Rules of Thumb in Real Options Applications

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Abstract

Real options theory suggests that managerial flexibility embedded within irreversible investments can account for a significant value in project valuation. Although the argument has become the dominant focus of capital investment theory over decades, yet recent survey literature in capital budgeting indicates that corporate practitioners still do not explicitly apply real options in investment decisions. The research purpose is therefore two-fold. First, we explore how real options decision criteria can be transformed into equivalent capital budgeting criteria under the consideration of uncertainty. Second, we propose heuristic investment rules in terms of capital budgeting practices to proxy for the inclusion of real options valuation and demonstrate how rules of thumb under incomplete information can approximate real options into capital budgeting techniques, providing the results close to optimality.

In this paper, the equivalent expressions of capital budgeting techniques under uncertainty are first derived, given various stochastic process assumptions such as geometric Brownian motion (GBM), mixed diffusion-jump (MX), and mean reversion (MR). These equivalent valuation techniques can be readily decomposed into conventional investment rules and “option impacts”, the latter of which describe the impacts on optimal investment rules with the option value considered. The loss functions of adopting a suboptimal investment rule are also analyzed.

Heuristic investment rules are then proposed to approximate real options valuation. The heuristic hurdle rate under a GBM, MX, or MR process can be conveniently derived from a “4-8 rule”, “3-9 rule”, or “2-10 rule”, respectively, with the information of volatility and discount rate. The simulation of random sampling reveals that these heuristic investment rules provide a robust performance close to the optimal rules with forecast errors minimized. The results indicate that these heuristic rules significantly improve prediction reliability compared to rules of thumb suggested by the related studies.

Keywords: real options, capital budgeting, geometric Brownian motion (GBM), mixed diffusion-jump, mean reversion, heuristic investment rules

JEL Classification: D81, G31

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1. Introduction

The literature on capital budgeting suggests two important facts in theory and practice: first, conventional capital budgeting techniques are shown to have various theoretical shortcomings yet still have widespread applications in practice\(^1\); second, real options techniques are often considered as relatively sophisticated analysis tools, yet most firms do not make explicit use of real options techniques to evaluate capital investments.\(^2\)

This paper aims to bridge the theory-practice gap by translating real options valuation into current capital budgeting practices. The research purpose is therefore two-fold. On one hand, we would like to explore how real options decision criteria can be transformed into equivalent capital budgeting criteria such as NPV, profitability index, hurdle rate, and (discounted) payback under the consideration of uncertainty. On the other hand, we would like to propose heuristic investment rules in terms of capital budgeting practices to proxy for the inclusion of real options valuation. We then demonstrate how rules of thumb under incomplete information can approximate managerial flexibility into capital budgeting techniques across different types of projects and provide results which are close to optimal investment decisions.

The literature on bridging the real options approach and capital budgeting techniques are seen in Dixit (1992), Ingersoll and Ross (1992), Boyle and Guthrie (1997), McDonald (1999), and Wambach (2000). Dixit (1992) suggests the optimal investment rule with the option of waiting to invest can be expressed as a constant hurdle rate for time-homogeneous cash flows in an infinite horizon framework. Ingersoll and Ross (1992) argue that management should set corporate hurdles rates above the cost of capital to recognize the gains of waiting. McDonald (1999) investigates whether arbitrage investment criteria such as hurdle rates and profitability indexes can proxy for the use of real options techniques. He finds that under the GBM assumption, a profitability index of 1.5 or alternatively a hurdle rate of 20% can


\(^2\) See Busby and Pitts (1997) and Graham and Harvey (2001).
provide a reasonable approximation to the optimal trigger across different characteristics of projects. Along the line of research, Boyle and Guthrie (1997) and Wambach (2000) propose a similar approach to equivalent investment rules for payback and hurdle rate under the option of waiting, assuming that the underlying process follows a GBM and that projects have time-homogenous cash flows. Compared to standard capital budgeting criteria, the modified investment rules tend to have a lower payback trigger or a higher hurdle rate trigger.

The rest of the paper is organized into the following sections: Section 2 illustrates how real options valuation can be approximated by capital budgeting techniques. Specifically, we show that conventional capital budgeting techniques can explicitly integrate the value of deferment options into become modified investment rules under alternative stochastic processes. Section 3 demonstrates how to estimate the opportunity cost of adopting a suboptimal investment policy. We show that the loss function of taking a suboptimal investment policy is asymmetric for a GBM or a mixed diffusion-jump process and approximately symmetric for a mean-reverting process. Section 4 develops the heuristic decision rules that can be applied to a wide range of investment projects given a specific stochastic process. Section 5 presents a simulation procedure of random sampling to test the robustness of the heuristic investment rules. The concluding remarks are given in Section 6.

2. Real Options and Capital Budgeting Criteria

The standard investment theory under uncertainty is to explore the optimal timing to pay an investment cost, $I$, in return for an irreversible project whose value, $V$, is a major source of uncertainty. Since the investment opportunity is normally assumed to exist infinitely in order to derive closed-form solutions, the investment timing problem turns out to be the optimal stopping problem in searching for the optimal investment trigger, $V^\ast$. In the presence of options of waiting, $V^\ast$ is found to be greater than $V$.\(^3\) In this section, we first derive modified capital budgeting criteria, e.g. NPV, profitability index, hurdle rate, and payback, in the explicit expressions of $V^\ast$ without specifying any particular stochastic process. We then assume a specific stochastic process such as GBM, mixed diffusion-jump, and mean reversion, to examine how the forms of modified capital budgeting criteria are influenced by the stochastic process.

\(^3\) See Brennan and Schwartz (1985), McDonald and Siegel (1986), Dixit (1989), Pindyck (1991), Ingersoll and Ross (1992), and others.
2.1 Modified Capital Budgeting Criteria

In conventional capital budgeting, the NPV rule states that the investment should be undertaken when the NPV is greater than zero. However, this criterion only works in the absence of real options. Under uncertainty, when the project is allowed to delay, the modified NPV rule should justify the loss of option of waiting when launching the investment opportunity. This means that the actual costs of initiating the project are not only the investment cost but also the opportunity cost due to the loss of options. We use the superscript * to denote the modified investment rules. Therefore, the new NPV rule under the consideration of options of waiting should be modified as shown below:

\[ NPV^* = V^* - I - F(V^*) \]  

(1)

where \( F(V^*) \) denotes the option value when project value equals \( V^* \).

It is important to note that the modified NPV rule should be less than the conventional NPV rule for the reasons that \( F(V^*) \) should be greater than zero when \( V^* > V \).

Profitability Index, denoted by \( \Pi \), is defined as the benefit/cost ratio or Tobin’s “q” ratio associated with the project. In the absence of managerial flexibility, the project is undertaken when PI is greater than 1. Since the project is taken at the point of \( V^* \) under the consideration of real options, the new PI rule should be changed as follows:

\[ \Pi^* = \frac{V^*}{I} \]  

(2)

\( \Pi^* \), by definition, can be interpreted as a unit optimal trigger and is greater than 1 due to \( V^* > I \).

Now we suppose the project can generate infinite cash flows once the project is undertaken. Expressed by instantaneous time-homogeneous cash flows\(^4\), \( \pi \), the

\(^4\) Time-homogenous cash flows can be seen as an equivalent transformation of non-time-homogenous cash flows, given other parameters held the same. Suppose there are project A and B under consideration. Both projects have two years of time horizon. These two projects are similar in every way except that the former has cash flows of $10 in both year 1 and 2 while the latter has $11 in year 1 and $8.9 in year 2. At the discount rate of 10%, they have exactly the same project value. Therefore, time-homogenous cash flows can represent non-time-homogenous cash flows with the advantage of easy mathematical treatment due to the linearity between project value and cash flows. With this
project value thus takes the form of the classic Gordon model:

\[
V = E\left(\int_0^\infty \pi_t e^{(\alpha - \mu)s} ds\right) = \frac{\pi}{\mu - \alpha}
\]  

(3)

where \(\alpha\) and \(\mu\) denote growth rate and discount rate, respectively, and \(\mu > \alpha\).

The convenience of the assumption of time-homogeneous cash flows provides a linear relationship between \(V\) and \(\pi\), which ensures that \(V\) and \(\pi\) follow the same stochastic process with the same drift and volatility. Consequently, \(\pi\) is still allowed to fluctuate at the next instant as new information arrives. Equation (3) thus can be rearranged as follows:

\[
\pi = V(\mu - \alpha)
\]

(4)

In capital budgeting, the hurdle rate rule states that the project should be undertaken when the internal rate of return exceeds the arbitrary hurdle rate. With the time-homogeneous cash flows assumption, the hurdle rate rule can be transformed into the equivalent cash flow rule. Let \(\gamma\) denote hurdle rate. By definition, the hurdle rate rule must satisfy the following relationship:

\[
\pi = I(\gamma - \alpha)
\]

(5)

By equating Equation (4) and (5), we obtain the expression of \(\gamma\) as follows:

\[
\gamma = \Pi(\mu - \alpha) + \alpha
\]

(6)

If the project is taken at the point of \(\Pi = 1\), then the hurdle rate is equal to the discount rate. However, if the project is taken at any point of \(\Pi > 1\), the hurdle rate is greater than the discount rate.\(^5\)

\(^5\) Proof: Suppose the project is taken at \(\Pi = 1 + z > 1\), where \(z\) denotes a number representing some arbitrary decision rule such that \(z > 0\). By substituting \(\Pi = 1 + z\) into \(\gamma = \Pi(\mu - \alpha) + \alpha\), we obtain \(\gamma = \mu + z(\mu - \alpha)\). Since \(z > 0\) and \(\mu > \alpha\) by design, we know \(\gamma > \mu\).
To derive the modified investment rules for hurdle rate and cash flows, we first substitute Equation (3) into Equation (2) and rearrange the terms for \( \pi^* \). The modified cash flow rule, \( \pi^* \) can be expressed in terms of \( V^* \) or \( P^* \) as follows:

\[
\pi^* = V^*(\mu - \alpha) = \Pi^*I(\mu - \alpha) \tag{7}
\]

With \( \Pi^* \) taking the place of \( \Pi \) in Equation (6), the modified hurdle rate rule can be derived as follows:

\[
\gamma^* = \Pi^*(\mu - \alpha) + \alpha \tag{8}
\]

Note that the modified investment rules of \( \pi^* \) and \( \gamma^* \) should be greater than the conventional investment rules of \( \pi \) and \( \gamma \) in the existence of positive option values.

