Bargaining, Profit-Sharing and Irreversible Investment Decisions In An Oligopoly

Jyh-bang Jou
National Taiwan University

Tan Lee
Yuan Ze University

Correspondence:
Jyh-bang Jou
No. 1 Roosevelt Rd. Sec. 4,
Graduate Institute of National Development, College of Social Science,
National Taiwan University, Taipei 106, Taiwan, R.O.C.
Tel: (886-2) 23630231 ext. 3590
E-mail: jbjou@ccms.ntu.edu.tw
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Abstract

This article considers a firm in an oligopoly as a quasi-permanent organization of shareholders and employees. The firm undertakes incremental investment and the investment expenditures are fully sunk. The firm’s employees receive a fixed wage rate as well as share a portion of the firm’s profits. The profit-sharing rate is determined by the firm’s manager, who finds a cooperative game solution through mediating between the shareholders and employees. The firm’s manager will grant a lower profit-sharing rate to the employees and will install a smaller capacity when facing more competitors. However, the choice of capacity for the industry as a whole will increase as competition becomes more intense.

Key Words: Irreversible Investment, Profit-Sharing

JEL No: G31, J33

1. Introduction

Profit-sharing schemes are quite common for large firms. According to a 1987 survey by the Government Accounting Office (Kenz and Simester, 2001), for 326 fortune 1,000 firms, 54% of non-union and 39% of union firms had profit-sharing schemes.\(^1\) However, few theoretical studies investigate why firms employ these schemes. Two such studies are Aoki (1980) and Moretto and Rossini (1995),\(^2\) both of which regard a firm as a quasi-permanent organization of shareholders and employees. Aoki indicates that the relative bargaining power between shareholders and employees affects the distribution of profits between them. Moretto and Rossini argue that Aoki ignores two important characteristics of investment emphasized by the real options literature, i.e., the rewards from the investment may be uncertain over time and the investment expenditures may be sunk. They employ the entry-exit model of Dixit (1989) who assumes both that a firm irreversibly undertakes fixed-scale lumpy investment and that the firm can temporarily shut down its operation. They then investigate how the shut-down option as well as the relative bargaining power between shareholders and employees affect distribution of the firm’s profits. These two studies, however, focus on a monopolized firm. This article, instead, will focus on an oligopoly, thus being able to investigate how competitive pressure affects a firm’s profit-sharing decision in industry equilibrium.

This article considers an industry that is composed of a fixed number of identical firms. Each firm distributes some portion of its profits to employees whose skills and knowledge are more or less specific to the firm. Following the literature of real options (Dixit and Pindyck,\(^1\) Profit-sharing schemes were more widely employed than were employment stock ownership plans (ESOPs). Mitchell (1995) reported that for medium and large establishments in the U.S., 16% of total employees entered into profit-sharing plans, while only 3% entered into the ESOPs.\(^2\) See also Aoki (1982) who allows a firm to have junior and senior employees, and Moretto and Rossini (1997) who allow repeated bargaining.

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\(^2\) See also Aoki (1982) who allows a firm to have junior and senior employees, and Moretto and Rossini (1997) who allow repeated bargaining.
1994), this article assumes that the firm incurs sunk costs upon expanding capacity. The firm also faces demand function with a constant elasticity and employs linearly homogenous Cobb-Douglass production technology with capital and labor as its inputs. The uncertainty is characterized by a multiplicative demand-shift factor that follows a geometric Brownian motion process. At each instant, the firm’s manager acts on behalf of shareholders and makes the incremental investment decision. Immediately before making this decision, the firm’s manager also mediates between employees and shareholders so as to decide an appropriate profit-sharing rule between these two parties.

Our framework enables us to investigate how competitive pressure affects a firm’s incentive to invest. We find that, given a firm’s profit-sharing rate, increasing competition induces a firm to invest earlier. By contrast, by employing a model without profit-sharing, Grenadier (2002) finds that increasing competition induces a firm to invest later. Both this article and Grenadier’s assume that a firm in an oligopoly irreversibly undertakes increment investment. However, this article allows capital and labor to be substitutable each other, while Grenadier assumes that capital, the only input, produces output corresponding to a one-to-one relationship. Consequently, in our framework, capital exhibits diminishing marginal returns, which no longer holds in Grenadier’s framework. This divergence explains why an individual firm facing more competitors is more bold in investing in Grenadier’s framework than in our framework.

We also show that a firm facing more competitors will grant a lower share of profits to its employees. This is because in our framework, employees not only receive a fixed wage rate that is determined by the external labor market, but also receive a bonus out of the firm’s profits. However, a firm’s profits will be lower as more firms exist in the industry. Increasing competition thus exerts more harm on shareholders than on employees because the pie that shareholders can share shrinks. Consequently, as a mediator, the firm’s manager must grant a larger share to its shareholders, or equivalently, a lower share to its employees when facing more competitors.

The traditional literature of profit-sharing has focused on a different issue from this article. For example, Weitzman (1983, 1985) and Estrin and Wadhwani (1990) argue that profit-sharing schemes might affect labor productivity, the unemployment rate, and wage rate. By contrast, this article explores the determinants of optimal profit-sharing schemes and finds that a firm’s employees will receive a lower profit-sharing rate as either their bargaining power is weaker or the firm faces more competitors.

This article focuses on a model of continuous investment that stems from the real options model of Pindyck (1988). Pindyck’s model has been extended to investigate other issues such as price ceiling (Dixit, 1991), hiring and firing decisions (Bentolina and Bertola, 1990), dividend policies (Holt, 2003), and strategic interactions (Grenadier, 2002). Note that in Pindyck’s model, a firm has the option to temporarily suspend its operation if marginal profit flow becomes negative. By contrast, our article, like Bertola and Caballero (1994) and Abel and Eberty (1996), assumes that a firm does not have any option to shut down its operation because the firm produces at a level such that its marginal profit is equal to zero.

