The Performance of GARCH Models with Short-memory, Long-memory, or Jump Dynamics: Evidence from Global Financial Markets

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ABSTRACT

This study evaluates the performance of alternative volatility models, including EGARCH, FIEGARCH, EGARCH-jump, and EGARCH- skewed-t models, on model fitting, volatility forecasting, and Value-at-Risk (VaR) prediction. As compared with the simple EGARCH model, the EGARCH-jump model demonstrates significant improvements, outperforming every other model in almost all aspects, but the computation load substantially increases by the inclusion of jump dynamics. If less expensive volatility models are preferred, then alternatives may include the use of the EGARCH-skewed-t model for model fitting and VaR prediction, and the FIEGARCH model for volatility forecasting, since these models also demonstrate fairly good performance for these specific purposes. Furthermore, since the FIEGARCH model demonstrates relatively good performance with regard to U.S. stock market returns only, this suggests that the long-memory pattern captured by the fractionally-integrated volatility model may not be a global stylized fact.

Keywords: GARCH; Model fitting; Volatility forecasting; VaR prediction; International finance

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1. INTRODUCTION

The volatility of asset returns is a key input for both risk management and the pricing of derivatives.[1] Since volatility, unlike returns, is a latent measure of asset price dynamics, determining the ways for describing volatility has become an issue of crucial importance in the study of modern financial management.

Amongst the diverse stream of volatility modeling methods, the autoregressive conditional heteroskedasticity (ARCH) type models have emerged as the most popular, and probably, most successful. Following the initial introduction of the first ARCH model in Engle (1982), modifications were soon proposed by Bollerslev (1986) and Taylor (1986), who simultaneously, but independently, proposed a 'generalized ARCH' (GARCH) model to substantially improve the practicability of the model implementation. Thereafter, numerous extended models were soon introduced, and indeed, were shown to be plausible.[2]

All of these studies demonstrate that throughout the development of volatility modeling, a growing range of factors has been brought into consideration. However, as the model becomes more sophisticated, it also becomes increasingly complex, leading to a corresponding increase in the difficulty of model implementation. It is therefore natural to question whether the more complicated

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[1] The seminal work of Black and Scholes (1973) led to the development of 'option pricing theory', which was subsequently followed by an array of option pricing models including: the stochastic volatility (SV) model of Heston (1993), the GARCH model of Duan (1995), the long-memory GARCH model of Taylor (2000), the SV-jump model of Bates (1996), the GARCH-jump model of Duan et al. (2005), and the SV double-jump model of Duffie et al. (2000). Irrespective of the ways in which the frameworks are modified, the volatility of the underlying asset returns continues to play an extremely important role in deciding both option prices and hedging portfolios.

[2] These models include the 'asymmetric news impact' (short-memory) models, such as EGARCH (Nelson, 1991) and GJR-GARCH (Glosten et al., 1993), the 'long-memory' models, including FIGARCH and FIEGARCH (Bollerslev and Mikkelsen, 1996), and the 'GARCH-jump' models as proposed by Maheu and McCurdy (2004) and Duan et al. (2005, 2006).

model performs any better for a particular purpose, and if so, whether such an improved performance can be globally valid for different financial markets.

To the best of our knowledge, and in light of the extremely comprehensive review of the literature on volatility forecasting undertaken by Poon and Granger (2003), the above questions have yet to be clearly answered. Thus, the primary objective of this study is to attempt fill this gap in the literature by comparing the performance of short-memory, long-memory, and jump models, in terms of several purposes including model fitting, volatility forecasting,[3] and value at risk (VaR) prediction.[4] This study is expected to provide important implications and guidance for the implementation of volatility models in both practical and academic fields.

In this study we develop a 'nested volatility' model based upon the EGARCH framework. The model incorporates short-memory (EGARCH), long-memory (FIEGARCH), and jump (EGARCH-jump) dynamics to support an investigation into the performance of various models, in terms of model fitting, volatility forecasting, and VaR prediction, for the stock market indices of eight relatively large national markets. Since the specification of jumps or long memory essentially makes the unconditional distribution of financial asset returns skewed and leptokurtic, then alternatively we also include the skewed-t EGARCH model (Hansen, 1994), with the stylized facts being directly modeled in the stochastic innovation of a short-memory model.

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[3] In addition to the use of historical asset returns, some studies have also taken exogenous information, such as 'intraday returns', 'news arrival counts', and 'implied option volatility', as a means of supporting the accurate forecasting of volatility. These studies have reported obvious improvements, with 'implied option volatility' proving to be superior. Nevertheless, the focus of this study is necessarily placed upon endogenous forecasting models.

[4] Following the recent occurrence of several financial scandals in many well-known corporations, such as Enron and LTCM, monitoring the level of operational or investment risk has become an extremely serious task for those with the responsibility for risk management.

Our empirical results show that the inclusion of jumps in GARCH-type model specifications can substantially raise performance, in terms of model fitting, volatility forecasting, and VaR prediction, with the single exception of volatility forecasting for some indices. However, since the computation load of this model is much greater than those of the other models, if a less-expensive model is preferred, then 'second-best' choices are available for any one of the various purposes. We can, for example, employ the skewed-*t* model for both volatility model fitting and VaR prediction and use the long-memory model for volatility forecasting. It is also worth noting that the FIEGARCH model has relatively satisfactory performance with regard to U.S. stock market returns only, which suggests that the long-memory pattern captured by the fractionally-integrated volatility model may not be a global stylized fact. We leave further investigation of this issue for future research.

The remainder of this paper is organized as follows. Section 2 provides details of the volatility models employed in the empirical analysis. Section 3 presents the measures used to evaluate the performance of the alternative models, followed in Section 4 by a presentation of the data used for the empirical analysis. Section 5 presents the empirical analysis and a discussion of the results. Finally, Section 6 presents the conclusions drawn from this study.

2. THE EMPIRICAL MODELS

According to Taylor (2005) the GARCH-extension long-memory model cannot be recommended, because the returns process has an infinite variance for all positive values of the fractional integration parameter, d. However, with regard to the EGARCH-extension model, the logarithm of

conditional variance is covariance stationary for $d<0.5$. Therefore, to have a fair comparison among models, the analysis in this study is based upon the EGARCH framework of Nelson (1991).