The payback period rule has been one of the commonly used capital budgeting criteria.\(^6\) Payback period is referred to as the time horizon in which the sum of expected cash flows returns the investment cost. In general, the payback rule favors the projects with shorter payback periods although the potential benefit of this concept is heavily disputed in the literature. One argument in support of the use of the payback rule is that to those firms which are short of capital, the payback rule may help recover the initial investment cost earlier. If we express payback period by time-homogeneous cash flows \( \pi \), the payback rule must satisfy the following condition:

\[
I = \pi \int_0^P e^{\alpha s} ds \tag{9}
\]

where \( P \) denotes the payback period.\(^7\)

Solving Equation (9) for \( P \), we have the payback trigger as shown below:

---


\(^7\) If \( \pi \) is non-time-homogeneous, the payback rule satisfies the following condition:

\[
I = \int_0^P \pi e^{\alpha s} ds
\]
To derive the modified payback rule under the consideration of options of waiting, we replace the profitability index $\Pi$ in Equation (10) with the modified profitability index trigger $\Pi'$. The new payback rule now becomes:

$$
P' = \begin{cases} 
\ln \left[ 1 + \frac{\alpha}{\Pi' (\mu - \alpha)} \right], & \alpha \neq 0 \\
\frac{1}{\Pi' \mu}, & \alpha = 0
\end{cases}
$$

Since $\Pi'$ is greater than $\Pi$, it is easy to see that the modified payback period is shorter than the conventional payback period. This means that under uncertainty and flexibility, one should defer investment until the market turns out to be favorable such that the project has a payback period not only shorter than the conventional payback period, $P$, but also the modified payback period, $P'$.

One of the major critiques regarding the payback criterion is that the use of payback often ignores the time value of cash flows. The justification for this drawback is to introduce discount rate to become discounted payback, which is defined as the time horizon over which the present value of total expected cash flows equals the investment cost. Thus, the discounted payback criterion must satisfy the following condition:

$$
I = \pi \int_0^{P^D} e^{(\sigma - \mu)s} ds
$$

where $P^D$ denotes the discounted payback period.

Equation (12) is solved as follows:
\[ P^D = \frac{\ln \left[ 1 + \frac{1}{\Pi - 1} \right]}{\mu - \alpha} \]  \hspace{1cm} (13)

With \( \Pi^* \) in Equation (2) in the place of \( \Pi \) in Equation (13), the modified discounted payback trigger is given below:

\[ P^{D*} = \frac{\ln \left( 1 + \frac{1}{\Pi^* - 1} \right)}{\mu - \alpha} \]  \hspace{1cm} (14)

We have demonstrated how real option approach can be integrated into conventional capital budgeting rules. Equation (1), (2), (7), (8), (11), and (14) are the modified investment rules where the options of waiting are in place. To rationalize the benefits of options of waiting, management should defer investment until the project has a larger NPV than \( NPV^* \), a higher profitability index than \( \Pi^* \), a higher rate of returns than \( \gamma^* \), higher expected cash flows than \( \pi^* \), or alternatively a lower payback period than \( P^* \) or \( P^{D*} \). Since \( V^* \) is greater than \( V \), we can easily make a comparison between modified investment rules and conventional investment rules as shown in Table 1:

<table>
<thead>
<tr>
<th>Modified Investment Rules</th>
<th>( NPV^* )</th>
<th>( \Pi^* )</th>
<th>( \pi^* )</th>
<th>( \gamma^* )</th>
<th>( P^* )</th>
<th>( P^{D*} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comparisons</td>
<td>( \wedge )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \wedge )</td>
<td>( \wedge )</td>
</tr>
<tr>
<td>Conventional Investment Rules</td>
<td>( NPV )</td>
<td>( \Pi )</td>
<td>( \pi )</td>
<td>( \gamma )</td>
<td>( P )</td>
<td>( P^D )</td>
</tr>
</tbody>
</table>

Note: \( \wedge \) = less than; \( \checkmark \) = greater than

**Table 1  The Comparisons between Modified Investment Rules the Conventional Investment Rules**

It is important to note that these modified investment rules are expressed in terms
of $\Pi'$ such that one investment rule can be readily computed into another modified investment rules as long as (unit) optimal investment trigger is known, regardless of the stochastic process assumption. Also, as the gains of waiting become more significant, the difference between modified investment rules and conventional investment rules, e.g., $\Pi'$ and $\Pi$, respectively, are further magnified.

2.2 Modified Capital Budgeting Criteria under a GBM

Modern investment theory centers on searching for optimal investment trigger, $V^*$, such that the value of the investment opportunity, $F(V)$, is maximized. We assume that the project value, $V$, follows a GBM as follows:

$$dV = \alpha V dt + \sigma V dz$$

(15)

where $\sigma$ and $dz$ denote volatility and an increment of a standard Wiener process, respectively.

For a project whose value follows a GBM, the literature has shown that the solution of $F(V)$ is as follows:  \(^8\)

$$F(V;V^*) = AV^h$$

(16)

where $A = (V^* - I)V^{*-h}$

(17)

$$b_1 = \left( 1 - \frac{r - (\mu - \alpha)}{\sigma^2} \right) + \sqrt{\left( \frac{r - (\mu - \alpha)}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2}}$$

(18)

At the maximum of option values, the optimal investment trigger equals the sum of investment cost and the value of investment opportunity, which is also called the value-matching condition. By substituting Equation (16) into the value-matching condition, $V^*$ is solved as follows:

$$V_{GBM}^* = \left( \frac{b_1}{b_1 - 1} \right) I$$

(19)

\(^8\) See McDonald and Siegel (1986), Dixit (1989), and Pindyck (1991).
where \( V_{\text{GBM}}^* \) denotes the optimal GBM trigger.

Substituting \( V_{\text{GBM}}^* \) in Equation (19) into the modified capital budgeting criteria in Equation (1), (2), (7), (8), (11), and (14), we can obtain explicit expressions in terms of \( b_1 \) as follows:

\[
\Pi_{\text{GBM}}^* = 1 + \frac{1}{b_1 - 1} \quad (20)
\]

\[
\pi_{\text{GBM}}^* = (\mu - \alpha) I + \frac{1}{b_1 - 1} (\mu - \alpha) I \quad (21)
\]

\[
\gamma_{\text{GBM}}^* = \mu + \frac{1}{b_1 - 1} (\mu - \alpha) \quad (22)
\]

\[
P_{\text{GBM}}^* = \begin{cases} 
\ln \left( 1 + \frac{b_1 \alpha}{(b_1 - 1)(\mu - \alpha)} \right) / \alpha, & \alpha \neq 0 \\
1 - \frac{1}{\mu} + \frac{1}{b_1 \mu}, & \alpha = 0
\end{cases} \quad (23)
\]

\[
P_{\text{GBM}}^{D*} = \frac{\ln (b_1)}{\mu - \alpha} \quad (24)
\]

Note that these modified investment rules in Equation (20)-(24) have an important implication regarding how the options of waiting impact on conventional investment rules. For a stochastic process like a GBM, we can easily decompose the modified investment rules into two terms, one of which represents the conventional investment rules and the other term stands for the “option impact”, which accounts for the impact on these investment rules in the presence of options of waiting. For the modified investment rules \( \Pi_{\text{GBM}}^* \), \( \pi_{\text{GBM}}^* \), and \( \gamma_{\text{GBM}}^* \), the option impacts are \( 1/(b_1 - 1) \), \( (\mu - \alpha)/b_1 \), and \( (\mu - \alpha)/(b_1 - 1) \), respectively. Note that Equation (20) stands for unit optimal investment trigger, which makes \( 1/(b_1 - 1) \) become unit
option value at the point of \( V_{\text{GBM}}^* \). As the option value gets larger, these option impacts have a more significant, positive influence on the modified investment rules. For the modified payback rules, the option impacts are not readily observed. However, in the case of zero growth, i.e., \( \alpha = 0 \), the option impact of \( P_{\text{GBM}}^* \) is derived to be \(-1/b_1 \mu\), which indicates a lower optimal payback when there is a higher option value. The parameter \( b_1 \) has a number of important properties. First, \( b_1 \) must be greater than 1 when there is a positive option value. Second, since \( b_1 \) is in the denominator of \( V_{\text{GBM}}^* \), \( b_1 \) is inversely correlated with \( V_{\text{GBM}}^* \). Consequently, \( b_1 \) is inversely correlated with \( \Pi_{\text{GBM}}^* \), \( \pi_{\text{GBM}}^* \), and \( \gamma_{\text{GBM}}^* \), and is positively correlated with \( P_{\text{GBM}}^* \) and \( P_{\text{GBM}}^{D*} \).

### 2.3 Modified Capital Budgeting Criteria under a Mixed Diffusion-Jump

In this subsection, we extend the preceding analysis to the case in which project value (or cash flows) follows a mixed diffusion-jump process. This process is often specifically used to describe the situation in that the value of an investment opportunity can become worthless as potential competitors enter the market as first-movers.\(^{10}\) In other words, the preemptive competitive effect may lead to the project value appropriated by the competitors, which thus can be characterized by a mixed diffusion-jump process.\(^{11}\) A mixed diffusion-jump process is formalized as follows:

\[
dV = \alpha V dt + \sigma V dz - V dq
\]

where \( dq \) is the increment of a Poisson process with a mean arrival rate of \( \lambda \) and is expressed by

\[
\text{Since } \quad V^* = \left(1 + \frac{1}{b_1 - 1}\right) I, \quad F(V^*) = \frac{I}{b_1 - 1} \quad \therefore \quad F(V^*) = \frac{1}{b_1 - 1} > 0, \quad \therefore \quad b_1 > 1.
\]

\(^9\) Trigeorgis (1990) deals with the preemptive competitive effect by treating the competitors’ actions as dividends which are the proportions of the project value appropriated by the competitors. His analysis is limited by the assumption that the erosion effect can be completely anticipated and quantified by the firm, which appears to be less realistic.