Alchian and Demsetz (1972) first proposed the idea that profit-sharing can enhance productivity through mutual monitoring among workers. The literature on efficiency wage (see, e.g., Shapiro and Stiglitz, 1984) also exhibits a similar idea that greater compensation to workers can enhance productivity.
Recent articles on the real-option approach to investment have allowed strategic interactions between firms, but focus on a firm that is controlled by the shareholders. For example, Grenadier (2002) mentioned before considers this firm that competes with the other firms in a Cournot-Nash environment. Some other articles allow this firm to compete with another firm in a Stackelberg environment. See, for example, Dixit and Pindyck (1994), Lambrecht (2001), and Weeds (2002).

This article is organized as follows. Section II constructs the basic model, followed by investigating the short-run output and the long-run investment equilibrium. Section III investigates the profit-sharing decision. Sections II and III also investigate the comparative-statics results regarding the impacts on the firm’s choices of capacity and profit-sharing rate resulting from changes in competitive pressure, the relative bargaining power between shareholders and employees, demand uncertainty, the price elasticity of demand, and the output elasticity of labor. Simulation analysis is also presented so as to make some of these theoretical results more vivid. The last section concludes and suggests for future research.

II. The Model

A. Short-Run Equilibrium

We extend the model of a firm’s incremental investment decision in an oligopoly as that in Grenadier (2002) by allowing a firm’s stockholders to share its profits with its employees. Suppose that an industry is composed of $N$ identical firms, indexed by $i=1,\ldots,N$. Let $l_i(t)$ denote the amount of firm $i$’s workers, and $k_i(t)$ denote its amount of capital stock. While Grenadier assumes that a firm’s production technology is such that each unit of capital yields one unit output per time period, we assume that a firm’s production technology is of a Cobb-Douglas form:

$$q_i(t) = l_i(t)^\gamma k_i(t)^{\gamma-1}, \quad 1 > \gamma > 0, \quad (1)$$

where $q_i(t)$ is firm $i$’s output. Suppose that the industry demand function is of a constant elasticity:

$$Q(t) = X(t)^{\frac{1}{\varepsilon}} P(t)^{-\varepsilon}, \quad \varepsilon > 0, \quad (2)$$

where $Q(t)$ is the aggregate industry demand, $g = 1/\gamma$, $h = 1/\varepsilon$, $f = hg/(h+g-1)$, $P(t)$ is output price, and $X(t)$ is a disturbance term that evolves a geometric Brownian motion as

$$dX(t) = \mu X(t) dt + \sigma X(t) d\Omega(t), \quad (3)$$

where $d\Omega(t)$ is an increment to a standard Wiener process.

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5 Suppose that $Z(t) = X(t)^\frac{1}{\varepsilon}$, then by Itô’s lemma, $Z(t)$ follows the process: $dZ(t) = \left[\frac{\mu}{f} + \frac{1}{2f}(\frac{1}{f} - 1)\sigma^2\right]Z(t)dt + \frac{\sigma}{f} Z(t)d\Omega(t)$. In what follows, for ease of exposition, we will investigate the impacts of the drift rate and the instantaneous volatility of $X(t)$ rather than those of $Z(t)$. 

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Suppose that the wage rate received by firm \( i \) ’s workers is set in the external competitive labor market, and is fixed at \( w \). In addition, from equation (1), \( l(t) = q_i'(t)k(t)^{-g} \). As a result, firm \( i \) ’s short-run variable cost \( w_l(t) \), denoted by \( C_i(t) \), is given by

\[
C_i(t) = wq_i(t)\frac{1}{1-g}k(t)^{-\frac{1}{g}}.
\] (4)

From equation (2), the inverse demand function is given by \( P(t) = X(t)^{\frac{1}{g}}Q(t)^{-\frac{g}{1-g}} \). Note that in the short-run equilibrium, aggregate demand should be equal to aggregate supply, i.e., \( Q(t) = \sum_{j=1}^{N} q_j(t) \). Firm \( i \) ’s short-run revenue, denoted by \( R_i(t) \), is equal to \( P(t) \) multiplied by \( q_i(t) \):

\[
R_i(t) = X(t)^{\frac{1}{g}}q_i(t)Q(t)^{-\frac{g}{1-g}}.
\] (5)

We assume that a firm is a quasi-permanent organization of shareholders and employees. Employees own skills and knowledge that are specific to the firm due to their quasi-permanent association with the firm. Employees thus have bargaining power regarding distribution of the firm’s profits, denoted by \( \pi_i(t) = R_i(t) - C_i(t) \). The firm’s shareholders are concerned only about the part of the firm’s profits attributed to them, denoted by \( \pi_s(t) \). Let \( \lambda_i(t) \) denote the portion of profits attributed to firm \( i \) ’s employees, where \( 0 \leq \lambda_i(t) \leq 1 \). The profits attributed to firm \( i \) ’s shareholders is thus equal to \( 1 - \lambda_i(t) \) multiplied by \( \pi_i(t) \), i.e.,

\[
\pi_s(t) = (1 - \lambda_i(t))(R_i(t) - C_i(t)).
\] (6)

We define firm \( i \) as a joint organization run by a manager who acts on behalf of shareholders when making output and investment decisions. The manager also mediates between employees and shareholders regarding distribution of the firm’s profits. Consequently, firm \( i \) ’s manager will choose an output level \( q_i(t) \) to maximize \( \pi_s(t) \), which yields the short-run equilibrium that is determined by equating marginal revenue and the marginal cost. Denote \( q_i^*(t) \) as firm \( i \)’s choice of output in the short-run equilibrium. Differentiating \( C_i(t) \) in equation (4) and \( R_i(t) \) in equation (5) with respect to \( q_i(t) \), equating the results, and imposing the equilibrium condition \( q_j(t) = q_i(t), \ j = 1, \ldots, N \), yields

\[
q_i^*(t) = (wg)\frac{1}{1+g}N^{-\frac{g}{1+g}}(1 - \frac{h}{N})^{\frac{1}{1+g}}X(t)^{\frac{1}{g}}k_i(t)^{\frac{g}{1+g}}.
\] (7)

Substituting \( q_i^*(t) \) into equation (6) yields the optimized value of \( \pi_s(t) \), denoted by

\[
\pi_s^*(k_i(t), X(t), \lambda_i(t)) = (1 - \lambda_i(t))a_sX(t)k_i(t)^{-g}, \quad (8)
\]

where \( a_s = [1 - \frac{h}{N}]^g - 1 \) is assumed to be smaller than one to ensure that \( \pi_s^*(\cdot) \) is increasing in \( k_i(t) \). Substituting \( q_i^*(t) \) into equation (4) yields the optimized value of short-run cost as given by

\[
C_i^*(t) = a_sX(t)k_i(t)^{-g} / m,
\]

where the term \( m = (1 - h/N)^g - 1 \), which is also equal to the firm’s profits over total wage.
payments.

