Product Market Competition, Insider Trading Regulation, and Optimal Managerial Contracts*

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Abstract

This paper intends to examine how allowing insider trading may impact a firm’s optimal managerial contract and hedging policy, thereby affecting the firm value. It also attempts to develop a theory of insider trading regulation based on firms’ concerns about inefficient competition that may take place in the product market. We consider an industry where firms engage in quantity competition and at least one firm is faced with uncertainty regarding its unit production cost. Risk averse firm managers may receive superior cost information after making output decisions, which allows them to earn trading profits when insider trading is not prohibited. Concerns for insider trading profits may lead the managers to over-expand outputs. We obtain the following results: (i) when there is a single firm faced with cost uncertainty, so long as the manager is not too risk averse, insider trading may be unanimously supported by the shareholders of the firm, even if the shareholders must bear all the trading loss caused by insider trading; (ii) when insider trading is allowed a firm that suffers from an adverse selection problem may end up having a higher market value than a rival that differs from the firm only because it does not have adverse selection problems; (iii) allowing insider trading tends to raise the power

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of the managerial incentive scheme; (iv) given a firm’s output decision, insider trading tends to create a negative correlation between a firm’s share price and the firm’s hedging activities—a firm tends to hedge more when the stock market is bearish than when it is bullish; (v) when managers are risk neutral and more than one firm is faced with cost uncertainty, allowing insider trading may result in a prisoner’s dilemma where all firms are indulging insider trading in equilibrium and yet insider trading regulation can make all shareholders better off; and (vi) index trading may help resolve the prisoner’s dilemma, and unlike in Subrahmanyam (1991), index trading can make both liquidity traders and insiders better off.

**Keywords:** Insider Trading, Insider Trading Regulation, Index Trading, Managerial Contract, Strategic Substitutes, Corporate Hedging.

1 **Introduction**

The finance literature has long recognized the efficiency roles of financial markets in allocating risks and resources (Arrow 1964; Debreu 1959; Duffie and Huang 1985), generating information about the profitability of new projects (Allen 1993; Khanna, Slezak, and Bradley 1994; Boot and Thakor 1997; Dow and Gorton 1997), and disciplining self-interested corporate managers (Scharfstein 1988; Stein 1988; Holmström and Tirole 1993), but it also suggests that under certain circumstances restricting some or all agents’ access to financial markets may improve economic efficiency. For example, restricting all agents’ access to a new market makes everyone better off in the economy analyzed in Newbery (1984), because trading in both the old and new markets would result in inadequate insurance for producers, and the changes in the latter’s production plans further hurt consumers. On the other hand, the moral hazard literature has shown that giving a risk-averse corporate manager access to the credit market may result in a Pareto dominated equilibrium, for the fact that the manager can freely trade in the credit market implies that the principal is faced with additional incentive compatibility constraints when designing the optimal managerial compensation scheme (Rogerson 1985; Chiappori, et al. 1994). Finally, there have been continual debates among legislators, market participants, and scholars in both law and economics regarding whether insider trading should be legally prohibited (Manove 1989; Laffont and Maskin 1990; Leland 1992; Fischer 1992; Fishman and Hagerty 1992;
Khanna, Slezak, and Bradley 1994), and if not, whether mandatory disclosure is appropriate (Admati and Pfeiderer 2000, 1991, Fishman and Hagerty 1995). Empirical evidence has shown that insiders consistently earn trading returns that exceed risk-adjusted benchmarks (Seyhun 1986; Meulbroek 1992), and for this reason the Securities and Exchange Commission (SEC) in the United States has regulated insider trading since 1934. Proponents for insider trading regulations argue that insider trading is harmful because it reduces liquidity in securities markets (Glosten 1989), creates perverse managerial incentives (Easterbrook 1985) and creates a perception of unfairness and loss of investor confidence in capital markets (Brudney 1979). Earlier critics of insider trading regulation contend that even if insider trading does incur social costs regulation is unnecessary because corporate shareholders would have the incentive of restricting it on their own (Carlton and Fischel 1983), and that allowing insider trading can improve informational efficiency in securities markets (Manne 1966), which is socially beneficial because efficient security prices lead to an efficient resources allocation.

The purpose of this paper is two-fold. First it intends to examine how allowing insider trading may impact a firm’s optimal managerial compensation scheme and hedging policy, thereby changing the value of the firm. Second, it attempts to develop a theory of insider trading regulation based on firms’ concerns about inefficient competitive outcomes in the product market.

We consider an industry with one incumbent firm and a new entrant, where the two firms must engage in quantity competition. The incumbent and the entrant are both equity-financed, and they differ in one aspect: being a new member to the industry, the entrant is faced with a random unit cost. A risk-averse professional manager must be hired to run the new entrant, and we assume with Fudenberg and Tirole (1986) that the output decision is unverifiable, so that the risk-neutral shareholders of the new entrant can affect the output decision only through the choice of the managerial incentive contract. We confine attention to the case where managerial compensation schemes are linear and the manager’s preferences can be represented by a mean-variance utility function.\(^1\) Although it is inessential to our analysis,

\(^1\)See Hart and Holmström (1987) for the prevalence of linear contracts. Holmström and Milgrom (1987) provides a rationale for the optimal managerial incentive contracts to be linear. When returns on feasible investment projects all follow Gaussian distributions, a mean-variance utility function defined on the space of wealth distributions is equivalent to
we assume that the incumbent is run by a risk neutral owner-manager, and that the incumbent makes its output decision before the entrant takes any actions.

We initially assume that there is no insider trading regulation, and the stock market opens after the manager makes the output decision and privately receives cost information. Our first result shows that when the manager is not too risk averse, insider trading can be value-enhancing even if the shareholders of the entrant firm must bear all the trading loss caused by insider trading. The idea is that when insider trading is allowed, the manager’s expected trading profit (which is essentially what matters when the manager is not very risk averse) increases with the firm’s output level, and hence allowing the manager to engage in insider trading encourages the manager to choose a high output ex-ante, and because the two firms’ output decisions are strategic substitutes, this forces the incumbent to concede in the output choice. On the other hand, as in Khanna, Slezak, and Bradley (1994), the shareholders can always recoup the manager’s trading gain by lowering the constant term in the managerial compensation scheme. Thus allowing insider trading enhances the entrant’s profit and results in a higher share price for the entrant firm. Assuming that the two firms differ only in the cost uncertainty, our second (somewhat surprising) result shows that in the absence of insider trading regulation a following firm that suffers from the adverse selection problem resulting from cost uncertainty may have a higher market value than a leading firm that is identical to the following firm but does not suffer from adverse selection problems.

We then examine how insider trading regulation may affect the slope of the managerial compensation scheme selected by the entrant firm (the power of the incentive contract). Our third result shows that allowing insider trading tends to raise the power of the managerial compensation scheme. When a negative exponential von Neumann-Morgenstern utility function defined on the terminal wealth. Alternatively, when risk is small, a mean-variance utility function defined on the space of wealth distributions provides a good approximation to a concave von Neumann-Morgenstern utility function defined on the terminal wealth; see for example section 4.2 of Gollier (2001).

See for example Bulow, Geanakoplos, and Klemperer (1985) for the original definition and applications.

For more discussions of high-powered and low-powered linear incentive contracts see section 1.4.2 of Laffont and Tirole (1993).
insider trading is prohibited, a low-powered incentive scheme is optimal, for it minimizes the risk-averse manager’s exposure to income uncertainty, thereby encouraging the manager to make an aggressive output decision. When insider trading is allowed, on the other hand, a high-powered incentive scheme may become optimal. When the distribution of the entrant’s unit cost is symmetric about the mean, this can happen only if the entrant’s shares are traded in a bear market, where liquidity (or noise) traders are more likely to sell than to buy. The idea is that the manager’s income consists of his share of the firm’s profit and the gain from insider trading, and these two components tend to be negatively correlated in a bear market. Since the manager cannot commit not to trade in the stock market, he will be exposed to the risk of the trading gain. A constant salary cannot hedge this risk; rather, a positive slope of the managerial compensation scheme makes the manager’s random salary a good hedging instrument for the trading gain. When the manager is not too risk averse, raising the slope of the managerial compensation scheme has another advantage: the output level chosen by the self-interested manager tends to exceed the first-best level, and raising the slope of the managerial compensation scheme reduces the manager’s incentive to over-expand the output.

We then investigate how allowing insider trading may alter a firm’s hedging policy. We show that when insider trading is allowed, given a firm’s output decision, the firm tends to hedge more when the market is bearish than when it is bullish. The intuition runs roughly as follows. Corporate hedging has value because the manager is risk averse, but when insider trading is allowed, corporate hedging may reduce the expected trading gain from insider trading. When the entrant’s shares are traded in a bullish (bearish) market, the manager’s salary and the trading gain are negatively (positively) correlated: in a bearish market where most investors hold a pessimistic belief about the future earnings the manager can make a high profit exactly when he sees a good state, but then his trading profit is positively related to his managerial salary. On the other hand, in a bullish market where most investors feel optimistic about the future earnings, a high trading gain occurs exactly in the state where the firm’s profit will be low, and in this case the trading gain is negatively correlated with the managerial salary. Insider trading thus partially takes over the role of costly corporate hedging in a

\footnote{Fischer (1992) considers optimal incentive contracts with insider trading, but without addressing issues relevant to product market competition.}
bullish market. Assuming that the cost of hedging is independent of liquidity (or noise) trading, we hence conclude that *ceteris paribus*, the firm tends to hedge more when the stock market is bearish than when it is bullish.

We then examine the case where both firms in the industry are faced with cost uncertainty and their shares are traded in the stock market after their managers simultaneously make output decisions and privately receive cost information. We show that unless the level of noise trading is sufficiently high and the shareholders bear only a small portion of the trading losses caused by insider trading, a prisoner’s dilemma can arise where both firms allow their managers to engage in insider trading in equilibrium, and yet all the shareholders would be made better off if insider trading could be prohibited. This happens because one firm’s indulging insider trading encourages its manager to expand output ex-ante, which results in a downward shift of the demand for the rival’s product. Thus one firm’s indulging insider trading creates a negative externality on its rival firm, and shareholders would be better off if both firms can commit not to allow insider trading. This is a version of the prisoner’s dilemma originally studied by Rapoport and Chammah (1965). In this dilemma, whether a firm expects its rival to allow insider trading or not, allowing insider trading is always the firm’s best choice. Hence insider trading cannot be prohibited in a non-cooperative way. Only a collective commitment such as a regulation can resolve the problem. Hence we obtain our fourth result that *insider trading regulation has value even though all firms choose to indulge insider trading in equilibrium*. This result disproves Carlton and Fischel’s argument that even if insider trading regulation has a social value, regulation is still unnecessary because firms would automatically prevent their managers from engaging in insider trading.