\(^{10}\) McDonald and Siegel (1986).
\[ dq \begin{cases} \phi & \text{with a probability of } \lambda dt \\ 0 & \text{with a probability of } 1 - \lambda dt \end{cases} \quad (26) \]

where \( \phi \ (0 \leq \phi \leq 1) \) stands for the constant percentage of loss in \( V \) should the jump event, i.e., competitive arrivals, occur.

Meanwhile, \( dq \) is assumed to be independent of \( dz \), i.e., \( E(dqdz) = 0 \).

With the same boundary conditions as in the GBM model, McDonald and Siegel (1986) and Dixit and Pindyck (1994, Ch. 5) have verified that if \( \phi = 1 \), the solutions of \( F(V) \) and \( V_{MX}^* \) are exactly the same as Equation (16) and (19), respectively, except that \( b_1 \) is replaced with \( b_2 \).\(^\text{12}\)

\[
b_2 = \left( \frac{1}{2} - \frac{r - (\mu - \alpha)}{\sigma^2} \right) + \sqrt{\left( \frac{r - (\mu - \alpha)}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2(r + \lambda)}{\sigma^2}}
\]

\quad (27)

The parameter \( b \) is similar to the parameter \( b_1 \) in the functional form except that the jump intensity \( \lambda \) gets added into the interest rate in the constant term. It is easy to see that \( b \) is equal to \( b_1 \) if \( \lambda = 0 \) and greater than \( b_1 \) if \( \lambda > 0 \). Since both \( b_1 \) and \( b_2 \) are inversely correlated with optimal investment trigger, for the same set of parameter values the relationship of \( b_2 \geq b_1 \) leads to the result of \( V_{MX}^* \leq V_{GBM}^* \), where the subscript MX denotes a mixed diffusion-jump process.

Since the optimal trigger under an MX process takes the same form as the one under a GBM with \( b_1 \) substituted by \( b_2 \), all the modified optimal investment rules are thus akin to those in the GBM model with \( b_2 \) in the place of \( b_1 \). The option impacts under an MX process are also similar to those under a GBM with \( b_1 \) replaced by \( b_2 \). Note that \( b_2 \) also bears the same properties as \( b_1 \). With other parameters held constant, the jump intensity is inversely correlated with \( V_{MX}^* \). This means that a higher \( \lambda \) leads to a lower \( V_{MX}^* \) and, in turn, a lower \( \Pi_{MX}^* \), \( \pi_{MX}^* \).

\(^{12}\) If \( \phi \neq 1 \), the value of the investment opportunity is still the form of \( F(V) = AV^b \). However, the solution needs to be found numerically together with the boundary conditions.
and $\gamma_{MX}$, and also a larger optimal payback of $P_{MX}^*$ and $P_{MX}^{D*}$.

### 2.4 Modified Capital Budgeting Criteria under a Mean-Reverting Process

Dixit and Pindyck (1994, Ch. 5) introduce a specific mean-reverting process for the ease of deriving an analytical solution. The mean-reverting process can be expressed as the following form:

$$dV = \eta(\bar{V}-V)dt + \sigma Vdz$$

where $\bar{V}$ and $\eta$ denotes the long-run mean and the speed of mean reversion, respectively.

The solution of an investment opportunity under a mean-reverting process is given as follows:

$$F(V; V_{MR}^*) = BV^g G(x; \theta, g)$$

where the subscript MR denotes a mean-reverting process,

$$\theta = \frac{1}{2} + \left( \frac{\mu - r - \eta \bar{V}}{\sigma^2} + \sqrt{\frac{(r - \rho + \eta \bar{V})}{\sigma^2} - \frac{1}{2}} \right)^2 + \frac{2r}{\sigma^2},$$

$$x = \frac{2\eta}{\sigma^2} V,$$

$$g = 2\theta + \frac{2(r - \mu + \eta \bar{V})}{\sigma^2},$$

and

$$G(x; \theta, g) = 1 + \frac{\theta}{g} x + \frac{\theta(\theta + 1)}{g(g + 1)} \frac{x^2}{2!} + \frac{\theta(\theta + 1)(\theta + 2)}{g(g + 1)(g + 2)} \frac{x^3}{3!} + \cdots.$$}

Since $G(x; \theta, g)$ is an infinite confluent hypergeometric function, there is no closed-form solution for $V_{MR}^*$. The coefficient, $B$, and $V_{MR}^*$ must be solved numerically together with the same boundary conditions as in the GBM model.

To obtain the modified investment rules under a MR process in the expression of
Equation (29), we first equate both of the unit optimal triggers under an MR process and under a GBM process as follows:

\[
\frac{B^*(V_{MR}^*)^\theta G^*}{I} = \frac{1}{b_1 - 1} \tag{30}
\]

where \( B^* \) denotes the coefficient \( B \) at the point of \( V_{MR}^* \) and \( G^* = G(x^*; \theta, g) \).

\[
b_1 = \frac{I + B^*(V_{MR}^*)^\theta G^*}{B^*(V_{MR}^*)^\theta G^*} \tag{31}
\]

We then substitute Equation (31) into the modified GBM investment rules in Equation (20)-(24) for the modified MR investment rules as follows:

\[
\Pi_{MR}^* = 1 + \frac{B(V^*)^\theta G^*}{I} \tag{32}
\]

\[
\pi_{MR}^* = (\mu - \alpha)I + \left[ B^*(V_{MR}^*)^\theta G^* \right](\mu - \alpha) \tag{33}
\]

\[
\gamma_{MR}^* = \mu + \frac{B^*(V_{MR}^*)^\theta G^*}{I}(\mu - \alpha) \tag{34}
\]

\[
P_{MR}^* = \begin{cases} 
\ln \left( \frac{I + \frac{1}{\alpha/\mu} \left[ B(V^*)^\theta G^* \right]}{\left( I + B(V^*)^\theta G^* \right)(\mu - \alpha)} \right) & , \alpha \neq 0 \\
\frac{\alpha}{\mu} \left( \frac{B(V^*)^\theta G^*}{I} \right) & , \alpha = 0
\end{cases} \tag{35}
\]

\[
P_{MR}^{D*} = \frac{\ln \left[ \frac{1 + B^*(V_{MR}^*)^\theta G^*}{B^*(V_{MR}^*)^\theta G^*} \right]}{\mu - \alpha} \tag{36}
\]

The modified investment rules under an MR process can also be decomposed
into the conventional investment rules and the “options impacts”. The option impacts for $\Pi_{MR}^*$, $\pi_{MR}^*$, and $\gamma_{MR}^*$ are $\left[ B(V')^\theta G^* \right]/I \cdot \left[ B'(V_{MR})^\theta G^* \right]/I$, respectively. The option impact for $P_{MR}^*$ when $\alpha = 0$ is $-\left[ B(V')^\theta G^* \right]/\left[ I + B(V')^\theta G^* \right] \mu$. As the options of waiting becomes larger, the option impacts are more positively significant for $\Pi_{MR}^*$, $\pi_{MR}^*$, and $\gamma_{MR}^*$, and more negatively significant for $P_{MR}^*$.

### 2.5 Optimal Investment Rules under Alternative Processes

In the preceding subsections, the formulae for modified capital budgeting triggers under alternative processes are explicitly given. Here we would like to further investigate the relationships between modified investment rules and conventional investment rules under alternative processes by conducting a numerical comparative analysis based on a set of reasonable parametrical values. These alternative processes of interest include GBM, mixed diffusion-jump, and mean-reverting processes. Since the solution of optimal investment trigger under a mean-reverting process is not closed-form, numerical analysis is necessary for comparing various investment triggers under alternative processes. Specifically, we focus on the effects on optimal investment rules of jumps and mean reversion.

As illustrated in Section 2.2, when jump size, $\phi$, equals 1, the mixed diffusion-jump model has a similar formula for option value and investment trigger to the GBM model, with jump intensity, $\lambda$, added into the parameter, $b_2$. Since $\lambda$ is in the numerator of $b_2$, it is easy to find out $b_1 \leq b_2$ for the same parameters values and, in turn, $V_{GBM}^{*} \geq V_{MX}^{*}$. Given $I = 100$, $\mu - \alpha = \delta = 5\%$, $r = 5\%$, Figure 1 displays the effects of various jump intensities on optimal investment triggers across increasing instantaneous volatilities. An important finding evident from the diagram is that the inclusion of jumps into consideration lowers the optimal investment trigger, $V_{MX}^{*}$. This negative effect on trigger price is even significant with $\lambda$ increased. This negative effect can be described in terms of modified investment rules: the modified rules such as $\Pi_{MX}^{*}$, $\gamma_{MX}^{*}$, and $\pi_{MX}^{*}$ are less than the GBM counterparts.
The optimal paybacks, $P_{MX}^*$ and $P_{D}^{D*}$, are greater than the GBM paybacks, $P_{GBM}^*$ and $D_{GBM}^{D*}$. The implication to management is that when the market is highly competitive or the first-mover advantage is significant, optimal investment decisions should be initiated sooner than those under a GBM process. Note that the GBM line actually represents the mixed-jump case in that $\lambda = 0$. The more significant the competitive preemptive effect is, the higher $\lambda$ becomes and thus the sooner the investment should be launched.