The EGARCH model of Nelson (1991), the FIEGARCH model of Bollerslev and Mikkelsen (1996), and the EGARCH-jump model modified from that of Maheu and McCurdy (2004), which respectively incorporate short-memory, long-memory, and jump dynamics in the volatility of asset returns, can all be nested in a model. By imposing constraints upon certain parameters, we are able to analyze the three different models within a nested framework.

The specifications of the variance and the distribution of asset returns are conditional on the information set I_{t-1}, which is comprised of previous returns $\{r_{t-i}, i \geq 1\}$. The returns process r_t is defined by:

$$
\begin{aligned}
& r_t = \mu_t + \varepsilon_{1,t} + \varepsilon_{2,t} \\
& \text{where } \varepsilon_{1,t} = \sqrt{h_{1,t}} z_t, \quad z_t \sim N(0,1) \\
& \varepsilon_{2,t} = J_t - E[J_t \mid I_{t-1}] = \sum_{k=1}^{n_t} Y_{t,k} - \theta \lambda_t, \quad Y_{t,k} \sim N(\theta, \delta^2).
\end{aligned}
\tag{1}
$$

Here, μ_t is the conditional mean, and $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are two stochastic innovations. The first component is a normally-distributed mean-zero innovation, while the second is specified as a compound Poisson process, which is the sum of n_t jumps. The arrival intensity is λ_t and jump size $Y_{t,k}$ is normally distributed with mean θ and volatility δ. The jump innovation $\varepsilon_{2,t}$, adjusted by $E[J_t \mid I_{t-1}] = \theta \lambda_t$, is also conditionally mean zero and contemporaneously independent of $\varepsilon_{1,t}$.[5]

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[5] Another alternative specification is to follow Duan et al. (2005). More specifically, $r_t = \mu_t + \sqrt{h_t}(z_{1,t} + z_{2,t})$ where $z_t \sim N(0,1)$ and $z_{2,t} = \sum_{k=1}^{n_t} Y_{t,k} - \theta \lambda_t$ with $Y_{t,k} \sim N(\theta, \delta^2)$.

To capture the time-varying jump effect upon volatility, the arrival intensity of jumps is assumed to follow a time-varying process:

$$\lambda_t = \lambda_0 + \rho\lambda_{t-1} + \gamma\xi_{t-1}. \tag{2}$$

The current jump intensity depends on both its previous value and the intensity residual. The intensity residual is defined as:

$$\xi_{t-1} \equiv E[n_{t-1} \mid I_{t-1}] - \lambda_{t-1} = \sum_{j=0}^{\infty} jP(n_{t-1} = j \mid I_{t-1}) - \lambda_{t-1}, \tag{3}$$

where $P(n_{t-1} = j \mid I_{t-1})$ is the ex-post inference on n_{t-1}, given the information set at time t-1, and $E[n_{t-1} \mid I_{t-1}]$ is the ex-post expected number of jumps occurring from t-2 to t-1. These will be defined later in this section, along with their likelihood function. A sufficient condition for λ_t to be positive for all t>1 is: $\lambda_0 > 0, \rho \geq \gamma$ and $\gamma \geq 0$.[6]

According to the specification in Equation (1), the conditional variance of returns is therefore the sum of two components:

$$h_t = Var(r_t \mid I_{t-1}) = Var(\varepsilon_{1,t} \mid I_{t-1}) + Var(\varepsilon_{2,t} \mid I_{t-1}). \tag{4}$$

The first component, $h_{1,t} = Var(\varepsilon_{1,t} \mid I_{t-1}),$ is specified as a generalized EGARCH process which incorporates short-memory, long-memory, and jump dynamics:

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[6] According to our numerical experiment, the infinity in the summation of equation (3) is proxied by 50 in our empirical implementation, which is large enough to make the summation converge. This proxy is also applied to the computation using equation (8).

$$\log(h_{1,t}) = \omega + (1 - \phi L)^{-1}(1 - L)^{-d}(1 + \psi L)g(z_{t-1}) \quad \text{and}$$

$$g(z_{t-1}) = (\alpha_a + \alpha_{a,j}E[n_{t-1} \mid I_{t-1}])z_{t-1} + (\alpha + \alpha_j E[n_{t-1} \mid I_{t-1}])(|z_{t-1}| - C), \qquad (5)$$

where L is a lag factor, $z_{t-1} = \varepsilon_{t-1} / \sqrt{h_{t-1}}$ with $\varepsilon_{t-1} = \varepsilon_{1,t-1} + \varepsilon_{2,t-1}$, and C is the conditional expected value of $|z_{t-1}|$.[7] In a similar fashion to the method proposed by Nelson (1991), this model allows for the asymmetric impact of good news and bad news, which is respectively controlled by α and α_a. In addition, as in Maheu and McCurdy (2004), we allow for the asymmetric feedback effects from past jumps, versus normal innovations, by adding α_j and $\alpha_{\alpha j}$.

Given the jump dynamics as specified above, the second component of conditional variance $h_{2,t}$ is equal to $(\theta^2 + \delta^2)\lambda_t$, which changes over time, since λ_t is time-variant. As the arrival of jumps follows a Poisson process, the conditional density of n_t is:

$$P(n_t = j \mid I_{t-1}) = \frac{\exp(-\lambda_t)\lambda_t^j}{j!}, \quad j = 0,1,2,\dots. \qquad (6)$$

Conditional on j jumps occurring at time t, the conditional density of returns is:

$$f(r_t \mid n_t = j, I_{t-1}) = \frac{1}{\sqrt{2\pi(h_{1,t} + j\delta^2)}} \exp\left(-\frac{(r_t - \mu_t + \theta\lambda_t - \theta j)^2}{2(h_{1,t} + j\delta^2)}\right). \qquad (7)$$

Therefore, the conditional density of returns for every observation is given as:

$$f(r_t \mid I_{t-1}) = \sum_{j=0}^{\infty} f(r_t \mid n_t = j, I_{t-1})P(n_t = j \mid I_{t-1}). \qquad (8)$$

Given the information set at time t, which was used for the computation in Equation (3), the

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[7] Following Bollerslev and Mikkelsen (1996), this study approximates $E(|z_t|)$ by the sample means of the absolute standardized residuals.

ex-post inference on n_t is then constructed as:

$$P(n_t = j \mid I_t) = \frac{f(r_t \mid n_t = j, I_{t-1}) P(n_t = j \mid I_{t-1})}{f(r_t \mid I_{t-1})}, \quad j = 0,1,2,\dots. \tag{9}$$

We utilize the maximum likelihood estimate (MLE) to estimate the parameters by maximizing the

log likelihood function $\sum_{t=1}^{T} \log(f(r_t \mid I_{t-1}))$.[8] Alternative sub-models are obtained as follows when

we impose constraints upon certain parameters.