Fishman and Hagerty (1992) and Khanna, Slezak, and Bradley (1994) have also provided theories showing that insider trading regulation has value even when firms all choose to indulge insider trading. Fishman and Hagerty (1992) show that, unlike what Manne suggests, allowing insider trading might as well lead to lower informational efficiency in securities markets because it discourages outsiders from engaging in information search, which leads to a

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5This conclusion is consistent with the compelling empirical evidence cited in Carlton and Fischel (1983), where the authors noted that no evidence of organizers of firms trying to ban insider trading either before federally mandated restrictions were put in place or during the time they had been in place but not enforced.
smaller number of informed traders and an uneven distribution of information among traders. Moreover, they show that the efficiency loss caused by informationally inefficient securities prices may not be fully internalized by the firm that allows its manager to engage in insider trading, and hence a firm may choose to indulge insider trading while knowing it is detrimental to the economy as a whole. Khanna, Slezak, and Bradley (1994), on the other hand, argue that a corporate insider differs from an informed speculator outside the firm, since when offering the managerial contract to the corporate insider the shareholders can recoup their expected trading loss to the corporate insider by lowering the latter’s salary, but they cannot do the same to a market professional. This creates a difference between the social value and the private value of insider trading regulation.

To develop a theory of insider trading regulation that is orthogonal to both Fishman and Hagerty (1992) and Khanna, Slezak, and Bradley (1994) we have assumed that there are no informed speculators outside the firms, and that the firms do not have to make decisions regarding new investment projects (so that there will be no resource misallocation resulting from inefficient security prices). Our theory differs from Fishman and Hagerty (1992) and Khanna, Slezak, and Bradley (1994) in one important aspect: in their papers the firms can benefit from insider trading and hence to those firms insider trading regulation may not be welcome, whereas in our model insider trading regulation is unanimously preferred by the firms (shareholders), for those firms suffer from insider trading but cannot prohibit insider trading unilaterally.

The possibility that allowing insider trading may lead to a Pareto dominated outcome for imperfectly competitive firms leads to the question of how to resolve the dilemma, if insider trading regulation is either infeasible or ineffective. We offer a theory of valuable index trading from this perspective. Subrahmanyam (1991) shows that when index trading is available there may exist an equilibrium where trade clusters on the basket (the portfolio of securities), and in this equilibrium the expected trading cost for liquidity traders is lower than without index trading. The insiders that possess security-specific private information loses part of their information advantage in index trading, and hence are made worse off in this equilibrium. By contrast, our fifth result shows that index trading may benefit all market participants, including insiders. The idea is that index trading reduces the
expected insider trading profit per unit of output, and hence it mitigates the managers’ incentives to over-expand outputs. We identify conditions ensuring that index trading enhances the values of the firms that would otherwise be trapped in the aforementioned prisoner’s dilemma.

Explicit modelling the interactions of financial and product markets allows us to establish a relationship between industry-wide parameters and the stock market performance. Our sixth result shows that an upward shift of demand curve, a reduction in demand elasticity, and a reduction in unit production cost can each lead to lower stock market liquidity, higher price volatility, and higher informational efficiency.

Our theory also sheds new lights on such issues as how firms may benefit from initial public offerings and conglomerate diversifications. In our model, for insider trading to render a commitment power in quantity competition in the product market, two necessary conditions must be satisfied. First, the firm’s shares must be publicly traded, and second, corporate insiders must possess superior information that public investors do not have. Imagine that firms engaging in Cournot competition in a given industry are mostly privately owned. Our theory suggests that by unilaterally turning into a publicly traded company and engaging in insider trading, a firm can gain a larger market share in the product market, since it convinces other firms that it will adopt a more aggressive reaction function against its rivals. The same reasoning can be applied to conglomerate takeovers or diversifications as well. A conglomerate takeover is traditionally considered risky, because the entrant must face great uncertainty regarding the demand and the production costs in the new business. Our theory suggests that by unilaterally entering into the new business and then engaging in insider trading, a firm can gain a larger market share in its original business. Hence our theory suggests that the insider trading motive can play a significant role in the processes of IPO and conglomerate diversifications, and that there will be a strong incentive to engage in insider trading during the period of time when a firm just completed an IPO or conglomerate diversification.

Although throughout this paper we consider only equity firms, our theory has implications regarding the relationship between financial leverage and insider trading restrictions. Imagine a situation where an entrepreneur endowed only with illiquid assets would like to raise funds from outside investors
to implement a positive-NPV project. If verifying the ex-post earnings is prohibitively costly, as in Bolton and Scharfstein (1990), then the optimal financial structure consists of default-free debt and costly equity participation; see Chen and Chen (2004). In the current paper, when insider trading is allowed, a firm tends to hedge more in a bearish market than in a bullish market, which gives rise to the implication that, other things being equal, a firm tends to have a higher worse-case profit in a bearish market than in a bullish market, implying a negative relationship between debt capacity and stock returns.

The rest of this paper is organized as follows. Section 2 lays out the incumbent-entrant model and examines how allowing insider trading may impact a firm’s optimal managerial compensation scheme and hedging policy. Section 3 considers the case where symmetric firms are trapped in a prisoner’s dilemma of output expansion, and discuss the values of insider trading regulation and of index trading. Concluding remarks are given in section 4.

## 2 Insider Trading and Optimal Managerial Compensation Scheme

Consider an industry with one incumbent firm (firm I) and a new entrant (firm E) producing a homogeneous good and engaging in quantity competition. The unit cost of production for firm E is random, denoted by $\tilde{c}$, which may equal $c$ and $\bar{c}$ with probabilities $a$ and $1-a$ respectively, where

$$1 > \bar{c} > \underline{c} \geq 0, \quad 1 > a > 0.$$  \hfill (1)

The unit cost of production is $ac + (1-a)\bar{c}$ for the incumbent. The inverse demand for the good is, in the relevant range,

$$P = 1 - q_I - q_E,$$  \hfill (2)

where $q_I$ and $q_E$ stand for the two firms’ output levels. The incumbent and the entrant are both equity-financed, and the incumbent is run by a risk-neutral owner-manager (also referred to as I). Firm I must choose $q_I$ before the entrant takes any actions. Firm E is established by a risk-neutral
entrepreneur (also referred to as E), who needs to hire a risk-averse professional manager to run the firm. Suppose that professional managers are homogeneous, endowed with a mean-variance utility function

$$E(\tilde{W}) - \rho \text{var}(\tilde{W}),$$

where $\tilde{W}$ represents the terminal wealth, and $\rho > 0$ is a constant. Let $u_0$ be managers’ common reservation utility. We assume with Fudenberg and Tirole (1986) that $q_E$ is prohibitively costly to verify, so that $E$ can only affect the output choice through the design of the managerial compensation scheme. We confine attention to the case where only linear schemes are feasible:

$$\tilde{W} = A + B\pi_E + \pi_T,$$

where $A \in \mathbb{R}$ and $B \in [0, 1]$ are the parameters defining the managerial compensation scheme, $\pi_E$ is firm $E$’s profit, and $\pi_T$ is the manager’s gain from engaging in insider trading. Correspondingly, $\pi_I$ will denote firm $I$’s profit. For simplicity, we assume that there is one share of firm $E$’s stock outstanding. Note that $\pi_T = 0$ when insider trading is prohibited. The above descriptions are common knowledge of $I$, $E$, managers, and all participants in stock trading.

The timing of relevant events is as follows.

- Firms and managers first learn about whether there are insider trading restrictions. We assume that either there are no restrictions at all, or insider trading is completely prohibited.
- Firm $I$ chooses $q_I$.
- Firm $E$, upon seeing $q_I$, offers a managerial compensation scheme $(A, B)$ and hires one of the managers. Refer to the hired manager as $M$.
- $M$, given $(q_I, A, B)$, chooses $q_E$.
- $M$ observes privately the realized unit cost of firm $E$.
- The stock market opens, and the two firms’ shares will be traded. For each stock, there are risk neutral market makers competing in posting bid and ask prices. After all bid and ask prices are posted, $E$, $I$, $M$ and public investors can submit orders. Assume that the public investors
may trade for liquidity reasons. With probability $b$ one of the liquidity traders may want to buy $l$ shares of firm E’s stock, with probability $s$ one of the liquidity traders may want to sell $l$ shares of firm E’s stock, and with probability $1 - b - s$, there is no liquidity demand. After orders are submitted, exactly one order will be selected at random to transact with the market maker that posts the price appearing to be most favorable to the trader that submits the order. (If there are multiple market makers posting that price, then one market maker will be selected at random.) This is the Glosten-Milgrom-Easley-O’Hara quote-driven trading mechanism; see Glosten and Milgrom (1985) and Easley and O’Hara (1992).

- The stock market closes, and the two firms’ profits $\pi_I$ and $\pi_E$ are realized.

The following Lemma characterizes M’s output choice and the corresponding optimal managerial compensation scheme for the case where insider trading is prohibited.

**Lemma 1** Suppose that insider trading is prohibited. Assume that if $M$ feels indifferent about $q_I$ and $q_E$, then he chooses the one that maximizes the initial value of firm E. Then given $q_I$, $M$ chooses

\[
q_E = \frac{1 - q_I - a\bar{c} - (1 - a)\bar{v}}{2[1 + \rho B^2 a(1 - a)(\bar{v} - \bar{c})^2]}, \quad \text{if } B > 0; \tag{5}
\]

and

\[
q_E = \frac{1 - q_I - a\bar{c} - (1 - a)\bar{v}}{2}, \quad \text{if } B = 0. \tag{6}
\]

In this case the optimal managerial compensation chosen by $E$ is such that $B = 0$.

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6We can alternatively interpret these as noise trades, submitted by a trader who mistakenly regards noise as relevant information; see for example Black (1989).

7Since there is a risk of non-execution according to our descriptions, a more complete analysis should consider sequential trading in a dynamic model. Here we content ourselves with the static model because introducing dynamic trading does not alter our main results regarding value-enhancing insider trading and the social value of insider trading regulation.
Proof. The manager seeks to
\[
\max_{q_E} A + Bq_E[1 - q_I - q_E - a\zeta - (1 - a)c] - \rho B^2q_E^2a(1 - a)(\bar{c} - \xi)^2; \tag{7}
\]
and the necessary and sufficient first-order condition gives
\[
q_E = \frac{1 - q_I - a\zeta - (1 - a)c}{2[1 + \rho B^2a(1 - a)(\bar{c} - \xi)^2]} \tag{8}
\]
In anticipation of \(q_E\) as the above function of \((A, B)\), \(E\) seeks to
\[
\max_{A,B} -A + (1 - B)q_E[1 - q_I - q_E - a\zeta - (1 - a)c] \tag{9}
\]
subject to
\[
A + Bq_E[1 - q_I - q_E - a\zeta - (1 - a)c] - \rho B^2q_E^2a(1 - a)(\bar{c} - \xi)^2 \geq u_0, \tag{10}
\]
\[
q_E = \frac{1 - q_I - a\zeta - (1 - a)c}{2[1 + \rho B^2a(1 - a)(\bar{c} - \xi)^2]}, \tag{11}
\]
and
\[
q_E = \frac{1 - q_I - a\zeta - (1 - a)c}{2}, \text{ if } B = 0. \tag{12}
\]
It can be easily verified that the optimal \(B = 0\). \|

Note that making the output choice does not incur private costs to \(M\). Lowering the power \(B\) reduces \(M\)'s exposure to risks, thereby encouraging \(M\) to expand output, which is beneficial since it forces the rival firm to concede in output competition. Hence in Lemma 1 the optimal scheme requires that the risk-averse manager bears no risk in equilibrium. The next Lemma shows that when insider trading is allowed the manager’s gain from insider trading is linearly increasing in \(q_E\).