Next, we compare $V_{GBM}^*$ and $V_{MR}^*$, where the subscript MR denotes a mean-reverting process. Schwartz (1997) and other real options studies on mean reversion argue that when mean reversion is ignored in project evaluation, investment tends to be excessively delayed due to a overpriced investment trigger. This argument can be confirmed in our numerical analysis as shown in Figure 2, which exhibits the sensitivity of trigger price by varying mean-reverting speed and
instantaneous volatility, given $I = \bar{V} = 100, \mu - \alpha = \delta = 5\%$, and $r = 5\%$.\footnote{Note that the long-run mean, $\bar{V}$, is set to be equal to 100 for two reasons. First, we are only interested in near at-the-money projects, i.e., $NPV = 0$, since real options are less influential in investment decision-making when involved with deep in-the-money or deep out-of-the-money projects. Second, as $\alpha$ may be substantially positive or negative in a disequilibrium setting, i.e., $V \neq \bar{V}$, to keep $\delta$ unchanged we need to adjust $\mu$, which may produce a very unrealistic discount rate complicating the analysis.} It is apparent that the GBM trigger, $V^*_\text{GBM}$, is considerably greater than the mean reversion triggers, $V^*_\text{MR}$, even for a very slow mean-reverting speed. However, compared to $V^*_\text{GBM}$, $V^*_\text{MR}$ is relatively insensitive to project volatility. This is possibly because mean reversion reduces the long-run volatility. Thus, even though instantaneous volatility is exactly equal in the models of both GBM and mean reversion, the long-run volatility under a mean reversion gets smaller as the mean-reverting speed becomes faster. (Sarkar, 2003)

Figure 2  The Optimal Triggers under a GBM and a Mean-Reverting Process as a Function of Volatility

Note: Other parameter values are $I = 100, \bar{V} = 100, \delta = 5\%, r = 5\%$. 
Figure 2 also indicates that when investment projects are characterized by a mean-reverting process, investments should be induced sooner than those under a GBM. Given the fact that \( V_{MR}^* \) is less than \( V_{GBM}^* \) for a reasonable set of parameters, it is obvious that the modified investment rules of mean reversion such as \( \Pi_{MR}^* \), \( \gamma_{MR}^* \), and \( \pi_{MR}^* \), are less than the GBM counterparts while the optimal paybacks of mean reversion, \( P_{MR}^* \) and \( P_{MR}^{D*} \), are greater than the GBM counterparts.

To compare the effects of jumps and mean reversion, Figure 1 and 2 are combined within the same frame into Figure 3, which displays the sensitivity of optimal trigger price to the changes in volatility among three alternative models. Figure 3 suggests that for a set of reasonable parameter values, both mean reversion and jumps have a significant influence on bringing down trigger price closer to the conventional triggers, indicating that investment under both cases should be launched sooner than that under a GBM. Furthermore, mean reversion may have a stronger power to induce investment than the competitive preemptive effect, given such a set of reasonable parameter values.

Note: Other parameter values are \( I = 100, \overline{V} = 100, \delta = 5\%, \ r = 5\% \).

**Figure 3 A Comparison of Optimal Trigger Price under Alternative Processes**
To summarize the finding of this subsection, we make a comparison among the modified investment rules under alternative processes in Table 2. Since the option impact under a GBM process is the most significant among three stochastic processes of interest, the GBM rules such as $\Pi_{GBM}^*$, $\pi_{GBM}^*$, and $\gamma_{GBM}^*$ are thus larger than any of the MX and MR counterparts, and the GBM paybacks such $P_{GBM}^*$ and $P_{GBM}^{Dv}$ are lower than the MX or MR payback rules, given a set of reasonable parameter values.

<table>
<thead>
<tr>
<th>GBM Rules</th>
<th>$\Pi_{GBM}^*$</th>
<th>$\pi_{GBM}^*$</th>
<th>$\gamma_{GBM}^*$</th>
<th>$P_{GBM}^*$</th>
<th>$P_{GBM}^{Dv}$</th>
</tr>
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<tbody>
<tr>
<td>Comparisons</td>
<td>∨</td>
<td>∨</td>
<td>∨</td>
<td>∧</td>
<td>∧</td>
</tr>
<tr>
<td>MX Rules</td>
<td>$\Pi_{MX}^*$</td>
<td>$\pi_{MX}^*$</td>
<td>$\gamma_{MX}^*$</td>
<td>$P_{MX}^*$</td>
<td>$P_{MX}^{Dv}$</td>
</tr>
<tr>
<td>Comparisons</td>
<td>∨</td>
<td>∨</td>
<td>∨</td>
<td>∧</td>
<td>∧</td>
</tr>
<tr>
<td>MR Rules</td>
<td>$\Pi_{MR}^*$</td>
<td>$\pi_{MR}^*$</td>
<td>$\gamma_{MR}^*$</td>
<td>$P_{MR}^*$</td>
<td>$P_{MR}^{Dv}$</td>
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<tr>
<td>Comparisons</td>
<td>∨</td>
<td>∨</td>
<td>∨</td>
<td>∧</td>
<td>∧</td>
</tr>
<tr>
<td>Conventional Rules</td>
<td>$\Pi$</td>
<td>$\pi$</td>
<td>$\gamma$</td>
<td>$P$</td>
<td>$P^{Dv}$</td>
</tr>
</tbody>
</table>

Note: ∧ = less than; ∨ = greater than

Table 2  The Comparisons Among the Modified Investment Rules under Alternative Processes

3. **The Cost of the Suboptimal Investment Rules**

In standard real options analysis, we are interested in deriving the optimal
investment rule and the option formula. In this section, we would like to investigate the effects of managerial flexibility when management adopts suboptimal or arbitrary investment rules. Specifically, our investigation centers on evaluating the loss function of adopting a suboptimal investment rule, which is equivalently to measure the cost of losing the option value when the suboptimal rule of investment is adopted.

Given that the project value process evolves as a GBM, Equation (16) provides the option formula when $V^*$ is used as optimal investment rules. It is important to note that $F(V)$ is maximized at the point of $V = V^*$. Let $V^S$ be a suboptimal investment trigger, where the superscript $S$ denotes the suboptimal investment trigger. When management adopts the suboptimal investment trigger, $V^S$, the option formula with respect to $V^S$ can be revised as follows:

$$F(V; V^S) = AV^h$$  \hspace{1cm} (37)

where $A = \frac{V^S - I}{(V^S)^h}$. 

For a mixed diffusion-jump process, the option formula is exactly the same as Equation (37) with $b_1$ replaced by $b_2$. For a mean-reverting process, the option value with respect to $V^S$ is given below:

$$F(V; V^S) = BV^\theta G(x; \theta, g)$$  \hspace{1cm} (38)

where $B = \frac{V^S - I}{(V^S)^\theta G^S}$. 

The above equations can be used to evaluate the option value associated with the suboptimal investment rule. Since the option value is maximized at the point of optimal trigger, it follows that $F(V; V^*)$ must be greater than $F(V; V^S)$ for all $V^S$. We define $\kappa$ as the cost of following a suboptimal investment rule, which is computed from the difference $F(V; V^*)$ and $F(V; V^S)$. $\kappa$ effectively stands for the loss of option value under the suboptimal investment rule and can be expressed as follows:

$$\kappa_i = F(V; V^*) - F(V; V^S), \; i = L, H$$  \hspace{1cm} (39)
where the subscript \( L \) and \( H \) denote a low suboptimal trigger and a high suboptimal trigger relative to optimal investment rule, respectively, i.e., \( V^*_L < V^* \) and \( V^*_H > V^* \).

Based on Equation (37), (38), and (39), the costs of adopting the suboptimal investment rules for three alterative models are exhibited in Figure 4, 5, and 6, respectively. These diagrams are graphed with a combination of different volatility and discount rate under the basic assumption of \( I = 100, \overline{V} = 100, \delta = 5\% \), and \( r = 5\% \). There are a few findings which can be readily drawn from these diagrams. First, we reconfirm the results in McDonald (1999) that the worst investment policy is to follow the traditional NPV rule which maximizes \( \kappa \) due to the zero option value. Our findings support this argument, which exists not only under a GBM but also under other alternative processes such as an MX or MR process. Second, the loss function of following the suboptimal investment rule is asymmetric for a GBM or an MX process. According to the diagram, it is obvious that a higher investment trigger, \( V^*_H \), is better than a lower trigger, \( V^*_L \), for the same percentage of deviating from \( V^* \). The reason is mainly because \( V^*_H \) has a lower loss in the option value, i.e. \( \kappa_L > \kappa_H \).

However, for a mean-reverting process, adopting \( V^*_H \) may cause a greater loss in the option value such that the cost of adopting a higher suboptimal trigger increases substantially. As a result, it is found that the cost of adopting a higher suboptimal trigger is approximately equal to the cost of adopting a lower suboptimal trigger, i.e., \( \kappa_L \approx \kappa_H \), for the same percentage of deviating from \( V^*_MR \) on both sides.\(^{14} \) This symmetric-looking loss function suggests that \( V^*_L \) and \( V^*_H \), under an MR process can be equally worse than the optimal trigger \( V^*_MR \). In other words, waiting too long

\(^{14} \) By transforming the mean reversion trigger diagram into frequency histogram, we are able to conduct the Jarque-Bera normality test. The result indicates that the MR trigger diagram is a non-Gaussian distribution statistically. However, this does not invalidate the above argument because a non-Gaussian distribution is not necessarily asymmetric. For reasonable parameters, it is found that \( \kappa_L \) and \( \kappa_H \) are approximately equal for the same difference from \( V^* \). For example, given \( \rho = 20\%, \eta = 0.03 \), and \( \sigma = 40\% \), an increase by 8% would lead to a loss of 14.29% in the option value while a decrease by 8% would lead to a value loss of 15.59%. Since both \( V^*_H \) and \( V^*_L \), under an MR process, suffer from a nearly equal loss in the option value, we say \( V^*_H \) is not necessarily better than \( V^*_L \). Of course, as the deviation from \( V^* \) becomes considerably large, McDonald’s argument may still hold.
or ignoring real options can be equally suboptimal as it may lead to a loss in the option value. Consequently, the best investment rule under an MR process is the optimal investment rule itself. However, if a simple heuristic decision rule is used to approximate $V_{MR}^*$, the rule should be as near $V_{MR}^*$ as possible in order to minimize the opportunity cost of adopting the suboptimal investment policy.