The Short-Memory Model (EGARCH): $\lambda_t = d = 0$

The conditional variance is thus equal to:

$$\log(h_t) = \log(h_{1,t}) = \omega + \phi(\log(h_{1,t-1}) - \omega) + g(z_{t-1}) + \psi g(z_{t-2}). \tag{10}$$

The Long-Memory Model (FIEGARCH): $\lambda_t = 0$

The conditional variance is thus given as:

$$\log(h_t) = \log(h_{1,t}) = \omega + \sum_{i=1}^{N} b_i(\log(h_{1,t-i}) - \omega) + g(z_{t-1}) + \psi g(z_{t-2}), \tag{11}$$

where $b_1 = d + \phi$, $b_i = \alpha_i - \phi \alpha_{i-1}$ for $i \geq 2$ and $\alpha_1 = d$, $a_i = (i - d - 1)/i \times a_{i-1}$ for $i \geq 2$. Following

Bollerslev and Mikkelsen (1996) and Taylor (2005), we set $N = 1000$.[9]

The Model with Jump Dynamics (EGARCH-jump): $d = 0$

The two components of conditional variance are:

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[8] Following the advice of Maheu and McCurdy (2004), we set the starting value of λ_t as being equal to its unconditional value, $\lambda_1 = \lambda_0/(1 - \rho)$, and $\xi_1 = 0$ for the estimation of the parameters.

[9] In theory, $n = \infty$. However, in the empirical implementation, we need to truncate at a specific number, which is a trade-off between sufficient memory length and data availability.

$$\log(h_{1,t}) = \omega + \phi(\log(h_{1,t-1}) - \omega) + g(z_{t-1}) + \psi g(z_{t-2}) \text{ and } h_{2,t} = (\theta^2 + \delta^2)\lambda_t. \tag{12}$$

It is already well documented that the distributions of financial asset returns are usually skewed and leptokurtic. While adding a jump component in the residual of the return process or specifying a long memory for the conditional volatility process essentially makes the unconditional distribution of asset returns skewed and fat-tailed, directly modeling the stochastic innovation as a more flexible distribution also provides a possible solution. Hansen (1994) proposed a skewed-t distribution for the GARCH model, in which the stochastic innovation is characterized by a skewness parameter κ and a kurtosis parameter η. The conditional variance process of a skewed-t EGARCH model is the same as that of the normal short-memory EGARCH model. Therefore, we need to estimate only two additional parameters.

The Short-Memory Model with a Flexible Distribution (EGARCH-skewed-t)

The conditional density is given as:

$$f(z_t \mid I_{t-1}) = \begin{cases} bc[1 + \dfrac{1}{\eta-2}(\dfrac{bz_t + a}{1-\kappa})^2]^{-\frac{\eta+1}{2}} & \text{if } z_t < -a/b \\[2mm] bc[1 + \dfrac{1}{\eta-2}(\dfrac{bz_t + a}{1+\kappa})^2]^{-\frac{\eta+1}{2}} & \text{if } z_t \geq -a/b, \end{cases} \tag{13}$$

where $\alpha = 4\kappa c\dfrac{\eta-2}{\eta-1}$, $b = \sqrt{1 + 3\kappa^2 - a^2}$ and $c = \dfrac{\Gamma(\dfrac{\eta+1}{2})}{\sqrt{\pi(\eta-2)}\Gamma(\dfrac{\eta}{2})}$ with two constraints: $4 < \eta < \infty$ and

$-1 < \kappa < 1$.

3. MEASURES OF MODEL PERFORMANCE

In order to compare the performance of the various models, we analyze them from three different aspects: model fitting, volatility forecasting, and VaR prediction.

3.1 Model Fitting

Since not all models are nested, then with regard to model fitting we use both the conventional likelihood ratio test and the Akaike information criteria (AIC) to compare the performance of the alternative volatility models.

The likelihood ratio, determined by two distributions f_1 and f_2 with respective dimensions of parameters p_1 and p_2, is given as:

$$LR = 2(LLF_2 - LLF_1) \sim \chi^2_{p_2 - p_1} \quad \text{for } p_2 > p_1, \tag{14}$$

where LLF_1 and LLF_2 are the log likelihood functions, and the likelihood ratio follows an χ^2 distribution with a degree of freedom of $p_2 - p_1$.

The Akaike information criterion is defined as:

$$AIC = -2(LLF / T) + 2(k / T), \tag{15}$$

where k and T are the respective number of parameters and number of observations. The best model fitting is provided by the model with the smallest value of AIC.[10]

3.2 Volatility Forecasting

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[10] Since we use the first 1,000 observations to filter the first conditional variance for the FIEGARCH model, the parameters are estimated by maximizing the likelihood function for a sub-period which excludes the first 1,000 observations. This estimation procedure is applied to all of the models in order to ensure a consistent standard of comparison.

While employing the likelihood-based tests for the evaluation of the in-sample performance of the models, the out-of-sample performance is evaluated by analyzing the precision of the volatility forecasting.

Without intraday returns, the natural volatility target is 'next squared excess returns', $y_{t+1} = (r_{t+1} - \mu_{t+1})^2$.[11] As the long-term forecasts hardly have satisfactory precision and require simulated samples to calculate the expectations of $\exp[g(z_{t+i})]$, we focus on the one-step-ahead prediction in this study. The one-step-ahead realized variance is forecasted by the conditional variance, $h_{t+1} = h_{1,t+1} + h_{2,t+1}$, which naturally has a closed-form solution for the alternative conditional models.

For the out-of-sample evaluation of the volatility forecasting performance for the various models, we leave the last 100 observations as the out-of-sample period, recursively calculate volatility forecasting performance, and then compute their mean squared forecasting error (MSE), which is defined as:

$$MSE = \frac{1}{100} \sum_{i=1}^{100} (h_{t+1} - y_{t+1})^2 . \tag{16}$$

The model with the smallest MSE will be the best model for volatility forecasting.