**Lemma 2** Suppose that insider trading is allowed and that in equilibrium the market makers adopt the same strategy in posting bid and ask prices. The random trading gain that \(M\) obtains in equilibrium is as follows.
\[
\pi_T = \begin{cases} 
1(1 - a_b)(\bar{c} - \xi)q_E, & \text{if } \bar{c} = \xi; \\
la_s(\bar{c} - \xi)q_E, & \text{if } \bar{c} = \xi, 
\end{cases} 
\tag{13}
\]
where
\[
a_b = \frac{a(2 - s)}{a(2 - s) + (1 - a)b}, \quad a_s = \frac{as}{as + (1 - a)(2 - b)}. \tag{14}
\]
Proof. The probability for the event that a buy order is executed is \( a(1 - \frac{s}{2}) + (1 - a) \frac{b}{2} \), and the probability for the event that a sell order is executed is \( a\frac{s}{2} + (1 - a)(1 - \frac{b}{2}) \). Hence the probability for the event that \( \hat{c} = c \) conditional on a buy order being executed is \( a_{b} \), and the probability for the event that \( \hat{c} = c \) conditional on a sell order being executed is \( a_{s} \). Since the market makers adopt the same strategy, the bid price must equal the expected value of \( l \pi_{E} \) conditional on the event that a sell order being executed, which is

\[
l_{q_{E}}[1 - q_{I} - q_{E} - a_{s}c - (1 - a_{s})\bar{c}]; \quad (15)
\]

and the ask price must equal the expected value of \( l \pi_{E} \) conditional on the event that a buy order being executed, which is

\[
l_{q_{E}}[1 - q_{I} - q_{E} - a_{b}c - (1 - a_{b})\bar{c}]. \quad (16)
\]

Now, since M’s optimal trading strategy is to buy \( l \) shares in state \( \hat{c} = \underline{c} \) and to sell \( l \) shares in state \( \hat{c} = \overline{c} \), the assertions follow.

Now we are ready to present our first main result.

**Proposition 1 (Value-Enhancing Insider Trading)** Given \( a, b, s, \underline{c}, \overline{c} \) and \( c \) satisfying

\[
1 > a\underline{c} + (1 - a)\overline{c} + 3l(\overline{c} - \underline{c})[a(1 - a_{b}) + (1 - a)a_{s}], \quad (17)
\]

there exists \( \rho^* \in (0,1) \) such that for \( \rho \in [0, \rho^*) \), allowing insider trading results in a higher initial firm value for the entrant.

Proof. It can be verified that the equilibrium correspondence is upper semi-continuous in \( \rho \), and hence to prove the assertion it suffices to consider the case \( \rho = 0 \). When insider trading is allowed and M is risk neutral, given \( (q_{I}, A, B) \), the optimal output choice is

\[
q_{E} = \frac{l(\overline{c} - \underline{c})[a(1 - a_{b}) + (1 - a)a_{s}] + B(1 - q_{I} - a\overline{c} - (1 - a)\underline{c})}{2B}. \quad (18)
\]

Note that the imposed condition ensures that \( q_{E} < 1 \). Rationally expecting this reaction function, the incumbent’s optimal output choice is

\[
q_{I} = \frac{1 - a\overline{c} - (1 - a)\underline{c}}{2} - \frac{l(\overline{c} - \underline{c})[a(1 - a_{b}) + (1 - a)a_{s}]}{2B}, \quad (19)
\]

13
Given \((A, B)\), the initial value of firm E is therefore
\[
\frac{3l(\tau - \underline{c})[a(1 - a_b) + (1 - a)a_s]}{4B} + \frac{1 - a\underline{c} - (1 - a)\tau}{4},
\]
(20)
\[
\times \left[ \frac{1 - a\underline{c} - (1 - a)\tau}{4} - l(\tau - \underline{c})[a(1 - a_b) + (1 - a)a_s] \right].
\]
Since \(a, b, s, \underline{c}\) and \(\tau\) satisfy
\[
1 > a\underline{c} + (1 - a)\tau + 3l(\tau - \underline{c})[a(1 - a_b) + (1 - a)a_s],
\]
(21)
the optimal \(B\) is
\[
B = \frac{3l(\tau - \underline{c})[a(1 - a_b) + (1 - a)a_s]}{1 - a\underline{c} - (1 - a)\tau},
\]
(22)
implying that when insider trading is allowed, the initial value of firm E is
\[
\frac{4}{3} \left[ \frac{1 - a\underline{c} - (1 - a)\tau}{4} \right]^2 > \left[ \frac{1 - a\underline{c} - (1 - a)\tau}{4} \right]^2,
\]
(23)
where the right side of the inequality is the initial value of firm E when insider trading is prohibited.

To see the intuition underlying Proposition 1, note that if \(M\) is risk neutral, then when insider trading is prohibited, the equilibrium output levels are respectively
\[
q_I = \frac{1 - a\underline{c} - (1 - a)\tau}{2}, \quad q_E = \frac{1 - a\underline{c} - (1 - a)\tau}{4}.
\]
(24)
By allowing its manager to engage in insider trading, firm E creates an upward shift in its reaction function against firm I’s output choice, which forces firm I to reduce \(q_I\) thereby raising \(\pi_E\). Proposition 1 depicts only the case where \(M\) is nearly risk neutral. The assertion fails in general if \(M\) is risk averse; see Proposition 6 below.

The next Proposition shows how allowing insider trading may change the ranking of firm values in the industry.

**Proposition 2 (Ranking of Firm Values)** Suppose that \(a, b, s, \underline{c}\) and \(\tau\) satisfy
\[
1 > a\underline{c} + (1 - a)\tau + 3l(\tau - \underline{c})[a(a - a_b) + (1 - a)a_s],
\]
(25)
Given \(a, b, s, \underline{c}\) and \(\tau\), there exists \(\rho^* \in (0, 1)\) such that for \(\rho \in [0, \rho^*)\),
• when insider trading is prohibited, the initial value of firm I is greater than the initial value of firm E; and

• when insider trading is allowed, the initial value of firm I is less than the initial value of firm E.

Proof. First consider the case where insider trading is prohibited. It is straightforward to show that the initial values of firms I and E are respectively

\[ \pi_I = \frac{\left[1 - a_c - (1 - a)\bar{c}\right]^2}{8} \]  

(26)

and

\[ \pi_E = \frac{\left[1 - a_c - (1 - a)\bar{c}\right]^2}{16}. \]  

(27)

If insider trading is allowed, the initial values of firms I and E would become

\[ \pi_I^* = \frac{8}{9}\left[1 - a_c - (1 - a)\bar{c}\right] \]  

(28)

and

\[ \pi_E^* = \frac{4}{3}\left[1 - a_c - (1 - a)\bar{c}\right]^2 \]  

(29)

respectively. The assertion now follows from a direct comparison of profits.

Proposition 2 delivers a rather surprising result: when insider trading is allowed, the firm that suffers from an ex-post adverse selection problem resulting from cost uncertainty ends up with a higher firm value than its counterpart that does not have adverse selection problems! The presence of the adverse selection problem creates an opportunity for the better informed manager to engage in insider trading, and in order to enhance his insider trading profits, the manager optimally raises the output given each possible output choice made by the rival firm. The more aggressive reaction function hurts the rival firm, and in equilibrium the rival firm ends up with a lower market value. Although allowing insider trading may incur trading losses to the shareholders, these losses can be compensated by reducing the manager’s salary by a constant ex-ante; see Khanna, Slezak, and Bradley (1994) for details. Hence the net effect of allowing insider trading is to force the rival to concede in quantity competition, which raises the firm’s own market value.

Our next result proves that allowing insider trading raises the power of the managerial incentive scheme.
Proposition 3 (Power of the Managerial Compensation Scheme) Define $\tilde{a}$ as a random variable such that it equals $1 - a_b$ and $a_s$ in respectively the state $\tilde{c} = \tilde{c}$ and the state $\tilde{c} = \tau$. Define

$$q_E^* = \arg\max_{q_E} q_E l(\tilde{c} - \zeta)E(\tilde{a}) - \rho q_E^2 l^2(\tilde{c} - \zeta)^2 \text{var}(\tilde{a}).$$

Let $B_1$ and $B_0$ denote the optimal $B$ chosen when respectively insider trading is and is not allowed. Then $B_1 \geq B_0$, and the inequality is strict if the following condition holds:

$$I^2(q_E^*)^2(\tilde{c} - \zeta)^2a(1 - a)(1 - a_b - a_s)^2$$

$$\geq a[q_E^*(1 - q_I - q_E^* + (1 - a)(\tilde{c} - \zeta)) + l(1 - a_b - a_s)q_E^*(1 - a)]^2$$

$$+ (1 - a)[q_E^*(1 - q_I - q_E^* - a(\tilde{c} - \zeta)) - l(1 - a_b - a_s)q_E^* a]^2.$$
Observe that $q^*_E$ is M's optimal output choice when M is faced with such $(A, B)$. Consider replacing the contract $(A, B)$ with $B = 0$ by $(A, B)$ with $B = 1$. The stated condition ensures that choosing $q^*_E$ under the contract $(A, B)$ with $B = 1$ will result in a higher utility for M than doing the same under the contract $(A, B)$ with $B = 0$. Since $q^*_E$ need not be M’s optimal output choice under the contract $(A, B)$ with $B = 1$, we conclude that the contract $(A, B)$ with $B = 1$ attains a higher value for the objective function in problem (P2) than the contract $(A, B)$ with $B = 0$. Since $A$ was chosen arbitrarily, we conclude that no contract $(A, B)$ with $B = 0$ can be optimal. Hence at optimum $B > 0$.

The stated condition holds only if $B\pi_E$ and $\pi_T$ are negatively correlated. Since $B > 0$, this requires that $\pi_T$ be lower in state $\bar{c} = \bar{c}$ than in state $\bar{c} = c$. It can be verified that when $a = \frac{1}{2}$, this requires that $s > b$; that is, the entrant’s shares are traded in a bear market. The intuition underlying Proposition 3 is as follows. When insider trading is prohibited, the firm profit is the sole source of M’s income. Efficient risk sharing requires that M be fully insured, as Lemma 1 shows. When insider trading is allowed, M cannot commit not to engage in insider trading after he learns the true state of the unit cost. This implies that M’s terminal wealth will be random, unless the gain from trading can be hedged by M’s salary. When the firm profit is negatively correlated with the trading gain, sticking to a low-powered managerial contract may not be a good idea. Raising the power of the incentive scheme amounts to hedging M’s exposure to the risk of trading gain by adding more risk in M’s salary.