![Graph showing the cost of adopting a suboptimal investment trigger under a GBM](image)

**Figure 4** The Cost of Adopting a Suboptimal Investment Trigger under a GBM
Figure 5  The Cost of Adopting a Suboptimal Investment Trigger under an MX Process

Figure 6  The Cost of Adopting a Suboptimal Investment Trigger under an MR Process
4. Developing Heuristic Investment Rules

Our basic strategy for developing heuristic investment rules is to choose one target capital budgeting instrument which can be best predicted from heuristic investment rules with minimum information requirements. Once the target instrument is estimated from heuristic investment rules, the estimated target instrument can be conveniently transformed into other equivalent capital budgeting rules according to Equation (2), (7), (8), (11), and (14). In this section, we describe the complete process for developing the heuristic investment rules under the consideration of managerial flexibility.

4.1 Base Case Analysis

The analysis starts by examining the relationships between key parameters and modified capital budgeting rules based on a set of reasonable parametrical values through graphical diagrams, which may shed light on how the modified capital budgeting rules are influenced by key parameters of interest. Specifically, we focus on examining the relationship of linearity and sensitivity between the modified capital budgeting rules and the key parameters. For a GBM process, the key parameters are referred to as $\sigma$, $\mu$, and $\alpha$, all of which have been considered as important determinants in real options literature. In addition to these parameters, we also consider $\lambda$ for a mixed diffusion-jump process and $\eta$ for a mean-reverting process, respectively.

Abundant investment literature has indicated that managerial flexibility under uncertainty plays a crucial role in investment decision-making when the project has a close-to-zero NPV or is near “at-the-money” in terms of options terminology. If the project is deep “in the money”, the presence of real options adds fewer strategic values to project evaluation for that the project will be taken anyway. Conversely, if the project is deep “out-of-the-money”, no option values can rescue the project from such a substantial loss in the NPV. Therefore, here we are more interested in the near at-the-money case, i.e. $V \approx I$.

Based on a preliminary analysis, we decide to choose the modified hurdle rate rule as a target instrument for two main reasons. First, the analysis results indicate that the relationship between $\gamma$ and key parameters is the closest to linear while such linear relationships between other capital budgeting instruments and key parameters are not readily observed. From the perspective of sensitivity, the optimal
hurdle rate also reacts to the changes in key parameters in a seemingly even way. Second, the literature on capital budgeting practices also indicates the hurdle rate rule is one of the most commonly used criteria by financial practitioners in making investment decisions.

Having chosen the hurdle rate rule, we graph the sensitivity of optimal hurdle rate, \( \gamma' \), relative to key parameters under GBM, MX, and MR processes in Figure 7, 8, and 9, respectively, given the base case parameters \( I = 100 \) and \( r = 8\% \). All these diagrams indicate that the modified hurdle rates for each of the three alternative processes respond sensitively to the changes in \( \sigma \) or \( \mu \), holding other parameters constant, in a nearly proportional manner, suggesting that \( \gamma' \) has a quasi-linear relationship with \( \sigma \) or \( \mu \). From Figure 7 and 8, it is found that \( \gamma'_{GBM} \) increases by 4% when \( \sigma \) increases by 10% or alternatively \( \gamma'_{GBM} \) increases by 8% when \( \mu \) increases by 10%, other parameters held constant. From Figure 9 and 10, \( \gamma'_{MX} \) increases by an average 3% when \( \sigma \) increases by 10% or alternatively \( \gamma'_{MX} \) increases by an average 10% when \( \mu \) increases by 10%, other parameters held constant. From Figure 11 and 12, it is also found that \( \gamma'_{MR} \) increases by 2%-4% when \( \sigma \) increases by 10% or alternatively \( \gamma'_{MR} \) increases by 10-12% when \( \mu \) increases by 10%.

Although the diagrams of sensitivity analysis reveal that the modified hurdle rate rules for all the alternative processes have a quasi-linear relationship with \( \sigma \) or \( \mu \), it is also obvious that \( \gamma' \) is relatively insensitive to the changes in other parameters such as \( \alpha \), \( \lambda \), \( \eta \) or \( V \). Under an MR process, when a higher long-run mean is in place, \( \gamma'_{MR} \) appears to be a decreasing non-linear function of \( \eta \). It is also important to note that that the sensitivity analysis is based on the base case parameter values only. Later, we shall further extend the analysis based on a larger set of sample data which contain a wider range of parameter values.
Figure 7  The Sensitivity of $\gamma_{GBM}$ to $\sigma$ and $\mu$ ($\alpha = 8\%$)

Figure 8  The Sensitivity of $\gamma_{GBM}$ to $\sigma$ and $\alpha$ ($\mu = 25\%$)
Figure 9  The Sensitivity of $\gamma_{MX}$ to $\sigma$ and $\mu$ ($\alpha = 8\%$ and $\lambda = 20\%$)

Figure 10  The Sensitivity of $\gamma_{MX}$ to $\alpha$ and $\lambda$ ($\mu = 25\%$ and $\sigma = 25\%$)
Figure 11  The Sensitivity of $\gamma_{MR}$ to $\sigma$ and $\mu$ ($\bar{V} = 100$ and $\eta = 0.02$)

Figure 12  The Sensitivity of $\gamma_{MR}$ to $\eta$ and $\bar{V}$ ($\mu = 25\%$ and $\sigma = 25\%$)
4.2 Upper and Lower Bounds of $\gamma^*$

The preceding base case analysis provides a foundation for developing the heuristic investment rules. Here we would like to show that $\gamma^*$ can be mathematically situated between the upper and lower bounds under a GBM or a mixed diffusion-jump process.\textsuperscript{15}

According to Equation (18) and (27), it can be easily find that $b_1$ and $b_2$ fall between the following bounds, respectively:

\begin{align}
1 + \frac{2(\mu - \alpha - r)}{\sigma^2} < b_1 < 1 + \frac{2(\mu - \alpha - r)}{\sigma^2} + \frac{\sqrt{2r}}{\sigma} & \quad (40) \\
1 + \frac{2(\mu - \alpha - r)}{\sigma^2} < b_2 < 1 + \frac{2(\mu - \alpha - r)}{\sigma^2} + \frac{\sqrt{2(r + \lambda)}}{\sigma} & \quad (41)
\end{align}

Substituting Equation (40) and (41) into Equation (22), respectively, we can derive the upper and lower bounds for the optimal hurdle rate under a GBM and an MX process, respectively:

\begin{align}
\mu + \frac{\sigma^2}{2} \left[ \frac{\mu - \alpha}{(\mu - \alpha - r) + \frac{\sigma\sqrt{r}}{\sqrt{2}}} \right] < \gamma^*_\text{GBM} < \mu + \frac{\sigma^2}{2} \left( \frac{\mu - \alpha}{\mu - \alpha - r} \right) & \quad (42) \\
\mu + \frac{\sigma^2}{2} \left[ \frac{\mu - \alpha}{(\mu - \alpha - r) + \frac{\sigma\sqrt{r + \lambda}}{\sqrt{2}}} \right] < \gamma^*_\text{MX} < \mu + \frac{\sigma^2}{2} \left( \frac{\mu - \alpha}{\mu - \alpha - r} \right) & \quad (43)
\end{align}

For some projects that $\sigma < \sqrt{2r}$ under a GBM and $\sigma < \sqrt{2(r + \lambda)}$ under a mixed diffusion-jump, Equation (42) and (43) can be further simplified into the following equations, respectively:

\begin{equation}
\mu + \frac{\sigma^2}{2} < \gamma^*_i < \mu + \frac{r(\mu - \alpha)}{\mu - \alpha - r}, \quad i = \text{GBM, MX} \quad (44)
\end{equation}

Equation (44) suggests that the optimal hurdle rate under uncertainty equals the risk-adjusted discount rate plus a certain percentage of “option impact” which is a

\textsuperscript{15} Since the option formula under a mean-reverting process is not closed-form, it is unlikely to derive the upper and lower bounds of the optimal hurdle rate.
function of key parameters. Thus, this analysis for upper and lower bounds of $\gamma$ is consistent with the derivations in Equation (22).

4.3 Regression Analysis

Since Figure 7, 9, and 11 imply that $\gamma$ has a seemingly linear relationship with $\sigma$ and $\mu$ under various stochastic processes, for the purpose of estimation we regress $\gamma$ on the key parameters based on the ordinary least square (OLS) method. The dependent variable of $\gamma$ is derived from the modified investment rules, given a stochastic process assumption. The data of input parameters are randomly generated from a pre-specified range of parameter values as the independent variables. Table 3 displays the ranges of input parameter values across different types of projects and stochastic processes.

<table>
<thead>
<tr>
<th>Stochastic Process</th>
<th>Parameters</th>
<th>Ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low-Growth</td>
<td>Mid-Growth</td>
</tr>
<tr>
<td>GBM</td>
<td>Project Value ($V$)</td>
<td>70-130</td>
</tr>
<tr>
<td></td>
<td>Investment Cost ($C$)</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Interest Rate ($\sigma$)</td>
<td>4%-10%</td>
</tr>
<tr>
<td></td>
<td>Volatility ($\sigma$)</td>
<td>11%-50%</td>
</tr>
<tr>
<td></td>
<td>Discount Rate ($\mu$)</td>
<td>11%-50%</td>
</tr>
<tr>
<td></td>
<td>Growth Rate ($\alpha$)</td>
<td>0-10%</td>
</tr>
<tr>
<td>MX*</td>
<td>Jump Intensity ($\lambda$)</td>
<td>0-0.25</td>
</tr>
<tr>
<td>MR*</td>
<td>Negative Growth ($V_0 &gt; \bar{V}$)</td>
<td>130-101</td>
</tr>
<tr>
<td></td>
<td>Zero-Growth ($V_0 = \bar{V}$)</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Positive-Growth ($V_0 &lt; \bar{V}$)</td>
<td>0.01-0.05</td>
</tr>
<tr>
<td></td>
<td>Mean Reversion Speed ($\eta$)</td>
<td>0.01-0.05</td>
</tr>
</tbody>
</table>

*Unless specified, the other parameter values in the MX and MR models are the same as those in the GBM model.

