copper mine in Castillo-Ramirez (1999).
4. B&S (1985) means the value of the copper mine in Brennan and Schwartz (1985).
5. The copper mine values of column (2) are derived by using DOS. The mine values include the embedded options.
6. The unit of the mine values is Million $.

Table 6. The values of the copper mine and the embedded options

The condition of last period	Copper mine values without Acceleration Options		Copper mine values with Acceleration Options		The values of Acceleration Options	
	Open (1)	Closed (2)	Open (3)	Closed (4)	Open (5)=(3)-(1)	Closed (6)=(4)-(2)
Initial copper price ($/pound)						
0.3	1.4316	1.4596	1.5685	1.5888	0.1369	0.1292
0.4	3.1705	3.1842	3.4245	3.4011	0.2540	0.2169
0.5	5.6334	5.6126	6.0347	5.8848	0.4013	0.2722
0.6	8.5732	8.5100	9.3009	8.9395	0.7277	0.4295
0.7	11.8812	11.7825	12.9734	12.3460	1.0922	0.5635
0.8	15.4991	15.3704	16.9921	16.0442	1.4930	0.6738
0.9	19.3837	19.2332	21.2625	19.9936	1.8788	0.7604
1.0	23.3784	23.2141	25.6842	24.0756	2.3058	0.8615

Note: The settings follow Table 4

Table 7. Interest rate effects

Short rate pattern	r_0	μ_r	κ_r	σ_r	The mine condition of the last period	
					Open	Closed
Downward sloping	0.05	0.02	0.2	0.040	16.0561	14.9810
				0.085	16.0901	15.0204
				0.130	16.2808	15.2140
			0.6	0.040	15.1728	14.1226
				0.085	15.1881	14.1416
				0.130	15.3324	14.2913
Flat	0.05	0.05	0.2	0.040	18.1600	17.1640
				0.085	18.1651	17.1668
				0.130	18.2540	17.2535
			0.6	0.040	18.2319	17.2403
				0.085	18.2195	17.2283
				0.130	18.2468	17.2567
Upward sloping	0.05	0.08	0.2	0.040	20.2836	19.3599
				0.085	20.2423	19.3120
				0.130	20.2744	19.3351
			0.6	0.040	21.2167	20.2766
				0.085	21.2116	20.2652
				0.130	21.2131	20.2663

1. Initial copper price =0.8 ($/pound).
2. The mine values of this table are derived by considering Acceleration Options.
3. The other settings follow Table 4.

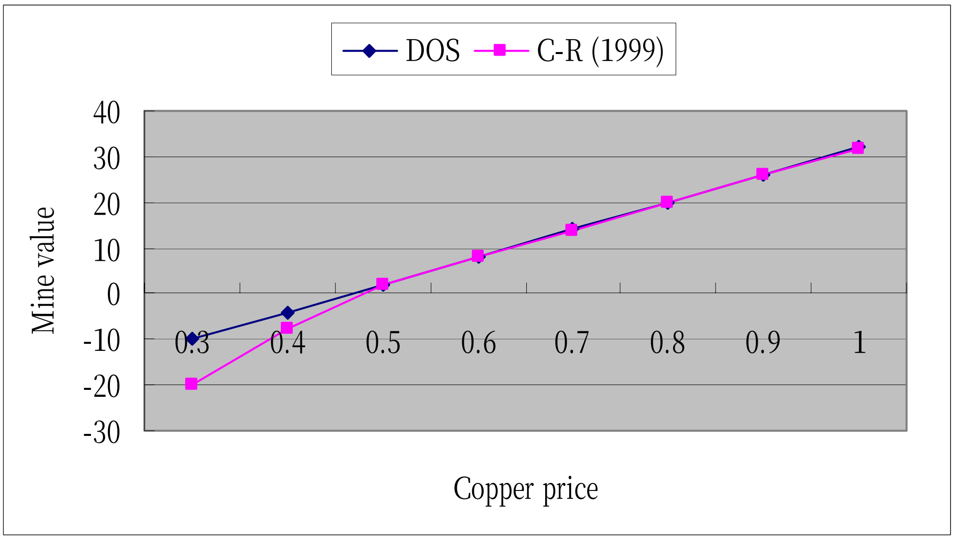

Figure 1. Copper mine value comparisons between DOS and C-R (1999)