To simplify the subsequent analysis, we shall assume throughout this section that $b = s = \frac{1}{2}$. With this simplification, we have

$$a_b = \frac{3a}{1 + 2a}, \quad a_s = \frac{a}{3 - 2a}.$$

The market for firm E’s common stock is referred to as 	extit{bull} if $a > \frac{1}{2}$ and as 	extit{bear} if $a < \frac{1}{2}$. We shall assume that there exists a hedging opportunity for firm E as follows. For all $\delta \geq 0$, by promising to pay the insurer $t\delta$ in state $\bar{c} = \bar{c}$, firm E can get a re-imbursement $\delta$ from the insurer in state $\bar{c} = \bar{c}$. Here we assume that $t > \frac{4a}{a}$, so that hedging is costly.\footnote{This may due to un-modelled adverse selection problems facing the insurer. Note that} We modify the timing of events as follows.
Firms and managers first learn about whether there are insider trading restrictions.

Firm I chooses \( q_I \).

Firm E, upon seeing \( q_I \), offers a managerial compensation scheme \((A, B)\) and hires one of the managers. Refer to the hired manager as M.

M, given \((q_I, A, B)\), chooses \( q_E \) first, and then given \((q_I, A, B, q_E)\) M chooses \( \delta \).

M observes privately the realized unit cost of firm E.

The stock market opens, and M engages in insider trading whenever it is allowed.

The stock market closes, and the two firms’ profits \( \pi_I \) and \( \pi_E \) are realized. Upon seeing the realization of \( \hat{c} \), the insurer and firm E pay each other according to the hedging contract \( \delta \).

**Proposition 4 (Corporate Hedging and Insider Trading)** When insider trading is allowed, if \( B > 0 \) then after making its output decision \( q_E \), firm E hedges more in the bear market than in the bull market. More precisely, let \( \delta^*(l) \) be the optimal hedging decision given the level \( l \) of liquidity trading, and we have

\[
\forall l > 0, \quad \delta^*(l, q_E) \geq \delta^*(0, q_E) \iff a < \frac{1}{2}.
\]

**Proof.** Define

\[
\Delta \equiv \bar{c} - c, \quad \mu \equiv ac + (1 - a)c, \quad \theta \equiv a(1 - a_b) + (1 - a)a_s.
\]

Given \((A, B, q_I, q_E, t)\) with \( B > 0 \), M seeks to

\[
\max_{\delta} B[q_E(1 - q_I - q_E - \mu) - at\delta + (1 - a)\delta] + t\Delta q_E \theta
\]

if \( t = \frac{1 - a}{\bar{c}} \), then firm E would certainly choose to be fully insured. Such an insurance opportunity rules out the contract problem that we are dealing with, and its existence is inconsistent with the real-world practice.

Recall that for a random variable taking values \( x \) and \( y \) with probabilities \( a \) and \( 1 - a \) respectively, its variance is \( a(1 - a)(x - y)^2 \).
The necessary and sufficient first-order condition with respect to $g$ gives

$$
\delta^*(l, q_E) = \frac{B(1 - a - at) + 2(1 + t)\rho a(1 - a)[Bq_E \Delta + l\Delta q_E(1 - a_b - a_s)]}{2(1 - t)^2\rho a(1 - a)}.
$$

Hence we have

$$
\delta^*(l, q_E) > \delta^*(0, q_E) \iff 1 - a_b - a_s > 0,
$$

where it can be verified easily that $1 - a_b - a_s$ is strictly decreasing in $a$, and

$$
1 - a_b - a_s = 0 \iff a = \frac{1}{2}.
$$

The intuition underlying Proposition 4 is as follows. When insider trading is allowed, the market makers expect $M$ to submit a buy order in state $\hat{c} = \underline{c}$ and a sell order in state $\hat{c} = \overline{c}$. Thus the adverse selection problem pertaining to an arrived buy order is considered less severe than that pertaining to an arrived sell order in a bear market. Accordingly, the ask premium demanded by the market makers is less than the bid premium in a bear market, implying that for $M$ the realized trading gain in state $\hat{c} = \underline{c}$ is higher than that in state $\hat{c} = \overline{c}$ in a bear market. Since $B$ is positive, $A + B\pi_E$ is also higher in state $\hat{c} = \underline{c}$ than in state $\hat{c} = \overline{c}$. This means that the managerial salary and the trading gain are positively correlated in a bear market, and this strengthens $M$’s incentive to engage in corporate hedging in a bear market. The same reasoning verifies that the managerial salary and the trading gain are negatively correlated in a bull market. In the latter case, insider trading serves partially as a hedging instrument for $M$’s salary, and this reduces his incentive to engage in the costly (recall that $t > \frac{1-a}{\rho}$) hedging activity.

**Proposition 5** Allowing insider trading shifts up firm $E$’s reaction function against $q_I$ if and only if

$$
\{\Delta[\theta + B(1 - a - at)(1 - a_b - a_s)]\}2(1 + t) + at\Delta B
$$

$$
> \{(1 + t)(1 - q_I - \mu) + \frac{B(1 - a - at)}{1 + t}\frac{1 - a - at}{2\rho a(1 - a)} + \Delta\}\{(1 - a_b - a_s)at\Delta B\}.
$$
Proof. Given \((A, B, q_I)\) with \(B > 0\), M seeks to

\[
\max_{q_E} B[q_E(1 - q_I - q_E - \mu) - at\delta + (1 - a)\delta] + l\Delta q_E \theta
\]

\[-\rho a(1-a)\{[Bq_E(1-q_I-q_E-c)-\delta+l\Delta q_E(1-a_b)]-[Bq_E(1-q_I-q_E-c)+\delta+l\Delta q_E a_s]\}^2,
\]

subject to\(^{10}\)

\[
\delta = \frac{B(1 - a - at) + 2(1 + t)\rho a(1 - a)[Bq_E \Delta + l\Delta q_E (1 - a_b - a_s)]}{2(1 + t)^2 \rho a(1 - a)}
\]

\[= G + H q_E,
\]

where

\[
G \equiv \frac{B(1 - a - at)}{2(1 + t)^2 \rho a(1 - a)} < 0, \quad H \equiv \frac{\Delta [B + l(1 - a_b - a_s)]}{1 + t}.
\]

We obtain

\[
q^*_E(l) = \frac{C + D l}{J + Kl}, \tag{36}
\]

where

\[
C = B(1 + t)(1 - q_I - \mu) + \frac{B^2(1 - a - at)}{1 + t} \left[\frac{1 - a - at}{2\rho a(1 - a)} + \Delta\right],
\]

\[
D = \Delta \theta + B(1 - a - at)(1 - a_b - a_s),
\]

\[
J = 2B(1 + t) + at \Delta B^2 > 0,
\]

\[
K = -(1 - a_b - a_s)at \Delta B.
\]

Note that \(q^*_E(0)\) gives firm E’s reaction function when insider trading is prohibited. Since \(q^*(l) > q^*(0)\) if and only if \(q^*_E\) is everywhere increasing in \(l\), we conclude that \(q^*(l) > q^*(0)\) if and only if

\[
DJ > KC.\|
\]

\(^{10}\)If we set \(\delta \equiv 0\), we obtain

\[
q^*_E = \frac{B(1 - q_I - \mu) + l\Delta \theta}{2B + 2\rho a(1 - a)\Delta^2[B + l(1 - a_b - a_s)]^2}.
\]

Here we can verify Proposition 1 once again. Note that if \(\rho\) is close to zero, \(q^*_E\) is higher when \(l > 0\) than when \(l = 0\).
Proposition 5 shows that, unlike in Proposition 1, when \( \rho \) is not small, allowing insider trading may not encourage the risk averse insider to expand output. When insider trading is allowed, the manager knows that he cannot commit not to exploit the superior cost information and engage in insider trading. In making the output decision, M knows that choosing a higher output will expose himself more to the risk of trading gain and the cost uncertainty, and that raises the need of engaging in costly hedging activity. When insider trading is allowed, how much hedging will be needed depends on whether the market is bearish or bullish (represented by the sign of \( 1-a_b-a_s \)). In a bearish market, because the trading gain and the managerial salary are positively correlated, and because hedging is costly (measured by \( 1-a-at = a(\frac{1-a}{a}-t) \)), expanding output is costly for the manager, although it certainly raises the expected gain from insider trading. Thus a complicated trade-off is needed to determine whether the output level should be raised beyond the optimal level for the case where insider trading is prohibited. Proposition 5 shows that one single condition summarizes all the above concerns: in case \( t \) is close to \( \frac{1-a}{a} \), then insider trading tends to encourage output expansion in a bearish market, for the increase in risk is not much a concern; in case \( t \) is considerably larger than \( \frac{1-a}{a} \), and if \( l \) is small so that the trading gain is rather limited, then the manager, who cannot commit not to engage in insider trading ex-post, rationally reduces output ex-ante when the stock market is bullish, a way to avoid ex-post costly corporate hedging activity.

**Proposition 6** Insider trading encourages firm E to hedge more in a bearish market if

\[
\min\left(\frac{\theta}{B(1-a_b-a_s)}, 2\Delta\rho a(1-a)\right) > a(t - \frac{1-a}{a}).
\]

Insider trading encourages firm E to hedge less in a bullish market if

\[
l \leq \frac{B}{a_b + a_s - 1}
\]

and

\[
\max\left(\frac{\theta}{B(1-a_b-a_s)}, 2\Delta\rho a(1-a)\right) < a(t - \frac{1-a}{a}).
\]

**Proof.** Suppose first that \( 1-a_b-a_s > 0 \) so that the market is bearish. Proposition 4 shows that given \( q_E \),

\[
\delta^*(l, q_E) = G + H(l)q_E \geq G + H(0)q_E = \delta^*(0, q_E).
\]
Now the above first inequality implies that the stated condition in Proposition 5 holds, so that \( q_E(l) > q_E(0) \). It follows that

\[
\delta^*(l, q_E(l)) = G + H(l)q_E(l) \geq G + H(l)q_E(0) \geq G + H(0)q_E(0) = \delta^*(0, q_E(0)).
\]

Next, suppose that \( 1 - a_b - a_s < 0 \) so that the market is bullish. Proposition 4 shows that given \( q_E > 0 \),

\[
\delta^*(l, q_E) = G + H(l)q_E \leq G + H(0)q_E = \delta^*(0, q_E).
\]

Now the above second inequality implies that the stated condition in Proposition 5 is violated, so that \( q_E(l) < q_E(0) \). It follows that

\[
\delta^*(l, q_E(l)) = G + H(l)q_E(l) \leq G + H(l)q_E(0) \leq G + H(0)q_E(0) = \delta^*(0, q_E(0)).
\]

Again the sublety in the two assertions arises from the fact that insider trading may or may not encourage the manager to expand output beyond the optimal output level when insider trading is not allowed. When \( t \) is close \( \frac{1-a}{a} \), insider trading always encourages the manager to engage in more hedging in a bearish market. In a bullish market where insider trading partially hedges the risk in the managerial salary, insider trading need not discourage the manager from hedging more; that would happen if \( t \) is considerably higher than \( \frac{1-a}{a} \), and if the liquidity trading is not very intense (\( l \) is small)—the manager knows that he cannot commit not to engage in insider trading ex-post, and when \( l \) is small he chooses to reduce the output level to reduce the risk in his future income, thereby resulting in little need of engaging in costly corporate hedging.

3 Insider Trading Regulation and Valuable Index Trading

In the preceding section, we have considered a risk-averse insider that trades under a quote-driven mechanism. Throughout this section, we shall consider only risk-neutral insiders, and risk-neutrality allows us to proceed the analysis under the more complicated order-driven mechanism initially studied by Kyle (1985).\(^{11}\) Consider a two-period economy with two firms 1 and 2 and two

\(^{11}\) We would have lost tractability had we adopted this trading mechanism in section 2. The complication stems from risk aversion and the fact that public investors must bear price risks when submitting market orders. In this case, no closed-form equilibrium can be found. See Subrahmanyam (1991a) for a numerical approach.
consumption goods X and Y. Good Y, taken as the numeraire, is perfectly storable to everyone, and we assume that throughout date 0 there is unlimited lending and borrowing for good Y at zero interest rate. Good Y is the sole required input for the production of good X. Good X are produced by two all-equity firms, referred to as firms 1 and 2, where at the beginning of date 0 firm $j$ is founded by entrepreneur $E_j$. The two firms are endowed with different production technologies: for firm 2 (respectively, firm 1), it takes $c$ (respectively, $\tilde{c} = c - \delta$) units of good Y to produce 1 unit of good X. One interpretation is that firm 2 is an incumbent firm and firm 1 is a new entrant. The incumbent’s production cost has become public information, but there is still uncertainty regarding the new entrant’s cost efficiency. Knowing the firm’s technology, $E_j$ hires manager $M_j$ by offering the latter managerial contract $\Gamma_j = (I_j, \tau_j, d_j)$, with $I_j$ being an amount of good Y to be delivered to $M_j$ at date 1 (a base salary), $\tau_j \in [0, 1]$ being a share of firm $j$’s date-1 profit (a bonus denominated in good Y), and $d_j = 1$ or 0 indicating respectively $M_j$ is and is not allowed to trade firm 1’s equity.\footnote{Hence we shall restrict attention to linear compensation contracts only; see Hart and Holmström (1987) for the prevalence of linear managerial contracts. See also Holmström and Milgrom (1987) for a rationale for the prevalence of linear managerial contracts that arises in a dynamic principal-agent model.}

In addition to the two entrepreneurs $E_1$ and $E_2$ and the two managers $M_1$ and $M_2$, there are $n$ hedgers, $C$ consumers and an unknown number of inactive agents in the economy. The two entrepreneurs, the two managers, the inactive agents and hedgers are assumed to consume only good Y. Consumers, on the other hand, derive satisfaction from consumption of both goods X and Y. The entrepreneurs wish to consume at date 0, but all other agents only consume at date 1. Let $y$ and $x$ be an agent’s date-1 consumptions in goods Y and X respectively. The common date-1 utility function for inactive agents and managers is $U_m(y) = y$. In our model, the stock market opens twice at date 0, and hedgers’ common date-1 utility function, when the stock market reopens at date 0, is $U_h(y) = -e^{-Ay}$, with $A > 0$ being the Arrow-Pratt measure of absolute risk aversion; see Arrow (1970) and Pratt (1964). (We shall be vague about hedgers’ preferences at the time the stock market opens for the first time, for insider trading only takes place when the stock market reopens.) For simplicity, we assume that consumers’ common date-1 utility function is quasi-linear. More precisely, for some constants
\[ a, b > 0, \]
\[ U_c(x, y) = U_{c1}(x) + U_{c2}(y), \quad U_{c1}(x) = ax - \frac{bC}{2}x^2, \quad U_{c2}(y) = y. \quad (37) \]

The assumption that \( U_{c1} \) is quadratic in \( x \) is made for tractability, and is inessential. Note that \( U_m(\cdot), U_h(\cdot), \) and \( U_c(\cdot) \) are von Neumann-Morgenstern utility functions, and hence cardinal; see for example Chapter 2 of Litzenberger and Huang (1988). To avoid degeneracy, we shall assume that production of firm 1 is sufficiently profitable; that is,
\[ a > c. \quad (38) \]

For both firms \( j \), after \( E_j \) hires \( M_j \), he sells the equity of firm \( j \) to consumers and inactive agents and then leaves the market. As a normalization, assume that each firm has one share outstanding. Let each consumer purchase \( f_j \) shares of firm \( j \)'s equity. Both firms' common stocks are then traded in the secondary stock market at date 0. Thus the riskless lending and borrowing opportunity and the two firms' stocks are the assets available for trading at date 0.\(^{15} \)

\(^{13}\)The specifications of agents' utility functions reveal the reason why we refer to them as respectively hedgers and consumers. From \( U_h \), we see that a hedger would like to hedge the risk of his terminal wealth (denominated in good \( Y \)), but he does not have a consumption decision to make at date 1. A consumer, on the contrary, has a preference which implies that only his date-1 consumption decision matters. To see this, note that by quasi-linearity the consumer's date-1 demand for good \( X \) is independent of his date-1 wealth denominated in good \( Y \). Hence the consumer would become a risk neutral uninformed investor in the stock trading at date 0, and being competitive, the consumer would make zero expected profits in the stock market equilibrium, as if he were absent from the stock trading. His date-0 welfare thus depends only on his date-1 consumption decision. This is why we refer to such an agent a consumer.\(^{14}\)

\(^{14}\)Although its shortcomings are well known and criticized by, for example, Ausubel (1990), quadratic function has frequently appeared in the existing work on insider trading. For example, Khanna, Slezak and Bradley (1994) assume a quadratic production function and Stein (1987) assume a quadratic utility function.\(^{15} \)

\(^{15}\)We shall assume no demand uncertainty, and hence firm 2's equity is a riskless asset, which is a perfect substitute for the riskless lending and borrowing opportunity.\(^{16} \)

\(^{16}\)Because of the zero interest rate, stock trading will be equivalent to forward trading. In fact, forward trading provides a more natural interpretation for investors' pre-trade positions specified below, which we assume can be negative (short positions). This equivalence can also be found in previous stock trading models that have been used to address issues regarding insider trading and market microstructure; see for example Greenwald and Stein (1991), Subrahmanyam (1991), and Spiegel and Subrahmanyam (1992).
After $E_1$ and $E_2$ leave the market, the stock market opens for the first time, and hedgers and inactive agents exchange firm 1’s shares for good Y. Assume that inactive agents are marginal traders so that the share price is equal to the expected date-1 earnings of firm 1. Then the stock market closes, and for personal reasons, inactive agents no longer participate in any economic activities throughout date 0. Let the position in firm 1’s shares that hedger $i$’s acquires from inactive agents be denoted by $\tilde{w}_i$, whose outcome is hedger $i$’s private information.

Then $M_1$ and $M_2$ must make output decisions. Initially we assume that $M_1$ and $M_2$ must simultaneously commit to output levels $q_1$ and $q_2$ respectively; these are the quantities of good X to be delivered to the spot commodity market at date 1. After output levels are committed, $M_1$ receives a private signal $\tilde{\delta} + \tilde{c}_1$ regarding the unit production cost $c + \tilde{\delta}$. Since $q_1$ is chosen before $M_1$ receives the private information, $q_1$ is independent of the realization of $\tilde{\delta} + \tilde{c}_1$. In an ideal situation where $q_1$ is contractible, $E_1$ should be able to implement any $q_1$ using a forcing contract; see for example Holmström (1979). Here rule out this possibility by assuming that the output decision is too complicated to be properly described (although our formulation below may give a false impression that it is simple), and hence $E_1$ can only implement $q_1$ using the incentive contract $\Gamma_1$.

After output decisions are made and $M_1$ receives his private information, we assume that the financial market opens for the second (and the last) time. Like in the first time, hedgers and consumers are present, but inactive agents are gone. On the other hand, depending on $d_j = 1$ or $d_j = 0$, $M_j$ may or may not be present when the stock market reopens. Let us recall these market participants’ pre-trade positions when the stock market reopens. Manager $j$’s endowments consist of a fraction $\tau_j$ of firm $j$’s ownership and a positive amount $I_j$ of good Y. Consumers’ endowments consist of a fraction $f_j$ of ownership of firm $j$ and an amount $I$ of good Y. For $i = 1, 2, \ldots, n$, hedger $i$ is endowed with a position $\tilde{w}_i$ in firm 1’s common stock (where $\tilde{w}_i < 0$ represents a short position) and a sure amount $I_h$ of good Y. Note that the existence of inactive agents results in a random supply for firm 1’s shares when the stock market reopens at date 0, even though the number of firm 1’s outstanding shares is known to be one.

Now we describe the trading mechanism and traders’ beliefs when the
stock market reopens at date 0. The hedgers and managers that wish to trade firm 1’s shares must simultaneously submit market orders, and for all \( i = 1, 2, \ldots, n \), and for all \( j = 1, 2 \), we let \( h_i \) and \( m_j \) be the market orders submitted by respectively hedger \( i \) and manager \( j \). Let \( \tilde{z} = \sum_{j=1}^{2} m_j + \sum_{i=1}^{n} h_i \) represent the net order flow coming from the managers and hedgers. On the other hand, consumers will act as Bertrand liquidity suppliers in the sense that they submit limit orders, and one of them will be selected to absorb the net order flow \( \tilde{z} \). For all \( k = 1, 2, \ldots, C \), let \( d_k(\cdot) \) be the demand schedule (or limit order) submitted by consumer \( k \). Bertrand competition among consumers results in a stock price \( P(\tilde{z}) \), which must equal the expected value of firm 1’s date-1 earnings conditional on \( \tilde{z} \). Let \( \chi_j \) be \( M_j \)'s expected stock trading profit before he receives the private information \( \delta + \tilde{e}_j \).

The above formulation of the date-0 stock market is similar to that in Spiegel and Subrahmanyam (1992). We shall follow Spiegel and Subrahmanyam and assume that, as agents’ common knowledge before \( M_1 \) and \( M_2 \) choose output levels,

17This is inessential; any agents endowed linear utility functions in good \( Y \) can serve as the market makers, as long as they do not have to trade for liquidity, hedging, or speculation reasons.

18In general \( d_k(\cdot) \) may be a multi-valued function, which allows consumer \( k \) to accept any trading quantity in a pre-specified set given a particular transaction price; see for example the discussions in section 3 of Kyle (1989).

19This is the order-driven mechanism first proposed by Kyle (1985) and extensively studied in Spiegel and Subrahmanyam (1992), Back (1992, 1993), and Back et al. (2000).

20Thus we assume that insiders and hedgers submit market orders and liquidity suppliers submit limit orders. Grossman and Miller (1988) and Rock (1987) point out that, since limit orders carry a risk of nonexecution, they will be avoided by traders with a high demand for immediacy or by those with valuable but perishable inside information. Our assumption is consistent with this view. Rochet and Vila (1994) allow a monopolistic insider to see the liquidity traders’ orders before submitting his own market order, and they argue that this is equivalent to assuming that the insider can submit limit orders. Back (1992) shows that in the continuous trading setting, a monopolistic insider can always observe the liquidity traders’ orders before submitting his own order, and hence with continuous trading assuming that the insider can only submit market orders is inessential. On the other hand, it makes little sense to talk about hedgers’ demand for immediacy in a one-shot model. For these reasons, we shall consider continuous trading in Section 3. For more discussions of limit order and market order, see for example Greenwald and Stein (1991), Brown and Zhang (1997), Chakravarty and Holden (1995), Handa and Schwartz (1996), Harris and Hasbrouck (1996), Parlour (1998), and Seppi (1997).