Table 3 The Range of Parameter Values in Random Sampling

In random sampling, a few restrictions are imposed on the possible parametric relationships in order to reflect model specifications and improve the validity of
Firstly, risk-adjusted discount rate is assumed to be greater than risk-free rate, i.e. $\mu > r$, due to the consideration of idiosyncratic project risk. Secondly, risk-adjusted discount rate should also be greater than growth rate, i.e. $\mu > \alpha$, due to the model restriction in Equation (3). Thirdly, $\sigma$, $\alpha$ and $r$ are sampled independently. In addition, to avoid multicollinearity, it is necessary to assume that discount rate and volatility either are uncorrelated or have a non-linear relationship with each other. The former implies that the project is uncorrelated with the market portfolio, which appears to be reasonable when the project is in a sector that is negligible relative to the market portfolio. Even though discount rate and volatility are correlated in some way, the literature indicates that their relationship is not necessarily linear. The capital asset pricing model (CAPM) contends that risk-adjusted discount rate is linearly and positively correlated with the coefficient of systematic risk, beta, not with volatility. In this situation, we can apply the sampling technique of two related variables suggested by Hertz (1979) when randomly sampling for $\sigma$ and $\mu$.

To maintain estimation stability, the data sets are divided into three subgroups based on different levels of growth rate. For the GBM and MX models, these three subgroups are low growth ($\alpha = 0\text{-}10\%$), mid-growth ($\alpha = 11\text{-}20\%$), and high growth ($\alpha = 21\text{-}30\%$) projects, with 5,000 sets of data points in each subgroup. As to the MR models, since growth rate is determined by the current level of project value relative to the long-run mean, scaled by the speed of mean reversion, i.e., $\alpha = \eta (\bar{V} - V_0)$, the projects under a mean-reverting process are broken down into three subgroups based on growth rate, including negative growth ($V_0 > \bar{V}$), zero growth ($V_0 = \bar{V}$), and positive growth ($V_0 < \bar{V}$), with 500 sets of data in each subgroup.

We consider two regression models, the first of which includes all the key parameters to compare the second model which includes only discount rate and volatility for the purpose of developing heuristic rules. The first regression model is

\[ Z_\sigma (\hat{\sigma}) = Z_\mu (\hat{\mu}) \]

Let $Z_\sigma$ and $Z_\mu$ denote the probability density functions of $\sigma$ and $\mu$, respectively. According to Hertz (1979), the method to build dependency between $\sigma$ and $\mu$ is to make their density functions equal, i.e. $Z_\sigma (\hat{\sigma}) = Z_\mu (\hat{\mu})$ in order to draw $\hat{\sigma}$ and $\hat{\mu}$ simultaneously. A more general approach is to introduce an arbitrary interval, $\xi$, such that $Z_\sigma (\hat{\sigma}) \leq Z_\mu (\hat{\mu}) + |\xi|$.

Since the optimal triggers under a mean-reverting process can only be solved numerically, computing the optimal investment trigger is computationally expensive. Consequently, the sample size for the MR model is relatively small compared to the GBM or MX models.
given as follows:

\[ \gamma_i = \beta_0 + \beta_1 \sigma_i + \beta_2 \mu_i + \beta_3 \alpha_i + \varepsilon_i \tag{45} \]

where \( \beta_0, \beta_1, \beta_2, \beta_3, \) and \( \varepsilon \) represent the intercept, the coefficients of \( \sigma, \mu, \alpha \), and the forecast error, respectively.

Equation (45) applies to both of the GBM and MR models. Note that growth rate under a mean-reverting process is defined as \( \alpha = \eta (\bar{V} - V) \). For the MX model, jump intensity \( \lambda \) is also considered an important key parameter and thus included in the regression model as an independent variable:

\[ \gamma_i = \beta_0 + \beta_1 \sigma_i + \beta_2 \mu_i + \beta_3 \alpha_i + \beta_4 \lambda + \varepsilon_i \tag{46} \]

where \( \beta_4 \) represents the coefficient of \( \lambda \).

The second regression model includes volatility and discount rate only without the intercept. This specification is based on the findings revealed from the base case analysis that \( \gamma^* \) is highly responsive to \( \sigma \) and \( \mu \) but relatively insensitive to other parameters, and based on our intention to develop simple heuristic investment rules. We shall demonstrate that for reasonable parameter values the optimal hurdle rate can be estimated by \( \sigma \) and \( \mu \) with forecast errors minimized. The second regression model considered is therefore given as follows:

\[ \gamma_i = \beta_1 \sigma_i + \beta_2 \mu_i + \varepsilon_i \tag{47} \]

The regression results for the GBM, MX, and MR models are provided in Table 4, 5, and 6, respectively. There are some of the specification issues of regression needed further addressed. First, since almost all the regression results encounter the problem of heteroscedasticity, the White’s heteroscedasticity-corrected standard errors, instead of the OLS standard errors, are reported in these tables. Another concern in the regression results is that the residuals do not appear to be normally distributed. However, the normality assumption may be relaxed asymptotically with the sample size of 5,000 sets within each subgroup. Although the sample is divided into several
subgroups, the coefficients of independent variables appear to be very stable across three subgroups with a variation less than 1%. The t-test of significance indicates that all the coefficients of independent variables across three subgroups are individually statistically significant at the level of 5%. In addition, the extremely high F-statistics indicate a goodness of fit among all the models at the significance level of 5%. The results also show that the regression models have a high explanatory power in predicting the optimal hurdle rate due to the high r-squared coefficients.

<table>
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<td></td>
<td>Model 1</td>
<td>Model 2</td>
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Variables

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<tr>
<td></td>
<td>0.3798</td>
<td>0.3761</td>
<td>0.9055</td>
<td>0.1120</td>
<td>0.0048</td>
</tr>
<tr>
<td></td>
<td>(0.0016)</td>
<td>(0.0012)</td>
<td>(0.0013)</td>
<td>(0.0013)</td>
<td>(0.0052)</td>
</tr>
</tbody>
</table>

Note: The standard errors of the coefficients are in the parentheses. All the coefficients appear to be statistically significant due to a very large t-statistic and an extremely small p-value.

Table 4  The Regression Results in the GBM Model

According to Table 4, there appears to be a positive relationship between optimal hurdle rate and key parameters under a GBM process. For the first model which considers all the key parameters, the coefficients of $\sigma$, $\mu$, and $\alpha$ are estimated to be 0.3749, 0.9113, and 0.0846, respectively, suggesting that for a 10% increase in $\sigma$, $\mu$, and $\alpha$, the optimal hurdle rate increases by 3.75%, 9.11%, and 0.85%, respectively. The result confirms our finding in the base case analysis that $\gamma$ is relatively insensitive to the changes of $\alpha$. This result appears to be consistent across three different types of projects. It is interesting to see that for a mid-growth or high growth project the $\alpha$ term can be approximately offset by the intercept due
to its negative sign. This offsetting effect provides us quite an incentive to simultaneously drop the intercept and $\alpha$ out of the estimation model.

Table 5 presents the regression result, assumed that the underlying process is an MX process. The result in Table 5 is very similar to that in Table 4 in that $\gamma_{MX}$ is sensitive to the changes in $\sigma$ and $\mu$, but relatively insensitive to the changes in $\alpha$ and $\lambda$. In the regression including all data sets, the coefficients of $\sigma$, $\mu$, $\alpha$, and $\lambda$ are estimated to be 0.2935, 1.0193, -0.0152, and -0.1037, respectively. It is also important to note that the sign of $\alpha$ becomes negative with $\lambda$ included in the model assumption although the magnitude of the $\lambda$ coefficient is small. The negative sign of jump intensity across different types of projects is consistent with the argument that as the probability of competitive arrivals increases, the optimal hurdle rate should be lower such that investment can be initiated sooner than the situation of no competitive arrivals.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Low Growth Projects</th>
<th>Mid Growth Projects</th>
<th>High Growth Projects</th>
<th>All Projects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model 1</td>
<td>Model 2</td>
<td>Model 1</td>
<td>Model 2</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.9933</td>
<td>0.9855</td>
<td>0.9885</td>
<td>0.9778</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.0097</td>
<td>0.0143</td>
<td>0.0099</td>
<td>0.0137</td>
</tr>
<tr>
<td>F Statistic</td>
<td>185,401</td>
<td>169,336</td>
<td>107,353</td>
<td>110,157</td>
</tr>
<tr>
<td>N</td>
<td>5,000</td>
<td>5,000</td>
<td>5,000</td>
<td>5,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Intercept</th>
<th>Volatility ($\sigma$)</th>
<th>Discount Rate ($\mu$)</th>
<th>Growth Rate ($\alpha$)</th>
<th>Jump Prob. ($\lambda$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.0287</td>
<td>(0.0006)</td>
<td>(0.0015)</td>
<td>-0.0191</td>
<td>-0.0794</td>
</tr>
<tr>
<td></td>
<td>-0.0235</td>
<td>(0.0009)</td>
<td>(0.0014)</td>
<td>-0.0323</td>
<td>-0.1030</td>
</tr>
<tr>
<td></td>
<td>-0.0180</td>
<td>(0.0015)</td>
<td>(0.0019)</td>
<td>-0.0562</td>
<td>-0.1037</td>
</tr>
<tr>
<td></td>
<td>-0.0250</td>
<td>(0.0004)</td>
<td>(0.0021)</td>
<td>-0.0152</td>
<td>(0.0002)</td>
</tr>
</tbody>
</table>

* Note: The standard errors of the coefficients are in the parentheses. All the coefficients appear to be statistically significant due to a very large t-statistic and an extremely small p-value.