The payoff to employees as a whole include two parts: total wage payments $C'(t)$ given by equation (9), and the firm’s profits attributed to employees $\lambda(t)a,X(t)k(t)^{-\gamma}$. Summing these two parts yields the payoff to employees as a whole as given by

$$E'(t) = (\lambda(t) + \frac{1}{m})a,X(t)k(t)^{-\gamma}.$$  

(10)

B. Long-Run Equilibrium

In our framework, in the long-run, a firm’s manager, who acts on behalf of shareholders, must decide how much capacity to build initially and when to expand it later. We can solve the manager’s capacity choice problem in two steps. First, we can determine the value of an extra unit of capacity. Second, we can simultaneously determine the value of the option to invest in this unit, together with the decision rule for exercising this option.

Suppose that the investment costs on purchasing one unit of capital, denoted by a constant $P_x$, are fully sunk. Following Pindyck (1988) and Kandel and Pearson (2002), firm $i$’s net value attributed to its shareholders is given by

$$W'(k(t),X(t),\lambda(t)) = V'(k(t),X(t),\lambda(t)) + F'(k(t),X(t),\lambda(t)) - P_xk(t),$$

(11)

where on the right-hand side of equation (11), the first component is the value of the existing stock of investment, the second component is the value of the growth option, and the last component is the investment cost. Define the value of the next unit of capital as \( \Delta V'(k(t),X(t),\lambda(t)) = \frac{\partial V'(k(t),X(t),\lambda(t))}{\partial k(t)} \), and the value of the option to purchase the next unit of capital as \( \Delta F'(k(t),X(t),\lambda(t)) = -\frac{\partial F'(k(t),X(t),\lambda(t))}{\partial k(t)} \), given that $k(t)$ has already been installed. Using these definitions, we can write

$$W'(k(t),X(t),\lambda(t)) = \int_{k(t)}^{k(t)+1} \Delta V'(k(t),X(t),\lambda(t))dk(t) + \int_{k(t)}^{k(t)+1} \Delta F'(k(t),X(t),\lambda(t))dk(t) - P_xk(t).$$

(12)

Suppose that $\rho$ denotes the (risk-adjusted) discount rate applied to firm $i$’s shareholders, then $\Delta V'(k(t),X(t),\lambda(t))$ should be equal to the expected present value of the marginal return to capital, i.e.,

$$\Delta V'(k(t),X(t),\lambda(t)) = E\int_{\tau} \hat{\pi}_\tau(k(\tau),X(\tau)\lambda(\tau))d\tau$$

$$= a_\lambda(1-f)(1-\lambda(t))X(t)k(t)^{-\gamma}.$$  

(13)

Based on Appendix A,

$$\Delta F'(k(t),X(t),\lambda(t)) = b_\gamma(X(t)k(t)^{-\gamma})^\beta,$$  

(14)

where $b_\gamma$ is a constant to be determined, and $\beta_1$ and $\beta_2$ respectively are the larger and

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6 These sunk costs may come from the “lemons” problem, asset specificity, or government regulations (Dixit and Pindyck, 1994).
smaller roots of the quadratic equation

\[ \phi(\beta) = -\frac{1}{2} \sigma^2 \beta (\beta - 1) - \mu \beta + \rho = 0. \]  \hspace{1cm} (15)

Given that \( \beta, \beta_z = -2\rho / \sigma^2 \), \( \phi(\beta) \) in equation (15) can also be written as (see, Dixit, 1991)

\[ \phi(\beta) = -\frac{1}{2} \sigma^2 (\beta - \beta_z)(\beta - \beta_z) = \frac{\rho(\beta - \beta_z)(\beta - \beta_z)}{\beta_z}. \] \hspace{1cm} (16)

Firm \( i \)'s manager, who acts on behalf of the firm's shareholders regarding the firm's long-run investment decision, must choose the path for \( k(t) \) so as to maximize the firm's net value to its shareholders, \( W'(k(t), X(t), \lambda_i(t)) \). Given that the investment costs are fully sunk, firm \( i \)'s manager thus faces a one-sided control problem (Harrison and Taksar, 1983): The manager will invest whenever the demand-shift factor \( X(t) \) reaches a threshold level, denoted by \( X'^*(t) \). Both \( X'^*(t) \) and the constant \( b_e \) are solved through the conditions given by

\[ \Delta F'(k(t), X'(t), \lambda_i(t)) = \Delta V'(k(t), X'(t), \lambda_i(t)) - P_e, \] \hspace{1cm} (17)

\[ \frac{\partial \Delta F(k(t), X'(t), \lambda_i(t))}{\partial X(t)} = \frac{\partial \Delta V(k(t), X'(t), \lambda_i(t))}{\partial X(t)}. \] \hspace{1cm} (18)

Equation (17) is derived by setting the derivative of \( W'(\cdot) \) in equation (12) with respect to \( k_i(t) \) equal to zero. It is called the value-matching condition, which says that firm \( i \)'s manager will not expand capacity unless the option value to install one more incremental unit of capital (the left-hand side) equals the net value of the last incremental unit of installed capital (the right-hand side). Condition (18) is the smooth-pasting condition, which says that Condition (17) is required to be continuous and smooth at the critical exercise point \( X'^*(t) \). Otherwise, firm \( i \)'s manager could do better by adding capital at a different point. Solving equations (17) and (18) simultaneously yields

\[ b_e = \frac{a_s (1 - f)}{(\rho - \mu) \beta_z} \left( \frac{\beta_z - 1}{P_e} \right)^{\gamma - 1}, \] \hspace{1cm} (19)

\[ X'(t) = d_e (N_k(t))', \] \hspace{1cm} (20)

where \( d_e = w^{\frac{\gamma}{\gamma + 1}} \left( \frac{\rho - \mu}{\beta_z} \right) P_e v_s \left( \frac{1}{1 - f} \right)^{\gamma + 1} / m \), and \( v_s = \left[ 1 - \frac{h}{N} \right]^{\gamma / m} / m \).

The first important Proposition is derived from equation (20).

**Proposition 1**: (a) Given the capital stock for the industry as a whole, a firm will invest later when facing more competitors. (b) Given an individual firm's stock of capital, the firm will invest later when facing more competitors.

Proof: See Appendix B.

Pindyck (1993) constructs a two-period model, which, like our article, assumes that a firm faces an iso-elastic demand function and a Cobb-Douglas production function. Pindyck argues that competition has a depression effect on investment. Our result in Proposition 1(a)
complements Pindyck’s argument. However, our result in Proposition 1(a) is just opposite to the result of Grenadier (2002), which also investigates a firm’s incremental investment decision in an oligopoly in a real options framework. This divergence arises because we employ different assumptions about a firm’s technology from those of Grenadier. We assume that a firm employs Cobb-Douglas production technology with capital and labor as inputs. Instead, Grenadier assumes that a firm employs linear technology with capital as the only input. Our assumption implies that a firm’s marginal cost is increasing with its output, while Grenadier’s assumption implies that a firm’s marginal cost is equal to zero. As a result, with our assumption, as competition becomes more intense, the marginal return to capital, i.e., \( \frac{\partial \pi}{\partial k} \), is lower, thus triggering a firm to wait for a better state of nature to invest. By contrast, with Grenadier’s assumption, as competition becomes more intense, the marginal return to capital becomes higher, thus triggering a firm to invest sooner.

Proposition 1(b) indicates that increasing competition triggers a firm to invest later even if we hold the firm’s capital stock (rather than the capital stock for the industry as a whole) as constant. This also contrasts with Grenadier (2002), which will predict that, given a firm’s capital stock, increasing competition will exhibit an indefinite effect on a firm’s incentive to invest.

The optimal trigger strategy in equation (20) translates into an expression for the process of equilibrium capital, \( k'(t) \), or the “desired” level of capital coined by Bertola and Caballero (1994):

\[
k'(t) = (X(t)/d_X)^{\gamma}/N.
\]

Firm i’s optimal policy is to regulate the state variable \( k_i(t) \) at a lower barrier, denoted \( k'(t) \). If the currently installed capital \( k(t) \) is smaller than \( k'(t) \), the firm’s manager should invest so as to obtain \( k_i(t) = k'(t) \); otherwise no action should be taken. The firm’s manager should form expectations about the distant future regarding when and how much to invest because irreversible investment decisions will affect future cash flows, because the firm may be stuck with an excessive stock of capital in the initial period.

The second important Proposition follows from equation (21).

**Proposition 2:** A firm’s desired capital stock will be lower if (a) the profit-sharing rate attributed to employees is higher, (b) competition becomes more intense, and (c) demand

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7. Aguerrevere (2003), who assumes that a firm in an oligopoly faces a linear demand function and a constant marginal cost function, yields a result that is in line with Grenadier’s result.

8. Grenadier focuses only on a firm’s optimal timing decision given the whole industry’s capital stock. However, one can easily derive a firm’s investment timing, holding its own capacity as constant, through the following procedures. First, Substituting \( nq \) for \( Q \) in equation (21) (p.703) of Grenadier’s article, where \( n \) is the number of firms, and \( q' \) is the capital stock for a firm. Second, rewriting the investment trigger as \( X'(q') \). Finally, differentiating \( X'(q') \) with respect to \( n \) yields the result which indicates that the investment trigger will be increasing (decreasing) with competitive pressure if \( n\gamma - \gamma - 1 > (<) 0 \), where \( \gamma \) denotes the demand elasticity.
uncertainty is more significant.\textsuperscript{9}

Proof: See Appendix C.

The reason for Proposition 2 is as follows. Suppose that a firm’s manager attributes more profits to employees. The manager, who acts on behalf of the firm’s shareholders, will be less eager to invest because the marginal return from adding an extra unit of capital becomes lower.\textsuperscript{10} In Figures 1, 2, and 3, curve $Z, Z_1$ depicts the negative relationship between a firm’s desired capital stock and the profit-sharing rate attributed to employees. Similarly, when a firm faces more competitors, then the marginal return from adding an extra unit of capital will become lower, and thus the firm’s manager will install less capital. In Figure 1, curve $Z, Z_1$ shifting downward to curve $Z, Z_2$ indicates that a firm that faces more competitors will install less capital. In addition, the reason why increasing demand uncertainty reduces a firm’s incentive to invest is as follows (Aguerrevere, 2003): increasing uncertainty increases the likelihood of lower future demand, thus raising the cost of excess capacity. Therefore, more uncertainty reduces the level of optimal capacity. In Figure 2, curve $Z, Z_1$ shifting downward to curve $Z, Z_2$ depicts this depressing effect.

Proposition 2(a) also implies the following result:

\textbf{Corollary 1}: As compared to the case where profit-sharing plans do not exist, a firm that optimally chooses its profit-sharing schemes will install a smaller capacity.

The result of Corollary 1 is also shown by Figure 1 where the optimal capacity chosen by the former firm, $k_{0}^\ast$, is smaller than that chosen by the latter firm, $k_{1}^\ast$.

After solving the desired capital stock $k'(t)$ of a firm, we will respectively derive both the expected net firm values attributed to the shareholders and employees. We can then use these values to find the optimal profit-sharing rate in the next section. Substituting $b_{c}$ in equation (19) into equation (14), and then integrating the first and second terms on the right-hand side of equation (12) yields the optimized values of $V'(k_{1}(t),X(t),\lambda_{1}(t))$ and $F'(k_{1}(t),X(t),\lambda_{1}(t))$, respectively. Summing these two optimized terms, and then deducting the term $P_{1}k'(t)$ yields the optimized value of the firm’s net value to its shareholders, $W'(k_{1}(t),X(t),\lambda_{1}(t))$, as given by

$$W'(k_{1}(t)) = \frac{1}{\left(\beta_{1} - 1\right)} + \frac{1}{(f\beta_{1} - 1)} - 1]P_{1}k'(t) ,$$  \hfill (22)

\textsuperscript{9} The impacts of the price elasticity of demand and the output elasticity of labor on a firm’s incentive to invest, however, are indefinite.

\textsuperscript{10} We derive this result because we assume that profit-sharing schemes are unrelated to labor productivity. If these schemes enhance labor productivity, then the manager may be induced to invest more.
where \( \beta_i \) is required to be greater than one, which we assume here and in what follows.

Suppose that \( W^*(k(t), X(t), \lambda_i(t)) \) is firm \( i \)'s net value attributed to its employees and that the discount rate applied to firm \( i \)'s employees is also equal to \( \rho \). Integrating employees’ payoff given by (10) over time by applying the discount rate \( \rho \) yields

\[
W^*(k(t), X(t), \lambda_i(t)) = \int_0^\infty e^{-\rho \tau} \partial E^*E^*(k(\tau), X(\tau), \lambda_i(\tau))d\tau
\]

\[
= \frac{(\lambda_i(t) + 1/m)}{(\rho - \mu)} a_t X(t) k(t)^{1/\beta_i}.
\]

(23)

Evaluating equation (23) at \( X(t) = X^*(t) \) and \( k(t) = k^*(t) \) yields the optimized value of \( W^*(k(t), X(t), \lambda_i(t)) \) as given by

\[
W^*(k^*(t), \lambda_i(t)) = \frac{\beta_i}{(1 - \lambda_i(t)) (\beta_i - 1)} (\lambda_i(t) + 1/m) \frac{1}{(1 - f)} P_i k^*(t).
\]

(24)

It is worthy mentioning that in our framework, both a firm’s growth option value and the firm’s policy to invest is determined by the firm’s manager who acts on behalf of the firm’s shareholders only. Given the firm’s profit-sharing rate, the manager’s decision is best from the viewpoint of shareholders only, yet it is not best for the firm’s shareholders and employees as a whole. The firm’s manager, however, is unable to act on behalf of shareholders only when deciding a profit-sharing rule. This is because employees are able to exert bargaining power on the firm’s profits as they acquire firm-specific skills and knowledge.

III. Bargaining on The Distribution of Profits

We assume that the employees are homogeneous and thus they are renumerated equally by a firm. We also assume that a representative employee will bargain for the profit-sharing rate with the manager immediately before the manager makes investment decisions. The representative employee has a Von Neumann-Morgenstern utility function \( U_e(\cdot) \) defined on the domain of possible value of his expected lifetime earnings, which is equal to \( W^*(\cdot) \) in equation (24) divided by the amount of labor employed by firm \( i \) in the long-run equilibrium, \( l^*(t) \). The term \( l^*(t) \) is derived by substituting \( k(t) = k^*(t) \) into \( C^*(t) \) in equation (9) and then dividing the result by the wage rate \( w \). In other words, \( l^*(t) = a_t X(t) k^*(t)^{-1/\beta_i} / w m \). Similarly, the manager is assumed to have a single Von Neumann-Morgenstern utility function \( U_s(\cdot) \) defined on the net value of the firm attributed to shareholders, i.e., \( W^*(k^*(t)) \) in equation (22).

We assume that firm \( i \)'s manager mediates between employees and shareholders so as to find a cooperative game solution regarding the distribution of firm \( i \)'s profits. The equilibrium can be characterized as the result of the maximization of the generalized Nash product given by (Harsanyi, 1977):

\[
L = (U_e(W^*(\cdot)/l^*(t)) - \overline{U}_e)^a (U_s(W^*(\cdot)) - \overline{U}_s)^a,
\]

(25)
where \(0 < \theta < 1\), \(U_e(W^e) / \Gamma(t)\) and \(U_s(W^s)\) are respectively the utility functions for the representative employee and shareholders, and \(\bar{U}_e\) and \(\bar{U}_s\) are respectively the threat points for the employee and shareholders, i.e., the employee’s utility of alternative jobs available in the labor market, and the utility shareholders can get elsewhere. To find a closed-form solution for the cooperative game, we follow Moretto and Rossini (1995) by assuming that both the representative employee and shareholders have the constant relative risk averse functions given by \(U_e(W^e) = W^{e(1-\alpha_e)} / (1-\alpha_e)\), and \(U_s(W^s) = W^{s(1-\alpha_s)} / (1-\alpha_s)\), respectively, where the terms \(\alpha_e\) and \(\alpha_s\) are both between zero and one, and both represent the coefficients of relative risk aversion for the employees and shareholders, respectively. In addition, the threat points for both the representative employee and shareholders are normalized to be equal to zero, i.e., \(\bar{U}_e = \bar{U}_s = 0\).

Taking the logarithm of the objective function \(L\) defined in (25), differentiating \(\log L\) with respect to \(\lambda(t)\), and then setting the result equal to zero yields the optimal value of \(\lambda(t)\) as given by

\[
\lambda(t) = \frac{1}{1 + fr_e} \left[ fr_e - \frac{1}{m} \right],
\]

where \(r_e = (1 - \alpha_e) \theta / [(1 - \alpha_e)(1 - 0)]\) denotes the bargaining power of firm \(i\)’s employees relative to that of firm’s shareholders, and it is required that \(fr_e \geq 1/m\) to ensure that \(\lambda(t)\) is non-negative, which we assume here and in what follows. In Figures 1, 2, and 3, vertical line \(\lambda\) depicts \(\lambda(t)\) defined in equation (26), which indicates that the optimal profit-sharing rate is constant over time. Furthermore, neither demand growth expectations nor demand uncertainty will affect the optimal profit-sharing rate, yet some exogenous variables will do, as indicated by Proposition 3.

**Proposition 3:** A firm’s manager will grant a lower profit-sharing rate to employees as (a) the firm faces more competitors, (b) the firm’s employees have a relatively lower bargaining power, (c) the price elasticity of demand is larger, and (d) the output elasticity of labor is higher.

**Proof:** See Appendix D.

The reason for Proposition 3(a) is as follows. A firm’s profit relative to total wage compensation is equal to \(m = (1 - h/N)g - 1\). This ratio is decreasing with the number of firms \(N\), thus indicating that increasing competition will exert more harm on shareholders than on employees because employees always receive a fixed wage rate that is independent of product market structure. Consequently, as a mediator, a firm’s manager should grant a larger profit-sharing rate to shareholders, and thus a lower profit-sharing rate to employees as competition becomes more intense. This is shown by Figure 1 where line \(Y_1Y_1\) shifting leftward to line \(Y_2Y_2\). Furthermore, the result of Proposition 3(b) is in line with the bargaining literature such as Aoki (1980) and Moretto and Rossini (1995), i.e., as the bargaining power of employees becomes weaker, the manager will naturally grant a lower profit-sharing rate to them. This is indicated by Figure 3 where line \(Y_1Y_1\) shifting leftward to line \(Y_2Y_2\).
The intuition behind Propositions (3c) and (3d) is as follows. As mentioned before, we assume that \( f < 1 \) so as to ensure that capital is productive. This assumption also implies that the price elasticity of demand is greater than one. When this elasticity is increased from one, however, the firm’s profits will decline. Similarly, as the output elasticity of labor (\( \gamma \)) becomes higher, the output elasticity of capital (\( 1 - \gamma \)) will then become lower. This reduces the marginal return to capital, and thus lowers the firm’s profits. Consequently, greater price elasticity of demand and greater output elasticity of labor will do more harm on shareholders than on employees who can receive a fixed wage rate. The manager will thus distribute less profits to employees.

Previous study (Estrin, Pérton, Robinson, and Wilson, 1997) has shown that the profit-sharing premium, defined as profit-sharing bonuses over total wage payment, was of order 5%-10%. We, however, are interested in how this premium is affected by competitive pressure, as stated below.

**Proposition 4:** The profit-sharing premium is lower as competition becomes more intense.

Proof: See Appendix E.

The reason behind Proposition 4 is as follows. Appendix E shows that the profit-sharing premium, \( pr \), is equal to \( \lambda_t(t) \) (the profit-sharing rate attributed to employees) multiplied by \( m \) (the firm’s profits over total wage payment). Consequently, two effects arise as competition becomes more intense. First, given the profit-sharing rate, increasing competition drives down the firm’s profits. Consequently, profit-sharing bonuses relative to total wage payment will decline. Second, given the firm’s profits, increasing competition will result in a lower profit-sharing rate, as indicated by Proposition 3(a). Consequently, profit-sharing bonuses relative to total wage payment will decline. Proposition 4 follows because the above two effects reinforce each other.

Combining equations (21) and (26) yields comparative-statics results regarding the total effect of the exogenous variables such as competitive pressure (\( N \)) or the relative bargaining power (\( \alpha \)) on the desired capital stock, \( k(t) \).

**Proposition 5:**

(a) As the bargaining power of employees is decreasing relative to that of shareholders (either \( \theta \) or \( \alpha \) decreases or \( \alpha \) increases), a firm’s optimal capacity will rise.

(b) As competitive pressure increases (\( N \) is larger), a firm’s choice of capital stock will decrease, yet the desired capital stock for the industry as a whole will decrease.

Proof: See Appendix F.

Figure 3 explains the result of Proposition 5(a). As the bargaining power of employees relative to that of shareholders, \( \alpha \), is weaker, then shareholders can naturally ask for a larger share of profits, and the manager will also agree with this demand. This is shown by a shift from line \( Y' \) to line \( Y' \), where the profit-sharing rate to employees is reduced from \( \lambda \) to \( \lambda' \). With a larger profit-sharing rate to shareholders, the manager will install a larger capacity,
as suggested by Proposition 2(a). This is also shown in Figure 3 where the optimal capacity is increasing from \( k^*_i \) to \( k^*_j \).

We use Figure 1 to explain the result stated in Proposition 5(b). Suppose that the initial equilibrium point is \( E_i \), which is the intersection of vertical line \( Y_i Y_i \) (defined by equation (26)) and curve \( Z_i Z_i \) (defined by equation (21)). Increasing competition will affect a firm’s desired capital stock through two channels. First, as shown by Proposition 1(b), given the choice of the profit-sharing rate, as competitive pressure increases, the firm’s manager will invest later. Equivalently, the manager will install a smaller capacity, as suggested by Proposition 2(b). This is shown by a fall of the choice of capital stock from \( k^*_i \) to \( k^*_j \). We call this the “direct” effect. Second, as indicated by Proposition 3(a), as competitive pressure increases, given a firm’s choice of capital stock, the firm’s manager will grant a lower share of profit to the employees, which is depicted by a leftward shift from line \( Y_i Y_i \) to line \( Y_i Y_i \). The new equilibrium \( \lambda^*_i \) is lower than the old one, \( \lambda^*_i \). As indicated by Proposition 2(a), a lower profit-sharing rate to employees, in turn, induces the manager to install a larger capacity. This is shown by a rise of the optimal capacity from \( k^*_i \) to \( k^*_j \). We call this the “indirect” effect. Figure 1 shows that the new equilibrium of the choice of capital stock, \( k^*_i \), is smaller than the old one, \( k^*_i \). This is because as shown by equation (F1), when competitive pressure increases, the direct effect outweighs the indirect effect induced by a lower profit-sharing rate granted to employees. However, as competition becomes intense even though an individual firm reduces its desired capital stock, yet the desired capital stock for the industry as a whole is increasing due to more firms exist in the market.

We make our theoretical results more meaningful through numerical examples. The benchmark parameter values are chosen as follows: \( X(t) = w = P = 1 \), \( e = 1.15 \), \( y = 0.50 \), \( \alpha = \alpha = 0.5 \), \( \rho = 10\% \) per year, \( \mu = 0\% \) per year, and \( \sigma = 20\% \) per year. Given this benchmark parameter values, Figure 4 shows the responses of two endogenous variables, the profit-sharing rate to employees, \( \lambda(t) \), and the profit-sharing premium, \( pr^r \), to a change of \( N \) around the region \((1, 20)\). Figure 5 shows four endogenous variables, \( k^*_i(t) \), \( Nk^*_i(t) \), \( k^*_i(t) \), \( Nk^*_i(t) \), to a change of \( N \) around the region \((1, 20)\), where \( k^*_i(t) \) and \( k^*_i(t) \) respectively denote a firm’s optimal capacity without and with profit-sharing schemes.

Figure 4 shows that when only one firm exists, both the profit-sharing rate and premium to employees are substantial, with the former equal to 45% and the latter equal to 639%. However, both decline sharply as more firms stay in the market. When there are twenty firms in the market, the former becomes 0.70%, while the latter becomes 0.77%. Note also that the profit-sharing premium is located at the range indicated by Estrin et al. (1997), i.e., \((5\%, 10\%)\), when the number of firms is between seven and eleven.

Figure 5 clarifies why the effects of competitive pressure on a firm’s choice of capacity and the optimal capacity for the industry as a whole may be quite different when profit-sharing schemes exist than when otherwise. As indicated by Proposition 2(b), if competitive pressure increases, an individual firm will install a smaller capacity regardless of the existence of profit-sharing schemes. Furthermore, a firm will install a larger capacity if the schemes exist.
than if otherwise, as indicated by Corollary 1. Yet this difference shrinks quickly as competition becomes more intense because the profit-sharing to employees will also decline fast. However, we see that as competition becomes more intense, the optimal capacity for the industry as a whole will decrease if profit-sharing schemes do not exist, but will increase if profit-sharing schemes exist. Our results are thus suggestive: When investigating the effect of market structure on investment, empirical researchers must be cautious because their results by using firm-level data may be different from that by using industry-level data.

IV. Conclusion

This article combines the literature of irreversible investment with the literature of optimal profit-sharing. This article then investigates how competitive pressure, the relative bargaining power between employees and shareholders demand uncertainty, the price elasticity of demand, and the output elasticity of labor affect a firm’s incentive to invest and distribution of the firm’s profits. The main results are shown by Propositions 1-5, which can be empirically tested for future study.

Some extensions are possible in the future. First, it is possible to construct a model that allows a firm to resell its installed capital stock. The model will then resemble that in Pindyck (1988) and Kandel and Pearson (2002), both of which allow the existence of temporarily shut-down option. Second, it is possible to allow other parameters to vary over time. For example, product market environments may affect the wage rate, which is assumed to be fixed in this article. Finally, it is possible to consider two types of employees, senior and junior ones as indicated by Aoki (1982). These extensions deserve investigating for future research.
<Appendix A>

We derive \( F'(k(t),X(t),\lambda(t)) \) by using the contingent-claims analysis (see, e.g., Pindyck, 1988). Let us create a portfolio that is long the option value of adding extra unit of capital and short \( \partial F'(k(t),X(t),\lambda(t))/\partial X(t) \) units of output. Because the expected growth rate of \( X(t) \) is only \( \mu \), the short position requires a payment of \( (\rho - \mu)X(t)\partial F'(k(t),X(t),\lambda(t))/\partial X(t) \) per unit of time. The value of this portfolio is

\[
\Phi = \Delta F'(k(t),X(t),\lambda(t)) - X(t)\partial F'(k(t),X(t),\lambda(t))/\partial X(t) ,
\]

and its instantaneous return is

\[
d(\Delta F'(k(t),X(t),\lambda(t))) = \frac{\partial F'(k(t),X(t),\lambda(t))}{\partial X(t)} dX(t) - (\rho - \mu)X(t) \frac{\partial F'(k(t),X(t),\lambda(t))}{\partial X(t)} dt . \tag{A1}
\]

By Ito’s lemma,

\[
d(\Delta F'(k(t),X(t),\lambda(t))) = \frac{\partial F'(k(t),X(t),\lambda(t))}{\partial X(t)} dX(t) + \frac{\partial^2 F'(k(t),X(t),\lambda(t))}{\partial X(t)^2}(dX(t))^2 . \tag{A2}
\]

Hence the total return on the portfolio is

\[
\frac{1}{2} \frac{\partial^2 F'(k(t),X(t),\lambda(t))}{\partial X(t)^2}(dX(t))^2 - (\rho - \mu)X(t) \frac{\partial F'(k(t),X(t),\lambda(t))}{\partial X(t)} dt . \tag{A3}
\]

From equation (3) for \( dX(t) \), we know that \( (dX(t))^2 = \sigma^2 X(t)^2 dt \) so that the return on the portfolio becomes

\[
\frac{\sigma^2}{2} X(t)^2 \frac{\partial^2 F'(k(t),X(t),\lambda(t))}{\partial X(t)^2} - (\rho - \mu)X(t) \frac{\partial F'(k(t),X(t),\lambda(t))}{\partial X(t)} dt . \tag{A4}
\]

Observe that this return is riskless. To avoid arbitrage profits, this return must be equal to \( \rho \Phi dt = \rho \Delta F'(k(t),X(t),\lambda(t))dt - \rho X(t)\partial F'(k(t),X(t),\lambda(t))/\partial X(t)dt \), thus yielding the following equation for \( \Delta F'(k(t),X(t),\lambda(t)) \):

\[
\frac{\sigma^2}{2} X(t)^2 \frac{\partial^2 F'(k(t),X(t),\lambda(t))}{\partial X(t)^2} + \mu X(t) \frac{\partial F'(k(t),X(t),\lambda(t))}{\partial X(t)} - \rho \Delta F'(k(t),X(t),\lambda(t)) = 0 . \tag{A5}
\]

Noting that \( \Delta V'(k(t),X(t),\lambda(t)) \) as given by equation (13), the solution to equation (A5) is thus in the form \( (X(t)k(t)')^\phi \) (see, Bertola and Caballero, 1994), for a solution to

\[
\phi(\beta) = -\frac{1}{2} \sigma^2 \beta (\beta - 1) - \mu \beta + \rho = 0 . \tag{A6}
\]

Figure 6 illustrates \( \phi(\beta) \) as a function of \( \beta \).

The solution for equation (A5) must also satisfy the boundary condition
Accordingly, only the larger root to equation (A6) must be considered. We thus derive the solution of equation (A5) as given by equation (14).

<Appendix B>

Holding \( N k(t) \) as constant,
\[
\frac{\partial d(t)}{\partial N} = \frac{d_f f}{(N-h)} (1 - \frac{1}{N}) > 0 .
\] (B1)

This proves Proposition 1(a). Holding \( k(t) \) as constant,
\[
\frac{\partial(d N^{*})}{\partial N} = N^{*+1} (f_{d} + N \frac{\partial d}{\partial N}) > 0 .
\] (B2)

This proves Proposition 1(b).

<Appendix C>

Differentiating \( k'(t) \) in equation (21) with respect to \( \lambda_i(t) \), \( N \), and \( \sigma \) yields
\[
\frac{\partial^2 k'(t)}{\partial \lambda_i(t)} = \frac{-k'(t)}{(1-\lambda)^f} < 0,
\] (C1)
\[
\frac{\partial^2 k'(t)}{\partial N} = \frac{-k'(t)}{N} \left[ 1 + \frac{h(N-1)}{(h+N(g-1))(N-h)} \right] < 0,
\] (C2)
\[
\frac{\partial^2 k'(t)}{\partial \sigma} = \frac{-k'(t)}{f \beta_2 (\beta_2 - 1)} \frac{\partial \beta_2}{\partial \sigma} < 0,
\] (C3)

since \( \frac{\partial \beta_2}{\partial \sigma} = \frac{\partial \phi(\beta_2)/\partial \sigma}{-\partial \phi(\beta_2)/\partial \beta} > 0 \), where \( \partial \phi(\beta_2)/\partial \sigma = -\sigma \beta_2 (\beta_2 - 1) < 0 \), and \( \partial \phi(\beta_2)/\partial \beta > 0 \) as shown in Figure 6.

<Appendix D>

Differentiation \( \lambda'(t) \) in equation (26) with respect to \( N \) yields
\[
\frac{\partial \lambda'(t)}{\partial N} = \frac{-gh}{N^2 (1 + f r_{\alpha})} \left( 1 - \frac{h}{N} \right)^{-1} \frac{1}{(m-1)} < 0 .
\] (D1)

A decrease in \( \theta \) or \( \alpha_\ast \) or a rise in \( \alpha_\ast \) will result a fall in \( r_\ast \). Differentiating \( \lambda'(t) \) with respect to \( r_\ast \) yields
\[
\frac{\partial \lambda'(t)}{\partial r_\ast} = \frac{f}{(1+fr_{\alpha})^2} \left( 1 + \frac{1}{(m-1)} \right) > 0 .
\] (D2)

Differentiating \( \lambda'(t) \) with respect to \( \varepsilon \) and \( \gamma \) yields
\[
\frac{\partial \lambda(t)}{\partial \epsilon} = \frac{1}{(1 + \frac{1}{m})} [r_m(1 + \frac{1}{m}) \frac{\partial f}{\partial \epsilon} + \frac{1}{m^2} (1 + fr_m) \frac{\partial m}{\partial \epsilon}],
\]

(D3)

\[
\frac{\partial \lambda(t)}{\partial \gamma} = \frac{1}{(1 + \frac{1}{m})} [r_m(1 + \frac{1}{m}) \frac{\partial f}{\partial \gamma} + \frac{1}{m^2} (1 + fr_m) \frac{\partial m}{\partial \gamma}],
\]

(D4)

where \( \frac{\partial f}{\partial \epsilon} = -\frac{(g-1)gh^2}{(h+g-1)^2} < 0, \quad \frac{\partial m}{\partial \epsilon} = -(1 - \frac{h}{N}) \frac{gh^2}{N} < 0, \)

\( \frac{\partial f}{\partial \gamma} = -\frac{(h-1)hg^2}{(h+g-1)^2} < 0, \) and \( \frac{\partial m}{\partial \gamma} = -(1 - \frac{h}{N}) \frac{g^2}{N} < 0. \)

**<Appendix E>**

The profit-sharing premium, denoted by \( pr \), is derived by dividing \( E_i(t) \) in equation (10) by \( C_{i}(t) \) in equation (9), and then subtracting one. This yields

\[
pr = \lambda_i(t)m.
\]

(E1)

Substituting \( \lambda_i(t) = \lambda(t) \) into equation (E1), and calling the resulting \( pr \) as \( pr' \), and then differentiating \( pr' \) with respect to \( N \) yields

\[
\frac{dpr'}{dN} = \frac{\partial \lambda(t)}{\partial N} m + \frac{\partial \lambda(t)}{\partial N} \frac{\partial m}{\partial N} < 0.
\]

(E2)

**<Appendix F>**

Totally differentiating \( k'(t) \) in equation (21) with respect to \( r_m \) and \( N \) yields

\[
\frac{dk'(t)}{dr_m} = \frac{\partial k'(t)}{\partial \lambda(t)} \frac{\partial \lambda(t)}{\partial r_m} < 0.
\]

(F1)

\[
\frac{dk'(t)}{dN} = \frac{\partial k'(t)}{\partial \lambda(t)} + \frac{\partial k'(t)}{\partial \lambda'(t)} \frac{d\lambda'(t)}{dN}.
\]

(F2)

\[
= -\frac{k'(t)}{N} \left[ 1 + \frac{h(N-1)}{(h+g-1)(N-h)} - \frac{(h+g-1)}{g(h+g-1)} \right] < 0.
\]

Totally differentiating \( Nk'(t) \) with respect to \( N \) yields

\[
\frac{d(Nk'(t))}{dN} = k'(t) + N \frac{dk'(t)}{dN} = \frac{hk'(t)}{h + N(g-1)} \left[ \frac{1}{f} - \frac{N-1}{N-h} \right] > 0.
\]

(F3)
Reference
Figure 1: An increase in $N$

Figure 2: A rise in $\sigma$
Figure 3: A decrease in $\theta$ or $\alpha_e$, or a rise in $\alpha_e$.

Figure 4: The rate and premium of a firm’s profit-sharing as $N$ increases.
Figure 5: The firm and the whole industry’s desired capacity as $N$ increases

Figure 6: $\phi(\beta)$ vs. $\beta$