21The assumption of Gaussian distributions implies that the total cost of production,
Let us recapitulate the sequence of relevant events.

- At the beginning of date 0, the two firms are founded by $E_1$ and $E_2$ respectively, and their production technologies become common knowledge.
- $E_j$ designs and offers contract $\Gamma_j$ to $M_j$, and if $M_j$ rejects the offer then he gets a reservation utility $u_m$.
- $E_j$ sells the firm to consumers and inactive agents. The latter are risk neutral towards consumption in good $Y$.
- The stock market opens for the first time, and hedgers and inactive agents exchange firm 1’s shares for good $Y$. Short sale is allowed, and the stock price is equal to the expected date-1 earnings of firm 1.
- $M_1$ and $M_2$ given $\Gamma_1$ and $\Gamma_2$ choose $q_1$ and $q_2$ simultaneously. The Nash equilibrium $(q_1^*, q_2^*)$ will be referred to as the date-0 product market equilibrium.
- $M_1$ receives private information $\tilde{\delta} + \tilde{e}_1$.\footnote{The case where both managers receive private signals about $c + \tilde{\delta}$ will be considered in section 3.}
- The stock market reopens and the stock trading equilibrium will be referred to as the date-0 stock trading equilibrium.

$q_1(c_1 + \tilde{c}_2)$, can go beyond any positive level with a positive probability, which means that firm 1’s date-1 earnings and the return on firm 1’s common stock are both unbounded below. This is inconsistent with the limited liability assumption adopted in corporate finance literature, which holds that shareholders cannot be forced to commit funds in the future. Previous work that adopts the Gaussian distributions assumption and exhibits similar problems includes, just to name a few, Sun (1992), Wang (1993), Holmstrom and Tirole (1993), Wang (1994), Kyle (1985) and the numerous papers adopting a Kyle-paradigm or a noisy rational expectations equilibrium model that address issues in market microstructure. Recognizing these problems, we choose to maintain this assumption in order to obtain a linear equilibrium in closed form.

Assumption 1

\begin{equation}
(\tilde{\delta}, \tilde{e}_1, \tilde{e}_2, \tilde{w}_1, \tilde{w}_2, \cdots, \tilde{w}_n)
\end{equation}

are totally independent normal random variables with zero means. Let $\text{var}(\tilde{\delta}) = \psi$, $\text{var}(\tilde{e}_j) = \phi$ for all $j = 1, 2$, and $\text{var}(\tilde{w}_i) = \sigma_w^2$ for all $i = 1, 2, \cdots, n$.
Finally, at date 1, the spot commodity markets for goods X and Y open. Being price-takers, consumers submit Marshallian demands. Given the fixed supply \( q_1 + q_2 \) committed by the managers \( M_1 \) and \( M_2 \) at date 0, a pair of markets-clearing prices \((p_1, 1)\) are then chosen for goods X and Y. Let \( \pi_j \) be firm \( j \)'s profit at date 1. This equilibrium will be referred to as the date-1 spot market equilibrium.

The above sequential economy is closely related to the Radner economy with incomplete asset markets, where assets are traded at date 0 and spot commodities are traded at date 1; see Radner (1972). There are important distinctions here, however. There are non-price-takers in both the stock market and product market in our model, and the mechanism used to consummate trade in the stock market is not Walrasian. Hence instead of Radner equilibria, perfect Bayesian equilibria (PBEs)\(^{23}\) will be derived for our sequential economy. Note that both equilibrium concepts require that agents have perfect foresight regarding future events.

### 3.1 The Equilibrium without Insider Trading

Let us consider first the benchmark case where insider trading is prohibited; i.e. \( d_1 = d_2 = 0 \). We shall solve for the perfect Bayesian equilibria (or PBEs) of the above economy using backward induction. First consider the date-1 spot market equilibrium. It is straightforward to verify the next Lemma.

**Lemma 3** The date-1 inverse demand for good X is

\[
p(Q) = a - bQ. \tag{40}
\]

Given the output levels \( q_1, q_2 \) committed by the two firms, and given a consumer’s date-1 wealth (denominated in good Y), \( W \), the consumer’s date-1 utility is

\[
a\left(\frac{q_1 + q_2}{C}\right) - \frac{bC}{2} (\frac{q_1 + q_2}{C})^2 + W - [a - b(q_1 + q_2)](\frac{q_1 + q_2}{C}) = W + \frac{b(q_1 + q_2)^2}{2C}. \tag{41}
\]

Before we characterize the date-0 stock trading equilibrium, a formal definition for this subgame equilibrium is useful.

\(^{23}\)See for example Fudenberg and Tirole (1991) for a formal definition of this equilibrium concept.
Definition 1 A linear symmetric perfect Bayesian equilibrium (LSPBE) for the date-0 stock trading subgame is a tuple of equilibrium trading strategies \((m_1(\hat{\delta} + \hat{e}_1), \{h_i(\hat{w}_i); i = 1, 2, \cdots, n\}, \{d_k(P); k = 1, 2, \cdots, C\})\) plus a system of posterior beliefs obtained from Bayes Law and these prescribed trading strategies, such that \(m_1\) maps \(M_1\)’s private information \(\hat{\delta} + \hat{e}_1\) into his market order, \(h_i\) maps hedger \(i\)’s endowed position in firm 1’s shares into his market order, and \(d_k(\cdot)\) is the demand function (or limit order) submitted by consumer \(k\), and such that all the functions \(m_1(\cdot), h_i(\cdot), \) and \(d_k(\cdot)\) are linear functions and given all other agents adopt the prescribed trading strategies, each agent finds it optimal to also adopt the trading strategy prescribed for him.

Lemma 4 Suppose that insider trading is prohibited. Then in the date-0 stock trading equilibrium, \(m_j = 0\) for all \(j = 1, 2, \cdots, n\). For all \(k\), \(d_k(\cdot)\) is a multi-valued function such that

\[
\begin{align*}
    d_k(P) &= \left\{ \begin{array}{ll}
        +\infty, & \text{if } P < q_1[a - b(q_1 + q_2) - c]; \\
        \mathbb{R}, & \text{if } P = q_1[a - b(q_1 + q_2) - c]; \\
        -\infty, & \text{if } P > q_1[a - b(q_1 + q_2) - c].
\end{array} \right.
\end{align*}
\]  

(42)

Independent of \(\hat{z}\), the date-0 equilibrium price of firm 1’s common stock is \(P = q_1[a - b(q_1 + q_2) - c]\).

Proof. In equilibrium, consumers know that they are trade with the uninformed hedgers, and since Bertrand competition dictates that consumers make zero expected profits from stock trading, given the commonly expected \(q_1, q_2\), the equilibrium price of firm 1’s common stock must be independent of \(\hat{z}\),

\[
P = q_1 E[a - b(q_1 + q_2) - c + \hat{\delta}] = q_1[a - b(q_1 + q_2) - c].
\]  

(43)

If \(P > q_1[a - b(q_1 + q_2) - c]\) (respectively, \(P < q_1[a - b(q_1 + q_2) - c]\)), then it is optimal for consumers to sell (respectively, buy) an unbounded number of shares. This shows the optimality of \(d_k(\cdot)\). For risk averse hedgers, on the other hand, it is apparently optimal to clear their positions in the stock given that the stock price is equal to the mean earnings to be distributed at date 1.\(^{24}\) This shows that the optimality of \(h_i\). ||

\(^{24}\)This happens because any long or short position will result in a date-1 wealth for the hedger that is second-order stochastically dominated by the sure wealth obtained by clearing all the pre-trade position.
Now we can consider the date-0 product market equilibrium.\footnote{Since the interest rate is zero, stock trading is equivalent to forward trading at date 0. Note that with the Gaussian distributions assumption about $\tilde{w}_i$, after stock trading a consumer may end up with a negative wealth at date 1 when spot commodities are traded. This is a problem commonly shared by the previous work adopting the Gaussian distributions assumption.}

**Lemma 5** Suppose that insider trading is prohibited. Then in the perfect Bayesian equilibrium of the above economy, both firms choose output level $\frac{a-c}{96}$ at date 0, resulting in an expected firm profit $\frac{(a-c)^2}{96}$ at date 1, which implies that at date 0, a consumer’s equilibrium expected utility is $I + \frac{(f_1+f_2)(a-c)^2}{96} + \frac{2(a-c)^2}{96}C$.

**Proof.** Omitted. ||

### 3.2 The Equilibrium with Insider Trading

The analysis in this subsection follows Spiegel and Subrahmanyam (1992). Since $M_2$ does not possess superior information he will not trade firm 1’s shares.

**Lemma 6** Given the commonly conjectured $q_1, q_2$ committed by $M_1$ and $M_2$, a date-0 stock trading LSPBE $(m_1, \{h_i; i = 1, 2, \cdots, n\}, \{d_k(P); k = 1, 2, \cdots, C\})$ exists if and only if

$$A^2 n \sigma^2 q_1^2 (\psi + 2\phi)^2 > 4(\psi + \phi),$$

where in the LSPBE,

$$m_1(\tilde{\delta} + \tilde{e}_1) = \beta q_1(\tilde{\delta} + \tilde{e}_1),$$

$$h_i(\tilde{w}_i) = \gamma \tilde{w}_i, \ \forall i = 1, 2, \cdots, n,$$

and

$$d_k(P) = \frac{1}{k} [q_1(a - b(q_1 + q_2) - c) - P], \ \forall k = 1, 2, \cdots, C,$$

so that the equilibrium stock price is

$$P(\tilde{z}) = q_1(a - b(q_1 + q_2) - c) + \lambda \tilde{z},$$
with the constants $\beta, \gamma,$ and $\lambda$ being defined by

$$
\beta = \frac{\psi}{2(\phi + \psi)} \left( q_1 A \sigma_w (\psi + 2\phi) \sqrt{n} - 2\sqrt{\psi + \phi} \right),
$$

(49)

$$
\gamma = -\frac{q_1 \sqrt{\psi + \phi}}{2 \sqrt{n} \sigma_w (\phi + \psi)} \left( q_1 A \sigma_w (\psi + 2\phi) \sqrt{n} - 2\sqrt{\psi + \phi} \right),
$$

(50)

and

$$
\lambda = \frac{q_1^2 A \sqrt{\psi + \phi} ((\psi + 2\phi)^2 + \psi \phi + \frac{(n-1)(\psi + \phi)}{n})}{2 (\psi + 2\phi)^2 (q_1 A \sigma_w (\psi + 2\phi) \sqrt{n} - 2\sqrt{\psi + \phi})}. 
$$

(51)

**Proof.** The proof is similar to that for Proposition 1 of Spiegel and Subrahmanyan (1992), and is omitted. \|

Unlike in Kyle (1985), where noise trade is predetermined and independent of other parameters, here the hedging demand is endogenously determined, and parameters that violate the condition

$$
q_1^2 A^2 n \sigma_w^2 (\psi + 2\phi)^2 > 4 (\psi + \phi)
$$

(52)
depict a market environment where the adverse selection problem is too severe to retain any hedgers in stock trading. In that case no LSPBE can exist. Straightforward computations yield the following performance measures for the date-0 stock market.

**Corollary 1** The expected insider trading profit for $M_1$ is

$$
\chi_1 = \psi \sqrt{\psi + \phi} \frac{q_1 A \sqrt{n} \sigma_w (\psi + 2\phi) - 2 \sqrt{\psi + \phi}}{A ((\psi + 2\phi)^2 + \psi \phi + \frac{(n-1)(\psi + \phi)}{n})} = \theta q_1 - \theta'',
$$

(53)

where the constants $\theta, \theta'' > 0$.\(^{26}\) The post-trade earnings volatility for firm 1 is

$$
\frac{q_1^2 \psi (\psi + 2\phi)}{2 (\psi + \phi)}.
$$

(55)

\(^{26}\)It may appear that $\chi$ becomes negative when $A \downarrow 0$, but this is really not the case. Recall that for the LSPBE to exist, it is necessary that

$$
A > \frac{2 \sqrt{\psi + \phi}}{q_1 n \sigma_w (\psi + 2\phi)} > 0,
$$

(54)

and when the latter condition is satisfied, $\chi$ is non-negative.
The stock price variance is

$$\text{var}(P(z)) = \frac{q_1^2 \psi^2 (\psi + \phi)}{2(\psi + \phi)^2}. \quad (56)$$

In the sequel we shall sometimes assume that $A$ is very large, so that the condition that supports the LSPBE can be satisfied by a wide range of output choice $q_1$. When $A$ is sufficiently large the following limiting values for equilibrium coefficients provide good approximations. The limiting case is where the Spiegel and Subrahmanyam model looks most like the original Kyle (1985) model.\(^{27}\)

**Corollary 2** As $A$ tends to $+\infty$, we have

\[
\lim_{A \to +\infty} \beta = \frac{2 \sqrt{\psi + \phi} \sqrt{n} \sigma_w (\psi + 2\phi)}{q_1 \left[ (\psi + 2\phi)^2 + \psi \phi + \frac{(n-1) \psi (\psi + \phi)}{n} \right]}, \quad (57)
\]

\[
\lim_{A \to +\infty} \gamma = -\frac{2(\psi + \phi)(\psi + 2\phi)}{\left[ (\psi + 2\phi)^2 + \psi \phi + \frac{(n-1) \psi (\psi + \phi)}{n} \right]} < -1, \quad (58)
\]

\[
\lim_{A \to +\infty} \chi_1 = \frac{q_1 \sqrt{\psi + \phi} \sqrt{n} \sigma_w (\psi + 2\phi)}{\left[ (\psi + 2\phi)^2 + \psi \phi + \frac{(n-1) \psi (\psi + \phi)}{n} \right]^\frac{3}{2}}, \quad (59)
\]

and

\[
\lim_{A \to +\infty} \lambda = \frac{q_1 \sqrt{\psi + \phi} \left[ (\psi + 2\phi)^2 + \psi \phi + \frac{(n-1) \psi (\psi + \phi)}{n} \right] \psi}{2 \psi + 2\phi \sqrt{n} \sigma_w (\psi + 2\phi)^2}. \quad (60)
\]

\(^{27}\)The hedgers’ trading intensity $\gamma$ rises with $A$ in an intuitive manner; see Spiegel and Subrahmanyam (1992). When $A$ is large, $\gamma$ tends to rise with $n$ also: although adding one more hedger to the market raises the price risk facing all existing hedgers, but that would also mitigate the adverse selection problem and hence improve the terms of trade facing the hedgers, and when $A$ is large, the latter effect dominates, and hence $\gamma$ rises with $n$. Also, with $A$ being large, the higher price variability resulting from adding one more insider is not a major concern to the risk averse hedgers, for they are trading large quantities themselves. Thus with $A$ being large, adding one more insider is more likely to reduce $\lambda$ and raise the market depth via the competition among insiders. We shall analyze the case of two competitive insiders in section 3.

\(^{28}\)See section 1.1 of Spiegel and Subrahmanyam (1992) for a discussion of the hedgers’ overhedging behavior.
As in section 2, it can be easily verified that allowing insider trading encourages the manager to expand output ex-ante. Our next Proposition characterizes the relationships between industry-wide parameters and the stock market performance.

**Proposition 7** Suppose that $A$ is very large so that the condition $q_1^2 A^2 n \sigma_w^2 (\psi + 2\phi)^2 > 4(\psi + \phi)$ holds for all relevant $q_1$. Then in equilibrium a higher profitability $a - c$ of the industry or a smaller demand elasticity $b$ for good $X$ results in

- a lower liquidity (measured by the depth $1 / X$) in the date-0 stock market;
- a higher information efficiency (measured by the difference between pre-trade and post-trade earnings volatilities of firm 1) for the date-0 stock market;
- a higher stock price volatility;
- a higher expected trading profit for $M_1$.

The marginal impact of $a - c$ on the stock market performance is weakly decreasing in $a - c$. The marginal impact of $\frac{1}{b}$ on the stock market performance is weakly increasing in $a - c$.

**Proof.** Available upon request. ||

Proposition 4 shows that for a given industry, an upward shift of demand curve, a reduction in demand elasticity, or a reduction in unit production cost can each lead to lower stock market liquidity, higher price volatility, and higher informational efficiency. The intuition is that when $a - c$ is higher or $b$ is lower, the benefit of expanding output becomes higher, and output expansion implies a larger information advantage for $M_1$ in stock trading, which encourages $M_1$ to trade more aggressively (the insider’s trading intensity $\beta q_1$ is accordingly higher). This results in a more informative net order flow, and with rational expectations, consumers’ (common) limit order exhibits a larger elasticity, which translates into a lower market depth and a higher stock price volatility. The more aggressive insider trading also results in a more informative transaction price, so that the equilibrium informational efficiency is also higher.
That the marginal impact of $a - c$ on the stock market performance decreases with the level of $a - c$ can be understood as follows. When $a - c$ is small, an increase in $a - c$ leads to a higher marginal increase in $q_1$ because earnings reduction resulting from output over-expansion is not much a concern for $M_1$, and he focuses only on the expected insider trading profit. When $a - c$ is high, the earnings reduction resulting from output over-expansion becomes important, and hence the same amount of increase in $a - c$ leads to a lower marginal increase in $q_1$.

3.3 A Prisoner’s Dilemma of Output Expansion

Instead of assuming that the two firms produce a homogeneous good X, in this section we assume that firm 1 produces good X and firm 2 produces good Z. Consumers drive satisfaction from consumption in goods X, Y, and Z, but all other agents consume only good Y. For simplicity, we normalize the population of consumers to $C = 1$. Consumers have the following sub-utility function for goods X and Z:

$$U_{c1}(x, z) = a(x + z) - \frac{b}{2}(x^2 + z^2) - b'xz; \quad (61)$$

where the new parameter $b' > 0$ measures the substitutability of goods X and Z. It follows that the date-1 inverse demands for goods X and Z are respectively

$$p_x = a - bx - b'z, \quad p_z = a - bz - b'x. \quad (62)$$

Allowing firms 1 and 2 to produce heterogeneous goods in the current model represents an extension of the basic model that we have examined thus far. We shall also make the following alterations.

- We assume with Subrahmanyam (1991) that in addition to the two firms’ common stocks and the riskless lending and borrowing opportunity, there may exist a basket of securities which is also available for trading at date 0. Let this basket be an index for the earnings performance of the industry, which for simplicity is assumed to be $\pi_1 + \pi_2$.

- Both firms are faced with cost uncertainty at the time output levels must be determined. Let the unit cost for firm 2 be $c - \tilde{\delta}'$.

- Manager $M_2$ receives a private signal $\tilde{\delta}' + \tilde{e}_2$ at the same time $M_1$ receives signal $\tilde{\delta} + \tilde{e}_1$. 

34
• Instead of deriving liquidity traders’ demands by analyzing the hedgers’ optimization problem as in the previous sections, we assume with Subrahmanyam (1991) that there are four classes of liquidity traders: \( N \) non-discretionary traders that can only trade firm 1’s shares, \( N \) non-discretionary traders that can only trade firm 2’s shares, \( N_B \) non-discretionary traders that can only trade the basket, and \( L \) liquidity traders that must trade an equal number of both firms’ shares (in either the basket market or the stock market). Define \( n = N + L \) and \( n' = N_B + L \). Let \( A_1, A_2, B, \) and \( D \) represent the four classes of liquidity traders described above. Let each of these traders be endowed with a given quantity \( \gamma_j \tilde{w}_i \) of the type of securities that he must finish trading, where

\[
\gamma_j = \frac{q_j \sqrt{2(\psi'_j)^2(\psi' + \phi')}}{\sqrt{n''}\sigma_w(2\phi'_j + 3\psi'_j)} \frac{[3\psi'_j + 2\phi'_j]^2 \{q_j A_1 \psi'_j \sigma_w(\psi' + 2\phi' - 2\sqrt{2(\psi' + \phi')})\}}{q_j^2 A_1 \psi'_j \sqrt{2(\psi' + \phi')}[(\psi' + 2\phi')^2 + 2\psi'\phi' + \frac{2(2\phi'_j + 3\psi'_j)^2}{n''}]} \]

and note that the liquidity trader adjusts \( \gamma_j \) with the \( q_j \) that he expects, the population \( n'' \) of liquidity traders trading the asset that he will be trading, the variance \( \psi'_j \) of the intrinsic value of the traded asset, and the variance \( \phi'_j \) of the noise contained in the private information possessed by each insider trading the asset that he will be trading. In a more complete model, such an order strategy can be shown to be consistent with expected utility maximization.

• The random variables

\[
(\tilde{\delta}, \tilde{\phi}, \tilde{e}_1, \tilde{e}_2, \tilde{w}_i; i \in A_1 \bigcup A_2 \bigcup B \bigcup D)
\]

are totally independent normal random variables with zero means. Let \( \text{var}(\tilde{\delta}) = \text{var}(\tilde{\phi}) = \psi \), \( \text{var}(\tilde{e}_j) = \phi \) for all \( j = 1, 2 \), and \( \text{var}(\tilde{w}_i) = \sigma_w^2 \) for all \( i \in A_1 \bigcup A_2 \bigcup B \bigcup D \).

First we consider the case where the basket is not available for trading. The following Proposition shows that a prisoner’s dilemma of output expansion may arise when insider trading is allowed.

**Proposition 8** Suppose that the firms act simultaneously in the product market.
• Let \( x^* \) and \( z^* \) be the two firms' output decisions in equilibrium. In equilibrium \( d_1^* = d_2^* = 1 \), and the outputs, prices of goods \( X \) and \( Z \), and expected date-1 firm profits are respectively,

\[
x^* = z^* = \frac{a - c + \theta}{2b + b'}, \quad p_x^* = p_z^* = \frac{ba - (b + b') (\theta - c)}{2b + b'}.
\]

(65)

\[
E[\pi_1] = E[\pi_2] = \frac{(a - c + \theta) [b(a - c + \theta) - b' \theta]}{(2b + b')^2},
\]

(66)

where the parameter \( n \) in \( \theta \) and \( \theta'' \) is by definition equal to \( N + L \). In equilibrium, the expected insider trading profits for the two managers are

\[
\chi_1 = \chi_2 = \frac{\theta(a - c + \theta)}{2b + b'} - \theta''.
\]

(67)

• A regulation that prohibits insider trading makes \( E_1 \) and \( E_2 \) better off if and only if

\[
\frac{b [(a - c + \theta)^2 - (a - c)^2]}{(2b + b')^2} < \theta''.
\]

(68)

Ceteris paribus, the inequality holds when \( b' \) is sufficiently large.

Proof. Note first that, being uninformed about the rival firm’s production cost, a manager will not trade the rival firm’s shares. Now consider the first assertion. Entrepreneur \( E_1 \) seeks to

\[
\max_{x \in \mathbb{R}, d_1 \in \{0, 1\}} \left( xE[a - bx - b'z - c + \delta] + 1_{d_1 = 1}(d_1)[x\theta - \theta''] \right),
\]

(69)

if he conjectures that firm 2’s output choice is \( z \), where \( 1_A(d_1) \) is an indicator function for event \( A \), which equals 1 if \( d_1 \in A \) and 0 if otherwise. Similarly, entrepreneur \( E_2 \) seeks to

\[
\max_{z \in \mathbb{R}, d_2 \in \{0, 1\}} \left( zE[a - bz - b'x - c + \delta'] + 1_{d_2 = 1}(d_2)[z\theta - \theta''] \right),
\]

(70)

if he conjectures that firm 1’s output choice is \( x \). Since for all \( x > 0 \),

\[
x\theta - \theta'' > 0,
\]

(71)

it is easy to see that \( d_1^* = 1 \) is optimal. By symmetry, we conclude that \( d_2^* = 1 \) is optimal. Hence we can re-write the entrepreneurs’ problems as respectively

\[
\max_{x \in \mathbb{R}_+} x[a - bx - b'z - c] + [x\theta - \theta''],
\]

(72)
and
\[
\max_{z \in \mathbb{R}_+} z[a - bz - b'x - c] + [z\theta - \theta''].
\]
(73)

The Nash equilibrium outputs are therefore
\[
x^* = z^* = \frac{a - c + \theta}{2b + b'}.
\]
(74)

It follows that the date-1 prices of goods X and Z are
\[
p^*_x = p^*_z = \frac{ba - (b + b')(\theta - c)}{2b + b'}
\]
and the expected date-1 profits are
\[
E[\pi_1] = E[\pi_2] = \frac{(a - c + \theta)\theta(a - c - \theta) - \theta''}{(2b + b')^2}.
\]

Now consider the second assertion. It is easy to deduce that the date-0 value of each firm is
\[
\frac{b(a - c)^2}{(2b + b')^2} - u_m,
\]
in case insider trading is banned. Thus a regulation that prohibits the mangers from engaging in insider trading makes the entrepreneurs better off if and only if
\[
\frac{(a - c + \theta)(b(a - c - \theta) - b'\theta)}{(2b + b')^2} + \theta(a - c + \theta) - \theta'' - u_m
\]
\[
< \frac{b(a - c)^2}{(2b + b')^2} - u_m.
\]

Re-arranging, we obtain the equivalence condition
\[
\frac{b((a - c + \theta)^2 - (a - c)^2}{(2b + b')^2} < \theta''.
\]
(77)

This completes the proof.

The intuition behind Proposition 5 is as follows. When \(b'\) is large, an increase in consumption of good X reduces greatly the marginal utility from consuming good Z, and vice versa. In this situation one firm’s allowing its manager to engage in insider trading induces the manager to expand that firm’s output, which greatly reduces consumers’ valuation for the good produced by the rival, leading to a big reduction in the value of the rival firm. Note that no matter whether \(E_j\) chooses \(d_j = 0\) or \(d_j = 1\), choosing \(d_k = 1\) is optimal for \(E_k\). Hence the firms are trapped in a prisoner’s dilemma where firms cannot unilaterally disallowing insider trading. This provides a rationale for insider trading regulation.
3.4 The Value of Index Trading: A New Perspective

Now we assume that a basket is available for trading at date 0. At first, there exists an equilibrium where all traders disregard the basket at date 0. In this case the equilibrium properties (including the prisoner’s dilemma in insider trading) recorded in Proposition 5 remain valid. This equilibrium is supported by the off-the-equilibrium belief that only insiders would trade the basket, and hence consumer \( k \) submits the following limit order to the basket market:

\[
d_k(P) = \begin{cases} 
\mathbb{R}_-, & \text{if } P = +\infty; \\
\mathbb{R}_+, & \text{if } P = -\infty; \\
0, & \text{otherwise.}
\end{cases}
\] (78)

This game has a number of other PBE’s, among which Subrahmanyam emphasizes the one where trading clusters on the index, for it reduces the expected trading costs facing the liquidity traders. We shall show that in the current context where concerns for insider trading profits may result in output over-expansion, Subrahmanyam’s clustering-on-the-index equilibrium may benefit the insiders as well. To state our next Proposition, we define

\[
\kappa = 2\psi \sqrt{\frac{3\psi + \phi}{2}} \frac{\sqrt{n} \sigma_w (4\psi + 2\phi)}{\left((4\psi + 2\phi)^2 + 4\psi(\psi + \phi) + \frac{4(n' - 1)\psi(3\psi + \phi)}{n'} \right)}
\] (79)

and

\[
\kappa'' = 2\psi \sqrt{\frac{3\psi + \phi}{2}} \frac{2\sqrt{2(3\psi + \phi)}}{A \left\{ (4\psi + 2\phi)^2 + 4\psi(\psi + \phi) + \frac{4(n' - 1)\psi(3\psi + \phi)}{n'} \right\}}
\] (80)

where recall that \( n' = N_B + L \). Note that \( \kappa \) (respectively, \( \kappa'' \)) is obtained from \( \alpha \) (respectively, \( \alpha'' \)) by replacing \( n \) by \( n' \), \( \psi \) by \( 2\psi \) and \( \phi \) by \( \psi + \phi \).

**Proposition 9** With the basket traded at date 0, there exists a PBE where trade occurs only in the basket market. One supporting belief is that orders submitted for the two firms’ shares are coming from insiders with probability one, and hence consumer \( k \) submits the following limit order to the market.
for firm $j$’s stock:

$$d_\ell(P) = \begin{cases} 
\mathcal{R}_-, & \text{if } P = +\infty; \\
\mathcal{R}_+, & \text{if } P = -\infty; \\
0, & \text{otherwise}.
\end{cases} \quad (81)$$

In equilibrium, traders in classes $A_1$ and $A_2$ do not trade, and insiders’ and consumers’ trading strategies are as described in Lemma 6, with $n$ replaced by $n'$, $\psi$ by $2\psi$, and $\phi$ by $\psi + \phi$. In equilibrium, index trading enhances the date-0 value of both firms if

$$\frac{b[(a - c + \theta)^2 - (a - c)^2]}{(2b + b')^2} < \theta'', \quad (82)$$

and

$$\frac{b[(a - c + \kappa)^2 - (a - c)^2]}{(2b + b')^2} > \kappa''. \quad (83)$$

Proof. First observe that the supporting belief implies that the prescribed consumers’ limit order strategy is optimal, and given consumers’ limit order strategy, it is optimal for managers and class-$B$ and class-$D$ liquidity traders to trade only the basket. Now the game becomes the one described in Lemma 6, except that $n$ should be replaced by $n' = N_B + L$ and the traded asset promises to pay $\pi_1 + \pi_2$ rather than a single $\pi_j$. It follows that the noise in $M_1$’s signal is $-\delta + \tilde{e}_1$ and the noise in $M_2$’s signal is $-\delta + \tilde{e}_2$. Thus in applying the results obtained in Lemma 6 we should replace $\psi$ by $2\psi$ and $\phi$ by $\psi + \phi$. Moreover, rationally expecting a symmetric output choice by the two firms, the class-$B$ and class-$D$ liquidity traders correctly compute $\gamma = \gamma_1 = \gamma_2$, and given $\gamma$, each of them submits $\gamma \tilde{w}_i$.

Hence applying Lemma 6, we can obtain the date-0 value for both firms in the index trading equilibrium. Following Proposition 5, the last two asserted inequalities imply respectively that insider trading results in a lower value for each firm in the absence of index trading, and that with index trading, insider trading leads to a higher value for each firm.  

Proposition 6 bears on Subrahmanyam (1991) in an obvious manner. Subrahmanyam shows that insiders possessing security-specific private information loses much of their information advantage if they are forced to trade
the basket. In our model, this implies that insider’s incentive to over-expand output is mitigated, and with the output over-expansion problem being resolved, the firm value is enhanced.

4 Concluding Remarks

This paper has conducted an exploratory study regarding how insider trading may affect product market competition, stock market performance, and the optimal managerial compensation scheme. We considered duopolistic firms that engage in either Cournot or Stackelberg competition with at least one firm faced with an uncertainty in the marginal cost of production. If insider trading is not prohibited and if the cost uncertainty is removed for the firm manager earlier than it becomes public information, then ex-ante the manager has an incentive to expand output. Thus a firm’s allowing its manager to engage in insider trading forces its rival firm to reduce output in equilibrium, and hence creates a negative externality on the shareholders of the rival firm. When hedgers in the stock market exhibit a strong trading intensity, this externality can be so severe that it forces a rival firm faced with no cost uncertainty to cease production. On the other hand, if commitment value of insider trading in product market competition is sufficiently large, then insider trading may benefit the shareholders of the firm that allows its manager to engage in insider trading, even if the shareholders have to bear all the trading loss caused by insider trading. We have also shown that a prisoners’ dilemma in insider trading may arise if both firms are faced with ex-ante cost uncertainty and ex-post managerial information advantages. In this case, contrary to what was suggested by Carlton and Fischel (1983), although insider trading is indulged by every firm under the optimal managerial contract, a regulation that prohibits insider trading benefits all shareholders.

Finally, a theory of valuable index trading is developed, which shows that, contrary to the existing view that index trading tends to benefit liquidity traders but reduce insider trading profits, index trading may benefit insiders as well.

This research has exhibited several limitations. For example, we have disregarded the possibility of information acquisition by outside speculators, and hence our theory cannot distinguish inside speculators and outside speculators, nor can we capture the latter’s impact on informational efficiency,
as was the focus of Fishman and Hagerty (1992) and Khanna, Slezak, and Bradley (1994). Our analysis cannot address issues concerning consumer welfare. Even with the simplifying assumption of quasi-linear preferences the welfare implications of our analysis remain ambiguous. These problems define the directions in which the current research can be further extended and improved upon.
References