Table 5  The Regression Results in the MX Model
Table 6 exhibits the result of regression of the MR model. Note that in the MR model growth rate is seen as a variable which summarizes mean-reverting speed and long-run mean for directing a future trend. Consequently, a positive growth rate implies that the current project value is lower than long-run mean such that there exists an upward trend, and vice versa. When characterized by the mean-reverting property, a project with a positive growth rate should have a lower hurdle rate in order to launch investment sooner than that with a negative growth rate. This argument can be confirmed by the negative sign of growth rate in the result of regression. Table 6 also unveils an interesting finding that $\sigma$ under a MR process has the lowest coefficient in contributing to the hurdle rate trigger, compared to the preceding two alternative models. More specifically, in regression model 2 when project volatility increases by 10%, the hurdle rate triggers, $\gamma_{\text{GBM}}$, $\gamma_{\text{MX}}$, and $\gamma_{\text{MR}}$, increase by 3.5%, 2.5%, and 1.8%, respectively. Conversely, the coefficient of the risk-adjusted discount rate in the MR model is the highest among three alternative models. When the discount rate increases by 10%, $\gamma_{\text{GBM}}$, $\gamma_{\text{MX}}$, and $\gamma_{\text{MR}}$, increase by 9.1%, 9.5%, and 10.0%, respectively.

<table>
<thead>
<tr>
<th>Positive Growth $(V &lt; \bar{V})$ Projects</th>
<th>Zero Growth $(V = \bar{V})$ Projects</th>
<th>Negative Growth $(V &gt; \bar{V})$ Projects</th>
<th>All Projects</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-Squared</td>
<td>0.9084</td>
<td>0.9018</td>
<td>0.9920</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.0248</td>
<td>0.0258</td>
<td>0.0057</td>
</tr>
<tr>
<td>F Statistic</td>
<td>990.06</td>
<td>909.42</td>
<td>18,341</td>
</tr>
<tr>
<td>N</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.0409</td>
<td>-0.0433</td>
<td>0.0490</td>
<td>0.0208</td>
<td>(0.0083)</td>
<td>(0.0021)</td>
<td>(0.0264)</td>
<td>(0.0103)</td>
</tr>
<tr>
<td>Volatility $(\sigma)$</td>
<td>-0.1788</td>
<td>-0.1342</td>
<td>0.2508</td>
<td>0.1979</td>
<td>0.3991</td>
<td>0.1798</td>
<td>0.1566</td>
<td>0.1798</td>
</tr>
<tr>
<td>Discount</td>
<td>1.1791</td>
<td>1.2808</td>
<td>1.1046</td>
<td>0.9915</td>
<td>0.6154</td>
<td>1.0121</td>
<td>0.9556</td>
<td>1.0121</td>
</tr>
<tr>
<td>Rate $(\mu)$</td>
<td>-0.1584</td>
<td>-0.1528</td>
<td>-0.1735</td>
<td>-0.1687</td>
<td>-0.1687</td>
<td>-0.1687</td>
<td>(0.0057)</td>
<td>(0.0058)</td>
</tr>
<tr>
<td>Rate $(\alpha)$</td>
<td>(0.007)</td>
<td>(0.0166)</td>
<td>(0.0161)</td>
<td>(0.0057)</td>
<td>(0.0058)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Note: The standard errors of the coefficients are in the parentheses. All the coefficients appear to be statistically significant due to a very large t-statistic and an extremely small p-value.
To sum up, there are two important findings revealed from the regression results. First, project volatility and risk-adjusted discount rate are found to be two of the most influential factors in determining the hurdle rate trigger regardless of the stochastic process assumption. Other parameters such as growth rate and jump intensity have a relatively minor effect on the hurdle rate trigger although they are statistically significant. Secondly, the regression results indicate that the coefficients on the volatility term for the GBM, MX, and MR models are roughly 0.4, 0.3, and 0.2, respectively. It is interesting to see that as more parameters are required to specify a particular stochastic process, the coefficient on the volatility term decreases by 0.1. For example, in addition to the regular GBM parameters, we also need an additional parameter, \( \lambda \), to specify an MX process, so the coefficient of volatility decreases by 0.1. For an MR process, two additional parameters, \( \eta \) and \( \bar{V} \), are required such that the coefficient of volatility in the regression model decreases by 0.1 further. It is also important to stress that the regression models are simply used to provide an analytical foundation for developing heuristic investment rules in the later section.

4.4 Sensitivity Analysis

The objective of heuristic investment rules is to establish a simple but close-to-optimal paradigm for practitioners to apply in investment decision-making. To pursue a simple but near optimal decision rule, we next ask whether it is worth dropping some minor but statistically significant variables, e.g., growth rate and jump intensity, in an economic sense. In this subsection, we conduct a sensitivity analysis by varying these input variables.

For a specific stochastic process, we assume that there are five various types of projects with different combinations of discount rate and volatility: Project A - high discount rate \( (\mu = 35\%) \) and high volatility \( (\sigma = 40\%) \), Project B - high discount rate \( (\mu = 35\%) \) and low volatility \( (\sigma = 10\%) \), Project C - middle discount rate \( (\mu = 25\%) \) and middle volatility \( (\sigma = 25\%) \), Project D - low discount rate \( (\mu = 15\%) \) and high volatility \( (\sigma = 40\%) \), and Project E - low discount rate \( (\mu = 15\%) \) and low volatility \( (\sigma = 10\%) \). Assume that \( V = 100 \), \( I = 100 \), \( r = 8\% \). With the above information, Figure 13 presents the diagram of sensitivity of \( \gamma_{GBM} \) to the changes in growth rate, assumed that project value follows a GBM. The diagram indicates that \( \gamma_{GBM} \) in
general is insensitive to $\alpha$ for Project A, B, and C. As $\alpha$ changes by 1%, $\gamma'_{GBM}$ changes by only up to 0.1%. In contrast, $\gamma'_{GBM}$ is relatively responsive to the changes in $\alpha$ for Project D and E, both of which have a lower discount rate in common. When $\alpha$ increases by 1%, $\gamma'_{GBM}$ increases by about 0.9%.

Figure 13 The Sensitivity of $\gamma'_{GBM}$ to $\alpha$

With the same parameter values, the sensitivity of $\gamma'_{MX}$ to the changes in $\alpha$ is displayed in Figure 14 and 15, given $\lambda = 20\%$ and $\lambda = 40\%$, respectively. In spite of different values of $\lambda$, both figures show in a consistent manner that $\gamma'_{MX}$ in general is quite insensitive to $\alpha$ for Project A, B, C, and E. As $\alpha$ increases by 1%, the change in $\gamma'_{MX}$ ranges from -0.08% to 0.31%. For Project D with a low discount rate and a high volatility, $\gamma'_{MX}$ appears to be decreasing to $\alpha$. Specifically, as $\alpha$ increases by 1%, the change in $\gamma'_{MX}$ ranges from -0.17% to
-0.93%.

Figure 14 The Sensitivity of $\gamma_{MX}$ to $\alpha$ (Given $\lambda=20\%$)

Figure 15 The Sensitivity of $\gamma_{MX}$ to $\alpha$ (Given $\lambda=40\%$)
Extending the same analysis framework to the MX model, the sensitivity of $\gamma_{MR}^*$ to the changes in $\alpha$ is given in Figure 16. As shown from the figure, $\gamma_{MR}^*$ appears to decrease little with $\alpha$ for Project A, C, and E. Specifically, as $\alpha$ increases by 1%, the change in $\gamma_{MR}^*$ ranges from -0.41% to 0.04%. However, for Project D which has a low discount rate and a low volatility, the decreasing, convex relationship between $\gamma_{MR}^*$ and $\alpha$ becomes more obvious.

![Figure 16 The Sensitivity of $\gamma_{MR}^*$ to $\alpha$](image)

4.5 Heuristic Investment Rules

The preceding regression analysis indicates that both volatility and discount rate...
are statistically significant in predicting optimal hurdle rate trigger. It is also clear how the weights of both variables contribute to the changes in optimal hurdle rate. The insensitivity of hurdle rate trigger to growth rate suggests that growth rate is a minor factor which may be ignored in formulating heuristic investment rules, because the resulting forecast errors may be negligible in a GBM or an MX process.

Regression analysis and sensitivity analysis provide the evidence that $\alpha$ under an MR process may be relatively influential compared to alternative processes.

Based on the preceding analysis, we are able to formulate the heuristic investment rules for practitioners to follow under flexibility and uncertainty, given a specific stochastic process. It is found that the heuristic hurdle rate trigger under a GBM can be easily derived from a "4-8 rule", i.e. the sum of the weight of 0.4 times project volatility and the weight of 0.8 times risk-adjusted discount rate. The 4-8 rule can be mathematically expressed as follows:

$$\gamma_{GBM}^h = 0.4\sigma + 0.8\mu$$  \hspace{1cm} (48)

where the superscript $h$ denotes heuristic hurdle rate trigger.

In the similar ways of expression, the heuristic hurdle rate triggers for an MX process and an MR process can be conveniently derived from a "3-9 rule" and a "2-10 rule", respectively, as shown below:

$$\gamma_{MX}^h = 0.3\sigma + 0.9\mu$$  \hspace{1cm} (49)

$$\gamma_{MR}^h = 0.2\sigma + \mu$$  \hspace{1cm} (50)

Note that Equation (50) can only apply to the zero-growth case, i.e., $V = \bar{V}$. For a project which is characterized by mean reversion and disequilibrium, i.e., $V > \bar{V}$ or $V < \bar{V}$, heuristic hurdle rate trigger needs to be modified into a "2-10-(-2)" rule below:

$$\gamma_{MR}^h = 0.2\sigma + \mu - 0.2\alpha$$  \hspace{1cm} (51)
There are two interesting points which can be generalized from the above heuristic investment rules. First, the sum of the weights under all the stochastic processes of interest is greater than 1, which somewhat reflects the modified hurdle rates under the consideration of options of waiting, making a striking contrast to the hurdle rate without the options of waiting. Second, compared to a GBM process, additional key parameters in an alternative processes such as $\lambda$ in an MX process or $\eta$ and $\vec{V}$ in an MR process are not necessarily included in the heuristic investment rules due to their minor effects on the hurdle rate trigger. Practitioners can apply these heuristic investment rules by memorizing that the weight of volatility decreases by 0.1 and the weight of discount rate increases by 0.1 as more key parameters are required in an alternative process.