3.3 Value-at Risk (VaR) Prediction

VaR prediction is a means to evaluate the model performance in tail-density fitting,[12] and it is generally defined as the excess loss, given a probability, over a certain period. Since the VaR

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[11] For high-frequency returns, μ_t is usually very close to zero and can be ignored by assuming $\mu_t = 0$.
[12] As the out-of-sample VaR prediction requires a simulated sample, we focus on the in-sample prediction in this study.

method has been widely used as the primary tool for managing market risk, it also seems of interest to analyze the performance of the different models in the prediction of the next period's VaR.

The VaR forecast, conditional on information which is available at time t, is defined as $VaR_{t+1|t} = F(\alpha)\sqrt{h_{t+1|t}}$ with $F(\alpha)$ being the distribution function assumed by the alternative models, such that $\Pr(r_{t+1} < VaR_{t+1|t}) = \alpha$.[13] Therefore, given the variance forecast $h_{t+1|t}$ and the distribution assumption, it is natural to obtain the percentile β_{t+1} at which the realized return r_{t+1} is positioned. We use the violation rate as the criterion for evaluating the VaR prediction performance of the various models, with the violation rate R being defined as:

$$R = \frac{1}{T-1}\sum_{t=1}^{T-1} V_{t+1}$$
$$V_{t+1} = \begin{cases} 1 & \text{if } \beta_{t+1} < \alpha \\ 0 & \text{if } \beta_{t+1} \geq \alpha \end{cases} \tag{17}$$

Here, T is the number of observations in each sample.

A likelihood ratio test (Stulz, 2003) can be employed to test for the null hypothesis that a model is able to provide correct information for the next period's VaR prediction. If the true probability of exceedance is α, then the probability of observing n exceedances out of N is given by the binomial distributions as $\alpha^n(1-\alpha)^{N-n}$.

The following likelihood ratio (LR) statistic asymptotically follows a chi-square distribution with one degree of freedom.

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[13] Since the conditional mean for high-frequency returns is usually equal to zero, $\Pr(\varepsilon_{t+1} < VaR_{t+1|t}) = \alpha$ is almost equivalent to $\Pr(r_{t+1} < VaR_{t+1|t}) = \alpha$.

$$LR = 2[\ln(R^n(1-R)^{N-n}) - \ln(\alpha^n(1-\alpha)^{N-n})]. \qquad (18)$$

If the test fails to reject the null hypotheses, then in terms of inferring the next period's VaR, the distribution assumption for the data should be regarded as fairly reasonable. However, in practice a preferred model for predicting VaR should have a violation rate which is no greater than the threshold. Therefore, not only should a preferred model for predicting VaR pass the above test, but it should also have a violation rate which is no greater than the threshold.

4. DATA

The empirical investigation in this study is carried out on the stock market indices of eight relatively large markets (the U.S., Japan, the UK, Germany, France, Canada, Italy, and Spain). We obtain the daily values of the stock market price indices for each country from Datastream.[14] The sample period is from July 1990 to June 2005 and excludes holidays. There are, on average, about 3,785 observations.

Table 1 shows the summary statistics of the returns, defined by changes in the logarithms of these indices.

<Table 1 is inserted here>

As suggested in many of the prior studies, most of the index return series are negatively skewed, leptokurtic, and non-normally distributed, and some have a high first-lag autocorrelation coefficient.

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[14] The Datastream price indices offer broad coverage of the markets, in terms of market capitalization of at least 75% to 80% for each market. According to Bartram et al. (2006), it does not matter whether price indices or return indices are used, since the time series of the daily dividends do not vary much.

The exception is Japanese stock index returns, which exhibit positive skewness. In order to capture

the potential influence of a higher first-order autocorrelation in some of the index returns, we

specify the conditional mean as a moving average MA(1) process for our empirical implementation.

The autocorrelations of the absolute returns, as illustrated in Figure 1, allow us to undertake a

preliminary check of the persistence pattern of return volatility for each index, and indeed it seems

that all stock return volatilities exhibit long memory. We use the modified rescaled range (R/S)

statistic, proposed by Lo (1991), to run a further test for the existence of long memory,[15] and find

that, for all index returns, the null hypothesis (of short memory in volatility) is rejected at the 1%

significance level.[16] However, it is clear that the autocorrelations for both the U.S. and Canada

decay more slowly and smoothly, as compared to those of the remaining countries that decay much

faster. Therefore, the long-memory model may achieve different degrees of success for the stock

returns of different countries.

<Figure 1 is inserted here>

5. EMPIRICAL RESULTS

5.1 Parameter Estimation and Model Fitting

We use the MLE method to estimate the parameters of all models for the stock index returns across

countries. Table 2 presents the results for the U.S. and Japanese stock market indices.[17]

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[15] The Appendix provides details of the long-memory test.
[16] The R/S statistics are U.S. (5.57); Japan (2.34); UK (4.72); France (4.35); Germany (5.23); Canada (5.40); Italy (3.53); and Spain (3.65).
[17] The patterns of estimates for all other indices are very similar. Their results are available upon request.

<Table 2 is inserted here>

For the short-memory EGARCH model, the parameter estimates exhibit two conventional stylized facts, volatility clustering and news impact asymmetry, since φ is close to 1 and α_a is significantly negative. The sign and significance of the moving average term ψ in the conditional variance process are not consistent across each of the countries.[18]

In line with the results of our earlier long-memory tests on absolute returns, each series of stock index returns fit a long-memory FIEGARCH model. Similar to the findings of Bollerslev and Mikkelsen (1996) and Taylor (2000), the fractional integration parameter d estimated from the time series data is usually higher than 0.5 and is significant. The only one exception is Japanese returns, where d is equal to 0.4674.

The estimated results of the EGARCH-jump model are generally consistent with those of Maheu and McCurdy (2004). The inclusion of jump dynamics in asset returns substantially increases the level of the likelihood function for all countries (44.66-91.66), although the signs and significance of the parameters controlling the impact of jumps on volatility, α_{aj} and α_j, are not consistent across countries. Furthermore, the inclusion of the impact of jumps in the conditional variance process lowers the impact from the previous residual, α_j and α, resulting in the symmetry of this impact being ambiguous.

We also confirm that for all country returns, the means of the jump sizes are negative, with most of them being significant. This implies a negative conditional correlation between jump

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[18] This term is usually ignored in the specification of a GARCH-type model.

innovations and squared return innovations; i.e., large negative returns cause an immediate increase in variance, which Maheu and McCurdy (2004) referred to as the 'contemporaneous leverage effect'. This leverage effect is time variant due to the time-varying correlation, which is determined by the time-varying jump arrival intensity. The importance, in volatility modeling, of this time-varying jump arrival intensity is highlighted by the strong significance of parameters, ρ and γ.

As opposed to improving the specification of the conditional variance, we directly specify the return innovation as a skewed-t distribution, which is close to the true distribution. The estimates show that, in addition to volatility clustering and news impact asymmetry, the distribution of the return innovation across countries is both negatively skewed ($\kappa<0$) and leptokurtic ($\eta<<\infty$), which is consistent with the stylized fact of equity returns. The likelihood function is also remarkably greater than that of the EGARCH model. Although the increases are lower than those of the EGARCH-jump model, the largest increase (for stock returns in Canada) is as high as 73.39.

In order to statistically test model fitting performance, we utilize the AIC and the LR ratio test based on the EGARCH model. Table 3 presents the statistics.

<Table 3 is inserted here>

The test results are perfectly consistent across all indices for the EGARCH-jump model, with the AIC being the smallest and the LR statistic being the highest. This indicates that the inclusion of jump dynamics in returns dramatically improves the model fitting. Interestingly, although volatility shows a long-memory pattern, the FIEGARCH model does not necessarily result in any significant improvement

in model fitting. As shown by either the LR ratio or the AIC, the improvement in model fitting in the FIEGARCH model is clearly seen for U.S. stock market returns only.[19] However, it should be noted that, with only two additional parameters, the EGARCH-skewed-*t* model also provides obvious improvements to model fitting. Since the computation load is considerably higher for the EGARCH-jump model, then if we need a less expensive model for model fitting, the EGARCH-skewed-*t* model could prove to be an ideal alternative.

5.2 Volatility Forecasting

Since forecasts over a long horizon are unlikely to be satisfactory and require simulated samples, we measure the volatility forecasting performance of the various models by comparing their mean squared errors of the one-day-ahead forecasts. The forecast improvement - defined as the difference between the mean squared error of the EGARCH model and that of the evaluated model (as a percentage of the mean squared error of the EGARCH model) - is also provided to indicate the extent to which the evaluated model improves the precision of volatility forecasting, based on the EGARCH model's forecasting. Table 4 offers the results for the stock market index returns across countries.

<Table 4 is inserted here>

The EGARCH-jump model and the FIEGARCH model show the best global performance. The

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[19] In theory, the FIEGARCH model is nested with the EGARCH model. However, when implementing a FIEGARCH model, we have to truncate the memory at a specific time point since it is impossible to calibrate with infinitely lagged information. As a result, these two models may no longer be nested.

EGARCH-jump model has the lowest volatility forecasting error for Germany, Canada, and Italy stock returns, whilst the FIEGARCH model outperforms all other models for U.S., Japan, UK, and Spain stock returns. On average, the performance of the FIEGARCH model is probably better than that of the EGARCH-jump model, since the performance of the latter is more volatile. In terms of individual countries, the most dramatic reduction in forecasting errors for U.S. stock market returns was provided by these two models, with an improvement in excess of 15%.

Although the EGARCH-skewed-*t* model demonstrates very good performance in terms of model fitting, it does not necessarily provide any improvement to volatility forecasting. This could be due to the instability of the true skewness and kurtosis of the stochastic innovations.

In summary, when forecasting volatility over a short horizon, such as 1 day, the model that performs better is the one which incorporates longer historical information. However, the jump factor also plays a crucial role in the prediction of volatility, but with a higher variation. Interestingly, these two types of models performed almost equally well for U.S. stock market returns.

5.3 VaR Prediction

The VaR prediction performance of the various models is initially examined using LR tests. We then compare the percentage of cases where violations are no greater than the corresponding significance levels of those cases where the null hypothesis (of the correct prediction of next-period VaR) is not rejected by the LR tests. This is essentially because, in practice, a preferred model for VaR prediction should have a violation rate which is no greater than the threshold. Table 5 reports the

violation rates and the probability values of the LR statistics at the 1% and 5% significance levels.[20]

<Table 5 is inserted here>

Almost all of the models pass the LR tests for all country stock market returns at the 5% significance level. Half of the test statistics for the FIEGARCH model and virtually all of the test statistics for the EGARCH model reject the null hypothesis of accurate prediction of VaR at the 1% significance level. Of the violation levels in the EGARCH and FIEGARCH models, most of those occurring at the 5% significance level are greater than 5%, although all of the violations passed the LR tests. This finding is generally consistent with the stylized fact of the fat-tailed distribution of index returns.

Both the EGARCH-jump model and the EGARCH-skewed-*t* model pass the LR tests for all countries and at all significance levels. Since the LR tests indicate that both the EGARCH-jump model and the EGARCH-skewed-*t* model are able to accurately predict VaR, it is necessary to further verify whether they are practically acceptable. We find that, for the EGARCH-jump model, 87.5% of all violation rates are lower than the corresponding significance levels, while for the EGARCH-skewed-*t* model, only 62.5% of the violation rates are lower than the corresponding significance levels. Although these two models perform equally well in the statistical tests, the EGARCH-jump model outperforms the EGARCH-skewed-*t* model when the practical application of the model is considered, given that the violation rate is required to be no larger than the threshold.

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[20] Given the large sample size for every time series, in the computation of the LR statistic for the 10% significance level, our study suffers from $\ln(a \rightarrow 0) = -\infty$. Therefore, the LR statistic $= 2(-\infty + \infty)$ is unavailable.

6. CONCLUSIONS

Irrespective of the ways in which the approaches to derivatives pricing and risk management are developed or modified, volatility will continue to play a key role. With the specifications of GARCH-type models becoming more realistic, there is a corresponding increase in the complexity of the models. Therefore, it is natural to question whether a more complicated model can actually perform any better on a global scale, in terms of different markets and purposes.

In this paper we have developed a generalized EGARCH model that incorporates short-memory, long-memory, and jump dynamics, as a means of evaluating the performance of alternative volatility models including the EGARCH, FIEGARCH, EGARCH-jump, and EGARCH-skewed-*t* models, in terms of model fitting, volatility forecasting, and VaR prediction.

As compared to the simple EGARCH model, the EGARCH-jump model demonstrates significant improvement, outperforming all other models in all aspects, with the single exception of volatility forecasting for some indices. However, the computation load substantially increases by the inclusion of jump dynamics. If less expensive volatility models are preferred and we can tolerate a performance which is slightly less satisfactory, then alternatives may include the use of the EGARCH-skewed-*t* model for model fitting and VaR prediction, and the FIEGARCH model for volatility forecasting, since these models also demonstrate fairly good performance for these specific purposes.

It is also worth noting that since the relative satisfactory performance of the FIEGARCH model

is demonstrated for U.S. stock market returns only, this suggests that the long-memory pattern captured by the fractionally-integrated volatility model may not be a globally stylized fact. Our results should provide important implications and guidance for the implementation of volatility models in both practical and academic fields.

REFERENCES

Andrews, D.W.K. (1991), 'Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation', *Econometrica*, **59**: 817-58.

Bartram, S., S.J. Taylor and Y.H. Wang (2006), 'The Euro and European Financial Market Dependence', *Journal of Banking and Finance*, Forthcoming.

Bates, D. (1996), 'Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Market Options', *Review of Financial Studies*, **9**: 69-107.

Black, F. and M. Scholes (1973), 'The Pricing of Options and Corporate Liabilities', *Journal of Political Economy*, **81**: 637-59.

Bollerslev, T. (1986), 'Generalized Autoregressive Conditional Heteroskedasticity', *Journal of Econometrics*, **31**: 307-27.

Bollerslev, T. and H. Mikkelsen (1996), 'Modeling and Pricing Long Memory in Stock Market Volatility', *Journal of Econometrics*, **73**: 151-84.

Duan, J.C. (1995), 'The GARCH Option Pricing Model', *Mathematical Finance*, **5**: 13-32.

Duan, J.C., P. Ritchken and Z. Sun (2005), 'Jump Starting GARCH: Pricing and Hedging Options with Jumps in Returns and Volatilities', *Working paper*, Toronto University.

Duan, J.C., P. Ritchken and Z. Sun (2006), 'Approximating GARCH-jump Models, Jump-diffusion Processes and Option Pricing.' *Mathematical Finance*, **16**: 21-52.

Duffie, D., J. Pan and K.J. Singleton (2000), 'Transform Analysis and Asset Pricing for Affine Jump-diffusions', *Econometrica*, **68**: 1343-76.

Engle, R.F. (1982), 'Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of UK Inflation', *Econometrica*, **50**: 987-1007.

Giraitis, L., P. Kokoszka, R. Leipus and G. Teyssiere (2003), 'Rescaled Variance and Related Tests for Long Memory in Volatility and Levels', *Journal of Econometrics*, **112**: 265-94.

Glosten, R.T., R. Jagannathan and D. Runkle (1993), 'On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks', *Journal of Finance*, **48**: 1779-801.

Hansen, B.E. (1994), 'Autoregressive Conditional Density Estimation', *International Economic Review*, **35**: 705-30.

Heston, S.L. (1993), 'A Closed-form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options', *Review of Financial Studies*, **6**: 327-43.

Lo, A.W. (1991), 'Long-term Memory in Stock Market Prices', *Econometrica*, **59**: 1279-313.

Maheu, J.M. and T.H. McCurdy (2004), 'News Arrival, Jump Dynamics and Volatility Components for Individual Stock Returns', *Journal of Finance*, **59**: 755-93.

Nelson, D.B. (1991), 'Conditional Heteroskedasticity in Asset Returns: A New Approach', *Econometrica*, **59**: 347-70.

Poon, S. and C.W.J. Granger (2003), 'Forecasting Financial Market Volatility: A Review', *Journal of Economic Literature*, **41**: 478-539

Stulz, R.M. (2003), *Risk Management and Derivatives*, Mason: South-Western.

Taylor, S.J. (1986), *Modelling Financial Time Series*, Chichester: Wiley and Sons.

Taylor, S.J. (2000), 'Consequences for Option Pricing of a Long Memory in Volatility', *Working paper*, Lancaster University.

Taylor, S.J. (2005), *Asset Price Dynamics, Volatility and Prediction*, Princeton and Oxford: Princeton University Press.
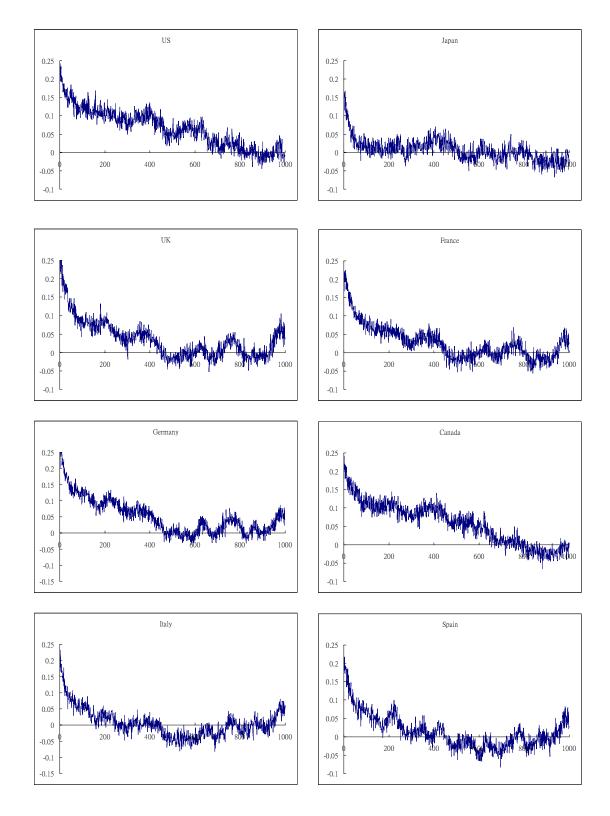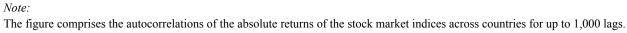
Figure 1: Autocorrelations of absolute returns

Table 1: Summary statistics

		U.S.	Japan	UK	France	Germany	Canada	Italy	Spain
Mean		0.0004	-0.0001	0.0004	0.0004	0.0002	0.0005	0.0003	0.0005
Max.		0.0537	0.0940	0.0543	0.0618	0.0555	0.0486	0.0692	0.0635
Min.		-0.0702	-0.0652	-0.0536	-0.0735	-0.0929	-0.0724	-0.0778	-0.0820
Std. Dev.		0.0103	0.0125	0.0095	0.0119	0.0116	0.0081	0.0130	0.0118
Skewness		-0.1174	0.1286	-0.1610	-0.1704	-0.4059	-0.6501	-0.1766	-0.2904
Kurtosis		6.9729	6.4026	6.2652	6.1110	7.3296	9.8360	5.6733	6.4823
Jarque-Bera		2484.06	1807.19	1701.35	1551.23	3703.90	7632.50	1149.43	1982.77
ACF	1	0.023	0.103	0.039	0.046	0.052	0.112	0.067	0.060
	2	-0.028	-0.055	-0.020	-0.016	-0.006	-0.006	0.023	-0.028
	3	-0.034	-0.026	-0.053	-0.045	-0.008	0.014	-0.014	-0.019
	4	0.007	0.005	0.033	0.014	0.031	-0.041	0.045	0.014
	5	-0.047	-0.022	-0.022	-0.037	-0.017	-0.013	-0.015	-0.013
Total No. of Obs.		3,764	3,730	3,793	3,801	3,802	3,783	3,794	3,818

Notes:
The table provides the summary statistics for the stock index returns across countries. The Jarque-Bera statistics are used for the normality test.

Table 2: Parameter estimates

	ω	φ	ψ	d	α_a	α_{aj}	α	α_j	λ_0	ρ	γ	$\theta(\eta)$	$\delta(\kappa)$	LLF
Panel A: U.S.														
EGARCH	-9.72	0.9818	-0.1757	–	-0.1326	–	0.1243	–	–	–	–	–	–	8936.63
	(-43.10)	(187.71)	(-0.96)		(-3.46)		(4.13)							
FIEGARCH	-9.94	0.9967	-0.1245	0.5336	-0.1561	–	0.1049	–	–	–	–	–	–	8958.79
	(-36.03)	(2386)	(-1.75)	(9.60)	(-5.80)		(5.32)							
EGARCH-Jump	-10.66	0.7148	0.0086	–	-0.1280	-0.0962	-0.1936	-0.0535	0.0389	0.9892	0.9391	-0.0011	0.0043	8984.37
	(-68.94)	(13.08)	(4.66)		(-3.48)	(-8.80)	(-2.28)	(-3.90)	(6.00)	(396.99)	(6.71)	(-6.58)	(17.93)	
EGARCH-Skewed t	-9.80	0.9833	-0.1264	–	-0.1241	–	0.1205	–	–	–	–	10.8815	-0.0776	8968.05
	(-46.85)	(242.73)	(-1.08)		(-5.13)		(6.64)					(4.36)	(-3.51)	
Panel B: Japan														
EGARCH	-9.24	0.9744	-0.0247	–	-0.0706	–	0.1721	–	–	–	–	–	–	8400.35
	(-64.91)	(131.71)	(-3.02)		(-4.37)		(8.16)							
FIEGARCH	-10.05	0.8537	-0.5140	0.4674	-0.0859	–	0.1876	–	–	–	–	–	–	8402.64
	(-8.58)	(4.91)	(-2.20)	(2.18)	(-4.21)		(5.30)							
EGARCH-Jump	-10.01	0.9823	0.0061	–	-0.1134	0.0473	0.0298	0.1293	0.0108	0.9406	0.2404	-0.0010	0.0140	8453.26
	(-51.75)	(237.68)	(3.44)		(-5.88)	(3.84)	(2.75)	(2.74)	(3.39)	(39.89)	(3.62)	(-1.64)	(9.59)	
EGARCH-Skewed t	-9.27	0.9803	-0.0844	–	-0.0762	–	0.1590	–	–	–	–	8.4622	-0.0112	8438.54
	(-60.88)	(165.11)	(-3.67)		(-5.06)		(7.20)					(6.48)	(-3.52)	

Note:

The panels present the parameter estimates, *t*-statistics, and log-likelihood functions of the alternative models for the stock market index returns across countries. The generalized model is specified as follows, with only the estimates of the variance parameters being reported:

$$r_t = \mu_t + \varepsilon_{1,t} + \varepsilon_{2,t}$$

$$\text{where } \varepsilon_{1,t} = \sqrt{h_{1,t}} z_t, \quad z_t \sim N(0,1)$$

$$\varepsilon_{2,t} = J_t - E[J_t \mid I_{t-1}] = \sum_{k=1}^{n_t} Y_{t,k} - \theta\lambda_t, \quad Y_{t,k} \sim N(\theta, \delta^2)$$

$$h_t = Var(r_t \mid I_{t-1}) = Var(\varepsilon_{1,t} \mid I_{t-1}) + Var(\varepsilon_{2,t} \mid I_{t-1}) = h_{1,t} + h_{2,t}$$

$$\text{where } \log(h_{1,t}) = \omega + (1-\phi L)^{-1}(1-L)^{-d}(1+\psi L)g(z_{t-1}) \text{ with}$$

$$g(z_{t-1}) = (\alpha_a + \alpha_{a,j} E[n_{t-1} \mid I_{t-1}])z_{t-1} + (\alpha + \alpha_j E[n_{t-1} \mid I_{t-1}])(|z_{t-1}| - C), \text{ and}$$

$$h_{2,t} = (\theta^2 + \delta^2)\lambda_t$$

Table 3: Tests for model fitting by model type

Tests for Model Fit	Model Type			
	EGARCH	FIEGARCH	EGARCH -Jump	EGARCH -Skewed *t*
Panel A: U.S.				
AIC	-6.4614	-6.4767	-6.4909	-6.4827
LR test based on EGARCH		44.32	95.48	62.84
Panel B: Japan				
AIC	-6.1490	-6.1499	-6.1826	-6.1755
LR test based on EGARCH		4.58	105.82	76.38
Panel C: UK				
AIC	-6.7300	-6.7305	-6.7629	-6.7438
LR test based on EGARCH		3.34	105.86	42.58
Panel D: France				
AIC	-6.2116	-6.2124	-6.2385	-6.2238
LR test based on EGARCH		4.08	89.32	38.20
Panel E: Germany				
AIC	-6.3047	-6.3053	-6. 3414	-6.3231
LR test based on EGARCH		3.56	116.78	55.58
Panel F: Canada				
AIC	-6.8889	-6.8937	-6.9497	-6.9402
LR test based on EGARCH		15.50	183.32	146.78
Panel G: Italy				
AIC	-6.1072	-6.1121	-6.1505	-6.1301
LR test based on EGARCH		15.68	134.94	68.04
Panel H: Spain				
AIC	-6.2555	-6.2563	-6.2982	-6.2852
LR test based on EGARCH		4.26	134.18	87.54

Notes:

The panels show the AIC and LR test statistics of the alternative models for the stock market index returns across countries. The LR test statistics are computed with respect to the log-likelihood functions of the EGARCH models.

Table 4:   Mean squared errors for volatility forecasting by model type

Volatility Forecast Period	Model Type						
	EGARCH	FIEGARCH		EGARCH-Jump		EGARCH-Skewed *t*	
Panel A: U.S. (1E-6)							
	0.0038	*0.0031*	*(18.42)*	0.0032	(16.05)	0.0037	(2.63)
Panel B: Japan (1E-6)							
	0.0209	*0.0206*	*(1.44)*	0.0213	(-1.91)	0.0208	(0.48)
Panel C: UK (1E-7)							
	0.0141	*0.0136*	*(3.55)*	0.0152	(-0.78)	0.0142	(-0.71)
Panel D: France (1E-7)							
	0.0466	0.0470	(-0.86)	0.0514	(-10.30)	*0.0462*	*(0.86)*
Panel E: Germany (1E-7)							
	0.0349	0.0341	(2.29)	*0.0320*	*(8.31)*	0.0345	(1.15)
Panel F: Canada (1E-7)							
	0.0222	0.0213	(4.05)	*0.0208*	*(6.31)*	0.0215	(3.15)
Panel G: Italy (1E-6)							
	0.0096	0.0098	(-2.08)	*0.0095*	*(1.04)*	0.0099	(-3.13)
Panel H: Spain (1E-7)							
	0.0533	*0.0520*	*(2.44)*	0.0576	(-8.07)	0.0529	(0.75)

Notes:

The table presents the mean squared errors of volatility forecasting for the stock market index returns across countries based upon the various models. The forecasting error is defined as the square of the difference between realized and forecasted volatility. Figures in parentheses refer to the percentage of forecast improvement based on the EGARCH model and are defined as the difference between the mean squared error of the EGARCH model and that of the evaluated model (as a percentage of the mean squared of the EGARCH model).

Table 5:   Tests for VaR prediction

Volatility Forecast Period	Model Type and Statistics							
	EGARCH		FIEGARCH		EGARCH -Jump		EGARCH -Skewed *t*	
	1%	5%	1%	5%	1%	5%	1%	5%
Panel A: U.S.								
Share of Total (%)	1.48	5.46	1.37	5.50	0.94	4.88	1.05	5.25
LR - *p* value	0.02	0.27	0.06	0.23	0.75	0.77	0.79	0.55
Panel B: Japan								
Share of Total (%)	1.32	4.69	1.32	4.58	0.84	5.05	0.88	4.84
LR - *p* value	0.11	0.45	0.11	0.30	0.58	0.87	0.51	0.69
Panel C: UK								
Share of Total (%)	1.36	5.55	1.32	5.73	0.86	4.62	0.75	5.16
LR - *p* value	0.07	0.19	0.10	0.08	0.44	0.34	0.16	0.70
Panel D: France								
Share of Total (%)	1.61	5.11	1.46	5.03	0.75	4.43	1.07	4.71
LR - *p* value	0.00	0.79	0.02	0.93	0.16	0.15	0.70	0.48
Panel E: Germany								
Share of Total (%)	1.36	5.42	1.36	5.42	0.96	4.93	0.89	4.89
LR - *p* value	0.07	0.30	0.07	0.30	0.84	0.85	0.55	0.78
Panel F: Canada								
Share of Total (%)	1.62	4.96	1.55	5.03	0.93	4.49	1.01	4.81
LR - *p* value	0.00	0.92	0.00	0.94	0.72	0.21	0.97	0.65
Panel G: Italy								
Share of Total (%)	1.43	5.51	1.18	5.37	1.00	5.37	0.93	5.33
LR - *p* value	0.03	0.22	0.34	0.37	0.99	0.37	0.70	0.42
Panel H: Spain								
Share of Total (%)	1.49	4.79	1.31	4.79	0.99	4.54	0.99	4.76
LR - *p* value	0.01	0.60	0.11	0.60	0.97	0.25	0.97	0.54

Notes:

The table presents the violation numbers, percentages, and the probability value of the LR test statistics under different significance levels for the various models for the stock market index returns across countries. If the true probability of exceedance is α, then the probability of observing n exceedances out of N is given by the binomial distributions, as: $\alpha^n (1-\alpha)^{N-n}$. The following LR statistic follows a chi-square distribution with one degree of freedom.

$$LR = 2[\ln(R^n(1-R)^{N-n}) - \ln(\alpha^n(1-\alpha)^{N-n})],$$

where R denotes the real violation rate.

Appendix

Let y_i be the absolute returns. The modified rescaled range statistic (R/S) proposed by Lo (1991) is:

$$Q(T,q) = \frac{1}{s_T(q)\sqrt{T}}[\max_{1 \le t \le T}(\sum_{i=1}^{t} y_i - \frac{t}{T}\sum_{i=1}^{T} y_i) - \min_{1 \le t \le T}(\sum_{i=1}^{t} y_i - \frac{t}{T}\sum_{i=1}^{T} y_i)],$$

where $s_T^2(q) = \sum_{j=-q}^{q} w_q(j)\hat{\gamma}(j)$, $w_q(j)$ is determined by the Bartlett kernel, q is the

bandwidth, and $\hat{\gamma}(j) = \frac{1}{T}\sum_{t=j+1}^{T}(y_t - \bar{y})(y_{t-j} - \bar{y})$ is the auto-covariance estimator.

Giraitis et al. (2003) extended the R/S statistic to test for long memory in volatility. Under the short-memory null hypothesis, Giraitis et al. (2003) demonstrated that:

$$Q(T,q) \overset{d}{\longrightarrow} range_{0 \le \tau \le 1} B^0(\tau),$$

where $range(f) = \max(f) - \min(f)$ and $B^0(\tau)$ is a standard Brownian bridge.

In this paper the R/S statistics are computed with $q = q^*$, where q^* is chosen by the data-dependent formula of Andrews (1991).