![Figure 17 The Procedure of Developing Heuristic Investment Rules](image-url)
The procedure of developing the heuristic investment rules is summarized in Figure 17. In the next section, a performance test for the robustness of these heuristic investment rules will be conducted.

Based on the heuristic hurdle rate, \( \gamma^h \), the equivalent capital budgeting triggers can be easily derived with the following equations:

\[
\Pi^h = 1 + \frac{\gamma^h - \mu}{\mu - \alpha}
\]

(52)

\[
\pi^h = I(\mu - \alpha) + \left( \frac{\gamma^h - \mu}{\mu - \alpha} \right) I(\mu - \alpha)
\]

(53)

\[
p^h = \begin{cases} 
\ln \left( 1 + \frac{\alpha}{\gamma^h - \mu} \right) / \alpha, & \alpha \neq 0 \\
1 - \frac{\gamma^h - \mu}{\mu - \alpha}, & \alpha = 0 
\end{cases}
\]

(54)

\[
p^{Dh} = \frac{\ln \left( \frac{\gamma^h - \alpha}{\gamma^h - \mu} \right)}{\mu - \alpha}
\]

(55)

5. Testing the Heuristic Investment Rules

To test the performance of the heuristic investment rules, Monte Carlo simulation is conducted by varying all the parameter values from which optimal hurdle rate (\( \gamma^* \)) and heuristic hurdle rate (\( \gamma^h \)) are computed simultaneously. The simulation of random sampling is implemented with Visual Basic codes.

There are several statistical measures used to gauge the performance of heuristic investment rules based on the criteria of association and forecast errors. The Pearson’s correlation, denoted by \( \rho \), is used to measure the linear association between \( \gamma^* \) and \( \gamma^h \). In addition, mean absolute error (MAE) and mean square
error (MSE) are also applied to measure forecast errors. MAE is to measure the center of overall absolute forecast errors, defined as a weighted average of absolute forecast errors with the relative probability as the weights. MSE is regarded as a criterion to measure the quality of heuristic hurdle rate in comparison to optimal hurdle rate such that we hope MSE minimized. MAE and MSE can be defined as follows, respectively:

\[
MAE = \frac{\sum_{i=1}^{n} |e_i|}{n} \quad (56)
\]

\[
MSE = \frac{\sum_{i=1}^{n} (e_i)^2}{n} \quad (57)
\]

where \(e\) denotes forecast error between \(g^*\) and \(h\), and \(n\) denotes sample size.

In the ideal situation where the heuristic hurdle rate can perfectly pinpoint the optimal hurdle rate, it is found that \(\rho_{g^*,h} = 1\) and \(\sqrt{e} = MSE = 0\). Consequently, \(\rho\) and the other two measures of forecast errors are expected to be as near 1 and 0, respectively, as possible in simulation. The \(F\) statistic is also reported to test goodness of fit at a significance level of 5%. Given the specific stochastic process assumption, i.e. GBM, MX, and MR, the scatter plots are given in Figure 18, 19, and 20, respectively, for the purpose of exhibiting the association between \(g^*\) and \(h\). In general, these scatter plots indicate that heuristic hurdle rate has a strong linear association with optimal hurdle rate. Figure 20 indicates that the “2-10 rule” works well on the “zero growth” projects, i.e., \(V_0 = \bar{V}\). For other mean-reverting type of projects in a disequilibrium, the “2-10 rule” tends to overestimate the hurdle rate when \(V_0 > \bar{V}\) and underestimate the hurdle rate when \(V_0 < \bar{V}\).
Figure 18 The Scatter Plot of $\gamma_{GRM}^b$ (Based on the 4-8 Rule) and $\gamma_{GRM}$
Figure 19  The Scatter Plot of $\gamma_{MX}^\alpha$ (Based on the 3-9 Rule) and $\gamma_{MX}^\alpha$. 

(A) Low-Growth Projects

(B) Mid-Growth Projects

(C) High-Growth Projects
Figure 20  The Scatter Plot of $\gamma_{MR}^\rho$ (Based on the 2-10 Rule) and $\gamma_{MR}^\rho$
Table 7  The Results of Monte Carlo Simulation in Testing Heuristic Investment Rules

Table 7 reports the results across various types of projects given the specified stochastic process. As shown in the table, $\rho^h$ is in general highly correlated with $\gamma^*$ under each of alternative processes. The $\rho$ coefficients range between 0.9830 and 0.9925 for the GBM model, between 0.9762 and 0.9909 for the MX model, and between 0.6146 and 0.9953 for the MR model. Also, $\gamma^h_{MR}$ shows a relatively lower correlation with $\gamma^h_{MR}$ when current project value deviates considerably from the long-run mean, i.e. $V_i \ne \bar{V}$, under a MR process. The above result seems to be consistent with the preceding scatter plots.

From the perspective of forecast errors, heuristic hurdle rate provides a very good approximation to optimal hurdle rate under a GBM and an MX process. MAE and MSE, in general, are minimal except in the MR case of $V_0 \ne \bar{V}$. MAE ranges
from 1.60% to 3.56% for the GBM model and from 1.22% to 1.35% for the MX model. In addition, a low MAE of 0.84% for the zero-growth MR projects imply that the 2-10 rule can be very close to optimal hurdle rate. The minimized MSE is also reported in Table 7, which also appears to be consistent with the preceding findings. The F test of goodness of fit, in general, is significant at the level of 5% across all types of projects except for the negative growth MR project.

The diagrams in Figure 20 (B) and (C) imply that the performance of the “2-10 rule” is unsatisfactory for the MR projects which deviate from the long-run mean, i.e., the growth rate is non-zero. It is apparent that the major reason for the poor performance is caused by the disequilibrium, i.e., $V_0 \neq \bar{V}$. Therefore, the result suggests that the performance of the heuristic investment rule in forecasting $\gamma_{MR}^*$ may be improved by including growth rate. By repeating the procedure of developing heuristics, a “2-10-and-minus 2” rule is proposed for the MR-type of projects. Based on the new rule, we display the scatter plot as well as the performance measures in Figure 21. With the new heuristic investment rule considering growth rate, it can be seen that the performance measures such as $\rho$, MAE, and MSE are considerably improved. The F test also indicates an overall goodness of fit at the significance level of 5%.

![Figure 21](image-url)
6. Concluding Remarks

In the paper, we have shown conventional capital budgeting rules can do more than just being “conventional”. Under the consideration of options of waiting, we obtain modified capital budgeting rules in a general expression and also for three alternative stochastic processes. Management can easily integrate these modified investment rules with their existing capital budgeting practices for making better investment decisions. These modified expressions of investment rules also permit us to readily analyze the option impacts on capital budgeting criteria. For a set of reasonable parameter values, the option impacts under a GBM can be the most significant among three alternative models.

The analysis on the cost of adopting a suboptimal investment rule suggests that the loss function of suboptimal investment behavior can be asymmetric for a GBM or a mixed diffusion-jump process and approximately symmetric for a mean-reverting process. In any case, the worst investment rule is the one that ignores the existence of managerial flexibility. On the other hand, it is shown that if a simple heuristic decision rule is used to approximate \( V^* \), the rule should be as near \( V^* \) as possible in order to minimize the opportunity cost of adopting the suboptimal investment policy.

We then develop the heuristic investment rules which provide a seemingly accurate approximation to optimal investment rules by a set of two parameters, volatility and discount rate, under the consideration of managerial flexibility and uncertainty. The basic idea behind the heuristics is that the optimal hurdle rate is highly linear and sensitive to the key parameters such as project volatility and discount rate. By using a combination of weights of volatility and discount rate, the heuristic hurdle rate can be easily estimated to provide a satisfactory performance close to optimality. The heuristic hurdle rate can be conveniently transformed into alternative capital budgeting instruments. Compared to profitability index of 1.5 or hurdle rate of 20% suggested by McDonald (1999), it is demonstrated that these heuristic investment rules can be a simple decision tool with a robust performance for corporate practitioners to make investment decisions.

Previous research in the area such as Boyle and Gutherie (1997), McDonald (1999), and Wamback (2000) tend to assume that the underlying value process follows a GBM. In the paper, the assumption of stochastic process is extended to the
other practical situations in that project value may evolve as a mixed diffusion-jump process (e.g. pharmaceutical industry) or a mean-reverting process (e.g. natural resource mining industry). As suggested in Lint and Pennings (1998), in the industry where a first-mover advantage is prominent, excessive waiting may diminish a profitable investment opportunity. Theoretically, this situation can be characterized by a mixed diffusion-jump model. In this paper, it is shown that for a moderate jump probability (0-20%) the justification of strategically entering the market can be achieved simply through adjusting the weights of volatility and discount rate. With regard to mean reversion, as argued by Schwartz (1997) and Slade (2001), the mean-reverting property in irreversible investments may decrease project volatility in the long-run and thus lead to a lower option value as well as a lower investment trigger. Our heuristic investment rules in fact seem to be consistent with this argument in that the weight of project volatility is reduced in comparison to the GBM model. Since the MR trigger is generally lower than the GBM trigger, the MR hurdle rate is thus lower than the GBM hurdle rate, other parameters being constant.

It is important to point out that the research in the paper does not argue that management should sacrifice optimal investment rules for heuristic investment rules. Instead the paper intends to provide simple investment rules of thumb, which, on one hand, approximate optimal investment trigger and, on the other hand, fits into existing capital budgeting practices. The major implication is that with these heuristic investment rules, corporate practitioners can accommodate sophisticated real options valuation to common capital budgeting practices without involving relatively complicated real options techniques.
Reference:


