A Dynamic Optimal Capital Structure Model with Costly Adjustment Mechanisms---A Real Option Perspective

Hsien-Hsing Liao∗  Yi-Hsuan Tung∗∗  Tsung-Kang Chen∗∗∗

Abstract

This paper develops a dynamic capital structure model with costly adjustment mechanisms. Based on an exogenous cash flow process, the model can endogenously determine the firm value and the claim value of a firm’s derivative security under optimal debt level. It considers costs of adjustment in capital structure, including a fixed and a proportional cost elements. The adjustment costs act as the recapitalization threshold and each of them has significant effects on the adjustment frequency and magnitude in capital structure. The numerical results of simulation analysis of the model are consistent with those expected in literature and intuition.

Key words: Optimal capital structure, Adjustment costs

∗ Corresponding Author.  Associate Professor, Department of Finance, National Taiwan University, Email: hliao@ntu.edu.tw, Phone/Fax:(886) 02-2363-8897, Address: Rm. 814, Building #2, College of Management, National Taiwan University, 85, Sec. 4 Roosevelt Road, Taipei 106, Taiwan
∗∗ Department of Finance, National Taiwan University, Email: r95723021@ntu.edu.tw
∗∗∗ Department of Finance, National Taiwan University, Email: r91723010@ntu.edu.tw
I. Introduction

The traditional view of optimal capital structure reflects tradeoffs between the benefits and costs associated with varying degrees of financial leverage. The main benefit of debt is the tax shield (Modigliani and Miller, 1963). The primary costs are those associated with financial distress and the personal tax expense bondholders incur when they receive interest income (Miller, 1977). However, there are two other important lines of capital structure theories, the pecking order theory and the Market timing theory. Nonetheless, a survey on chief financial officers by Graham and Harvey (2001) reports that over 80% of the respondents employ varying degrees of target debt ratios in their practice.\(^1\) Within the tradeoff theory framework, Goldstein, Ju and Leland (2001) develop an EBIT-based dynamic capital structure model. However, it does not consider the costs incurred when firms conduct adjustment activities in capital structure. In this study, we extend Goldstein, Ju and Leland (2001) to a cash flow-based dynamic capital structure model with costly adjustment mechanisms suggested by Leary and Roberts (2005), including a fixed and a proportional cost elements in capital structure adjustments.

There are substantial debates on the idea of a target debt ratio. Fama and French (2002) suggest that firms seem slowly to rebalance their leverage to their long-run mean or optimal level. Baker and Wurgler (2002) document that current capital structure is strongly related to historical market values and firm times equity issuances with high market valuations and has persistent

\(^1\) The survey reports that 37% of their respondents have a flexible target debt ratio, 34% have a somewhat tight target or range and 10% have a strict target. They also show that managers are concerned with the costs and benefits of debt financing, such as credit ratings and cash flow volatility, and they also regard tax shields as “important” or “very important” to almost half of those CFOs surveyed.
effects on capital structure. They conclude that capital structures are the cumulative outcome of historical market timing efforts rather than the results of a dynamic optimizing strategy. Welch (2004) observes that stock price changes have a strong effect on market leverage ratios. Other literatures have disparate estimated results, such as Flannery and Rangan (2005) who find that firms do target a long run capital structure and that a typical firm converges toward its long-run target at a rate of more than 30% per year. Kayhan and Titman (2007) indicate that after controlling for the changes in stock prices and other timing and pecking order effects, changes in debt ratios are still partially explained by movements towards a target debt ratio.

In literature, most empirical tests do not consider the adjustment costs of rebalancing the capital structure, that is, they implicitly assume that moving to the target debt ratio is costless so firms can continuously adjust their leverage toward an optimal level in the absence of adjustment costs. In reality, however, there are such costs so that the firms do not respond immediately to capital structure shocks. When the costs of adjustments outweigh the benefits, firms will wait to recapitalize, resulting in the speed at which leverage reverts to its target is often characterized as slow. Therefore, the periods of financing inactivity, induced by the presence of adjustment costs, have several implications for the dynamic behavior of capital structures and empirical studies seeking to understand corporate financial policy.

There is a growing theoretical literature that considers adjustment costs to explain the relatively slow movement towards optimal leverage, such as Fischer, Heinkel and Zechner.
(1989a). Their paper develops a model of dynamic capital structure choice in the presence of recapitalization costs. They find that even small recapitalization costs lead to wide swings in a firm's debt ratio over time. Leary and Roberts (2005) show that the presence of adjustment costs has significant implications for corporate financial policy. They reexamine the conclusions of Baker and Wurgler (2002) and find that the persistence in capital structure revealed by their empirical examinations is more likely due to adjustment costs. They also find that the effect of Baker and Wurgler's key market timing variable on leverage attenuates significantly as adjustment costs decline, proving that adjustment costs appear to dictate the speed at which firms respond to leverage shocks. Furthermore, in Leary and Roberts (2005), the adjustment costs can be separated into two components: the fixed cost and the proportional cost, which relate to the size of adjustments. If the adjustment cost includes only fixed cost, a firm would make one large adjustment when the leverage reaches the boundary, that is, the firm issues or retires debt to return leverage to its optimal level as those stated in Fischer, Heinkel and Zechner (1989a). The intuition for this policy is that once the benefits from adjustment outweigh the costs, the firm will make as complete an adjustment as possible because the cost and size of the adjustment are independent. But if adjustment cost only contains a proportional one, the frequency to revert the capital structure would increases, and the degree of the adjustment is somewhere acceptable. Under this policy, the firm makes the size of adjustment as small as possible because of cost-minimization. In reality, the adjustment costs should consist of both fixed and
proportional ones, so the adjustment for this cost function results in an optimal leverage range.

However, these studies do not consider that a firm’s optimal capital structure may change over time due to economic shocks and it may move away from their initial (optimal) leverage. If the optimal leverage changes over time, the estimates of the adjustment speed toward the target could be biased. The optimal capital structure is dynamic because it is variable instead of constant and is influenced by economic changes and corporate actions. There are several theoretical dynamic capital structure models have been developed, such as Fischer, Heinkel and Zechner (1989a), Goldstein, Ju, and Leland (2001), and Titman and Tsyplakov (2007), which include an optimal leverage determined by the various trade-offs between the costs and benefits of leverage level, changing over time, and depending on firm value. Titman and Tsyplakov (2007) consider a continuous time model of a firm that can dynamically adjust both its capital structure and its investment choices. It assumes a firm’s product unit market price follows a lognormal process which is not general enough for most firms because a firm’s product price is not a random process and does not change randomly over time.

In this study, by incorporating the EBIT-based model by Goldstein, Ju and Leland (2001) with costly adjustment mechanisms, we develop a dynamic capital structure model that allows us to determine the value of a firm, value of debt, value of bankruptcy, optimal coupon and leverage endogenously, and to quantify the benefits and costs associated with movements both towards and away from optimal capital structure. In addition, we assume that the adjustment costs
include both fixed and proportional costs when a firm dynamically rebalancing its capital structure. A firm chooses to rebalance its capital structure only if net benefit exceeds adjustment cost which act as the recapitalization threshold. Besides, the firm would adjust to a leverage level which maximizes the net benefit of firm, instead of adjusting to the theoretical optimal leverage level. The firm can issue or repurchase the debt and equity to do adjustments, and different ways of adjustments may incur different adjustment costs. In addition, we assume that the closer to the optimal level the current leverage is, the larger the adjustment cost is. With our model we examine the time-series as well as the cross-sectional variations in dynamic optimal capital structure.

The rest of the article proceeds as follows. Section II develops the one-time cross-section variation of theoretical capital structure model. In Section III, we determine optimal dynamic capital structure and rebalancing strategy with costly adjustments. In Section IV, we present the numerical results of simulation analysis of our model, and last Section we conclude this paper.

II. Description of the model

The model developed in this paper endogenously determines the firm value and the claim of firm value’s derivative security, which follows the EBIT-based model of dynamic capital structure (Goldstein, Ju and Leland (2001)).

Here we consider a simple model of firm value dynamics. A firm’s cash flow is specified by the stochastic process as in (1),
\[
\frac{d\delta}{\delta} = \mu_p dt + \sigma dZ^p
\]  

where the growth rate, \( \mu_p \), and the volatility of cash flow, \( \sigma \), are constants.

Both the risk premium, \( \theta \), and the risk-free rate, \( r \), are constants. It is well known that any asset of the firm can be priced by discounting its expected cash flows under the risk-neutral measure. In particular, the value of the claim to the entire cash flow, is

\[
V(t) = E_t^Q \left( \int_0^\infty e^{-\rho s} \delta_s ds \right) = \frac{\delta_t}{r - \mu}
\]  

where \( \mu = \mu_p - \theta \sigma \) is the risk-neutral drift of the cash flow rate:

\[
\frac{d\delta}{\delta} = \mu dt + \sigma dZ^0
\]  

Here, we refer to \( V \) as the value of the firm. Since \( r \) and \( \mu \) are constants, both \( V \) and \( \delta \) share the same dynamics:

\[
\frac{dV}{V} = \mu dt + \sigma dZ^0
\]  

It follows that

\[
\frac{dV + \delta dt}{V} = r dt + \sigma dZ^0
\]  

Thus, the total risk-neutral expected return on the claim is the risk-free rate. The existence of an equivalent martingale measure guarantees that every contingent claim of this firm will receive fair expected return for the risk.

We assume a simple tax structure that includes personal as well as corporate taxes. Interest payments to the investors are taxed at a personal rate \( \tau_i \), effective dividends are taxed at \( \tau_d \), and corporate profits are taxed at \( \tau_c \), with full loss offset provisions.
Considering a debtless firm with current firm value $V_0$, both equity and government have a claim to the firm’s cash flow. Assume that the current management refuses to take on any debt and that no takeover is likely. Then, the current firm value is divided between equity and government as

$$E = (1 - \tau_{\text{eff}}) V_0$$  \hspace{1cm} (6)$$

$$G = \tau_{\text{eff}} V_0$$  \hspace{1cm} (7)$$

where the effective tax rate is \((1 - \tau_{\text{eff}}) = (1 - \tau_d)(1 - \tau_c)\).

Now, consider an otherwise identical firm whose management decides to choose a static leverage level that will maximize the wealth of current equity holders. It will issue a perpetual bond, promising a constant coupon payment $C$ to debt holders as long as the firm remains solvent. Due to the issuance of perpetuity, the threshold at which the firm chooses to default is time-independent. We define this threshold as $V_b$. If the firm value reaches $V_b$, then an amount $\alpha V_b$ will be lost due to bankruptcy costs.

In general, any claim of firm’s derivative security must satisfy the partial differential equation (PDE) as follows:

$$\mu VF_r + \frac{\sigma^2}{2} V^2 F_{rr} + F_r + P = rF$$  \hspace{1cm} (8)$$

where $P$ is the cash flow. Because of the issuance of perpetual debt, all claims will be time-independent. Thus the PDE reduces to an ordinary differential equation:

$$\mu VF_r + \frac{\sigma^2}{2} V^2 F_{rr} + P - rF = 0$$  \hspace{1cm} (9)$$
The solution is

\[ F = A_0 + A_1 V^{-y} + A_2 V^{-x} \]  

(10)

where

\[
\begin{align*}
  x &= \frac{1}{\sigma^2} \left( \mu - \frac{\sigma^2}{2} \right) + \sqrt{\left( \mu - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} > 0 \\
  y &= \frac{1}{\sigma^2} \left( \mu - \frac{\sigma^2}{2} \right) - \sqrt{\left( \mu - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} < 0
\end{align*}
\]

and \( A_0, A_1, \) and \( A_2 \) are constants, determined by boundary conditions. Note that \( x \) is positive, while \( y \) is negative, so the first term explodes as \( V \) becomes large. Hence, in this section, \( A_1 \) equals zero for all claims of interest.

Before proceeding, it is convenient to define \( p_B(V) \) as the present value of a claim that pays \$1 contingent on firm value reaching \( V_B \). Since such a claim receives no intermediate cash flows, we know from equation (10) that \( p_B(V) \) will be of the form

\[ p_B(V) = 0 + A_1 V^{-y} + A_2 V^{-x} \]  

(11)

According to the boundary conditions

\[ \lim_{V \to \infty} p_B(V) = 0, \quad \lim_{V \to V_B} p_B(V) = 1 \]

given

\[ P_B(V) = \left( \frac{V}{V_B} \right)^{-x} \]  

(12)

While the firm is solvent, equity, government, and debt share the cash, \( \delta \), through dividends, taxes, and coupon payments, respectively. We define the value of this claim as \( V_{solv}(V) \) as long
as firm value remained above $V_B$. From equations (10), we know the form of $V_{\text{solv}}(V)$ is

$$V_{\text{solv}} = V + A_1 V^{-y} + A_2 V^{-x} \quad (13)$$

Taking into account the boundary conditions:

$$\lim_{V \to \infty} V_{\text{solv}} (V) = V, \quad \lim_{V \to V_B} V_{\text{solv}} (V) = 0$$

we find that

$$V_{\text{solv}} = V - V_B P_B (V) \quad (14)$$

The value of the claim to interest payments during continued operations, $V_{\text{int}}$, can be obtained in a similar form. As long as the firm remains solvent, the coupon payment is $C$ before the recapitalization. Thus, $V_{\text{int}}$ is of the form

$$V_{\text{int}} = \frac{C}{r} + A_1 V^{-y} + A_2 V^{-x} \quad (15)$$

Again, the boundary conditions are

$$\lim_{V \to \infty} V_{\text{int}} = \frac{C}{r}, \quad \lim_{V \to V_B} V_{\text{int}} = 0$$

Then we obtain

$$V_{\text{int}} = \frac{C}{r} \left[1 - P_B (V)\right] \quad (16)$$

The present value of the default claim $V_{\text{def}}(V)$ can be written as

$$V_{\text{def}} = 0 + A_1 V^{-y} + A_2 V^{-x} \quad (17)$$

with the boundary condition:

$$\lim_{V \to \infty} V_{\text{def}} = 0, \quad \lim_{V \to V_B} V_{\text{def}} = V$$

we can get the present value of the default claim as
When the cash level falls below promised interest payments, equity has the right to raise funds to avoid bankruptcy. However, for a sufficiently poor state of the firm, shareholders will optimally choose to default. At this point, the remaining firm value is divided up between debt, government, and bankruptcy costs.

Note that the sum of the present value of the claim to funds during solvency (eq. [14]) and the present value of the claim to funds in bankruptcy (eq. [18]) is equal to the present value of firm, \( V \). Hence, value is neither created nor destroyed by changes in the capital structure; rather, it is only redistributed among the claimants. This invariance result is consistent with the “pie” model of Modigliani and Miller (1958), except that in this framework the claims of government and bankruptcy costs are also part of that pie.

Separating the value of continuing operations and default claim between equity, debt, government, and bankruptcy costs as follows:

\[
E(V) = (1 - \tau_{eff})(V_{solv} - V_{int}) \tag{19}
\]

\[
D(V) = (1 - \tau) V_{int} + (1 - \alpha)(1 - \tau_{eff}) V_{def} \tag{20}
\]

\[
G(V) = \tau_{eff} (V_{solv} - V_{int}) + \tau V_{int} + (1 - \alpha) \tau_{eff} V_{def} \tag{21}
\]

\[
BC(V) = \alpha V_{def} \tag{22}
\]

A. Default Level

When the firm has the financial distress, which means the cash is less the promised coupon
payment to the debtholders. In poor situation, the firm goes into bankruptcy, comparing with
the debtholders get the residual firm value and the equityholders receive nothing. We can
obtained the bankruptcy level $V_B$ by invoking the smoothing-pasting condition (see more details
in Appendix A)

\[
\frac{\partial E}{\partial V}_{\gamma=v_B} = 0
\]  

(23)

Solving, we find

\[
V_B(C) = \lambda \frac{C}{r}
\]  

(24)

where

\[
\lambda = \frac{x}{x+1}
\]  

(25)

Observe that the firm value, $V_B$, at which bankruptcy occurs

a) is proportional to the coupon, $C$;

b) is independent of the current firm value, $V$;

c) decreases as the risk-free interest rate, $r$, rises.

B. Optimal Coupon level

The objective of management is to maximize shareholder wealth. That is, current equity
holders receive fair value for the debt claim sold. We assume the manager chooses the optimal
coupon level that maximizes the value of asset (see more details in Appendix B)

\[
\frac{\partial \nu}{\partial C}_{\gamma=v_0} = 0
\]  

(26)

where
\[ v(V) = D(V) + E(V) \]

By differentiating equation (26) with respect to \( C \) and setting the equation to zero, we find that the optimal coupon level chosen when current firm value is \( V_0 \), is

\[
C^* = V_0 \left( \frac{r}{\lambda} \right) \left( \frac{1}{1 + x} \left( \frac{A}{A + B} \right) \right)^{\frac{1}{2}}
\]

(27)

where

\[
A = \tau_{eff} - \tau_i
\]

\[
B = \lambda \alpha (1 - \tau_{eff})
\]

Hence, we can determine the optimal coupon level with firm value over time. This study put its emphasis on optimal coupon level dynamic, or optimal leverage dynamics, which has been less discussed in literature. The optimal coupon level relates to the firm value, which base on the cash flow of the firm; the cash flow of the firm is directly related to its operating performance, which is mainly affected by both corporate management policies and industrial cycle. When the industrial cyclical effect changes, not only the firm value would change, but also the index, \( x \), would change, then the optimal coupon level changes.

**C. Adjustment cost function**

There are two directions of rebalancing the capital structure: raise the leverage or lower the leverage. In this paper, we define that leverage ratio is debt value divide by asset value. If the leverage moves away from the optimal leverage, the firm has several forms of adjustments: debt issuance and equity repurchase for the leverage lower than optimal level; equity issuance and debt
repurchase for the leverage higher than optimal level.

Besides, we consider that the adjustment cost of rebalancing the capital structure includes two parts, the fixed cost and the proportional cost. The fixed cost is relative to size of current asset; the proportional cost is relative to the amount of issuance or repurchase; furthermore, the cost may change in different current leverage level, when the current leverage is closer to the optimal level, the adjustment of proportional cost would be larger. The adjustment cost function \( AC \) as follows:

\[
AC = (\varepsilon_0 D_0 + \omega_0 E_0) + [\varepsilon_1 (D_1 - D_0) + \omega_1 (E_1 - E_0)](\frac{L_0 - L}{L_0 - L})^k
\]

(28)

where

\[
L_0 = \frac{D_0}{V_0}, L_1 = \frac{D_1}{V_1}, L' = \frac{D'}{V'}
\]

and \( \varepsilon_0, \omega_0, \varepsilon_1, \omega_1, k \) are coefficient of adjustment cost, which depend on firm specific effect and industrial cyclical. Here, \( \varepsilon_F \) and \( \omega_F \) are coefficient of fixed cost of debt and equity; \( \varepsilon_P \) and \( \omega_P \) are coefficient of proportional cost of debt and equity, which are relative to the form of adjustment, for example, issuing the debt and repurchase the debt may have different \( \varepsilon_P \) and \( \varepsilon_F \). \( D_0 \) and \( E_0 \) are the valuation of debt and equity before adjustment; \( D_1 \) and \( E_1 \) are the valuation of debt and equity after adjustment. The formulas are as following expressions:

\[
D_1 = D[V_0, C_1, V^g(C_1)] = (1 - \tau^g) \frac{C_1}{r}[1 - P^g(V_0, C_1)] + (1 - \alpha)(1 - \tau^g)P^g(V_0, C_1)
\]

\[
E_1 = E[V_0, C_1, V^g(C_1)] = (1 - \tau^g)[V_0 - V^g(C_1)P^g(V_0, C_1) - \frac{C_1}{r}(1 - P^g(V_0, C_1))]
\]

\[
D_0 = D[V_0, C_0, V^g(C_0)] = (1 - \tau^g) \frac{C_0}{r}[1 - P^g(V_0, C_0)] + (1 - \alpha)(1 - \tau^g)P^g(V_0, C_0)
\]

\[
E_0 = E[V_0, C_0, V^g(C_0)] = (1 - \tau^g)[V_0 - V^g(C_0)P^g(V_0, C_0) - \frac{C_0}{r}(1 - P^g(V_0, C_0))]
\]
Lastly, we mention the concern that if the current leverage is closer to the optimal level, the adjustment of proportional cost would be larger, so we express this concern as the distance between the current leverage and the leverage after adjustment divides by the distance between the current leverage and the optimal leverage. Then, the expression to the power of $k$, the adjustment exponent, which stress the importance of this concern; the larger $k$ means the concern is insignificance.

### III. Dynamic Capital Structure Strategy

We determine the optimal capital structure strategy for a firm that has the option to adjust its leverage level. As in Leary and Roberts (2005), we find that there exists the optimal leverage range where the firm will maintain its debt level. Due to the existence of adjustment cost, the firm does not induce a response as the leverage ratio deviate from the optimal level slightly. The firm will recapitalize only if the benefit outweighs the cost of adjustments.

Actually, our objective is to find out the coupon level, $C_1$, which maximizes the firm benefit after recapitalization minus adjustment cost.

$$\text{Max}_{C_1} \{(v_1 - v_0) - AC, \ 0 \}$$

(29)

where $v_1$ is the asset value after adjustment and $v_0$ is the asset value before adjustment, as follows:

$$v_1 = D_i + E_i = D[V_0, C_i, V_g(C_i)] + E[V_0, C_i, V_g(C_i)]$$

$$v_0 = D_0 + E_0 = D[V_0, C_0, V_g(C_0)] + E[V_0, C_0, V_g(C_0)]$$
We develop the model as follows. First, we determine the value of all claims at period 0. We then calculate the theoretical optimal coupon level which maximizes the asset value endogenously, as equation (27). We also point out the default threshold, and bankruptcy cost. As time pass, the value of the claims would change, result in the leverage deviate from the optimal capital structure.

There are two situations that we choose to recapitalize:

1. When the firm value touch to the default value.
2. When the benefit of recapitalization is larger than cost.

In first situation, which is on the vary brink of bankruptcy, we force the firm recapitalized to the optimal level because both equity and debt holders want to avoid bankruptcy occurrence, which consistent with Leland and Toft (1996).

At each subsequent time period, we determine whether the firm value, $V$, is larger than the bankruptcy level, $V_B$, at each period. If $V < V_B$, then calculate the optimal coupon level which maximizes the shareholder wealth, and adjust the size of coupon to the optimal coupon level. Or else, $V > V_B$, comparing with the current coupon level, we have an option, after considering the adjustment cost and incremental firm benefit, to determine if rebalancing the capital structure and whether should the firm adjust to. According to equation (28), we can calculate the adjustment cost of recapitalization at different coupon level and find the maximum of benefit minus cost, as equation (29). In other words, we determine that the coupon level at each period,
then we can get the value of debt and equity; also, we know the leverage ratio after adjustment.

IV. Numerical Results

The base case parameter values that we use in our numerical analysis are displayed in Table 1; most parameters are similar to Goldstein, Ju and Leland (2001). To test and verify if our model is useful for the firm to increase the asset value, we simulate 1000 random paths for the cash flow ten years quarterly, which randomly generates the information embedded in the various periods. The results are showed in Figure 1, which illustrated that the firm do capitalization would increase the asset value over time. Otherwise, we focus on how parameter values affect the firm’s optimal capital structure and adjustment behavior. The following are main parameters we discussed: the volatility of cash flow, risk-free rate, the bankruptcy cost, the structure of adjustment cost.

[Insert Table 1 here]

[Insert Figure 2 here]

Comparing with different volatility of cash flow, we observe that the larger volatility of cash flow result in lower leverage ratio. It is not surprising because the firm with stable cash flow has higher debt capacity. Besides, we find that the firm under different volatility of cash flow has different frequency of adjustment; due to the stable cash flow, the firm with small volatility of cash flow will do recapitalization more frequently. The comparative results are displayed in Figure 2 to Figure 5.
Then, we consider the effect of the risk-free rate on capital structure. According to the equation (27), we know that the optimal coupon level is a proportion of risk-free rate, and is also relative to the firm value, which is an inverse proportion of the risk-free rate minus the growth rate, as equation (2). The two forces cancel out most of the risk-free rate effect on capital structure, so the optimal leverage just up slightly under high risk-free rate environment, as in Figure 6. And the frequency of adjustment is increasing and the size of adjustment coupon is decreasing as the risk-free rate increasing, but both are not significant. The detail results are displayed in Figure 7 to Figure 9.

Furthermore, we discuss the bankruptcy cost of the firm, and set three different scenarios of bankruptcy cost illustrated in Figure 10. The good firm with lower bankruptcy cost has higher optimal level; and the bad firm with higher bankruptcy cost has lower optimal level. Besides, the good firm with lower bankruptcy cost recapitalizes frequently and has larger size of adjustment coupon, because it has higher debt capacity, which can enjoy the benefit of higher debt ratio. The results are showed in Figure 11 to Figure 13.

Finally, the simulation results of our adjustment cost function are consistent with our assumption, based on Leary and Roberts (2005), both the fixed cost and the proportional cost are
significant. In Figure 14, it shows that the frequency of adjustment is large when there is no fixed cost; the speed of adjustment goes down with fixed cost increasing. On the other hand, it also illustrated that the proportional cost has an effect on the frequency of adjustment in Figure 15. Comparing with the size and frequency of recapitalization in Figure 16 and 17, we find that the firm recapitalization is directed by the type of adjustment cost, so that the fixed cost results in the size of adjustment coupon large at once and return the leverage not so frequently; the proportional cost results in the firm do recapitalization all the time when the leverage deviates the optimal level, and the size of adjustment follows the degree of deviation. Eventually, we verify the speed of adjustment would sharply down when there exists large fixed cost or proportional cost, as in Figure 18 and 19.

[V. Conclusion]

We modify a EBIT-based Model proposed by Goldstein, Ju, and Leland(2001), and develop dynamic capital structure model with costly adjustment mechanisms. We consider costs of capital adjustments, including a fixed cost element and a proportional one, which have significant effect on the frequency and size of adjustments in capital structure by a firm. In addition, our model is able to consider multiple types of adjustments, such as issuing and repurchasing either debt or equity. Based on an exogenous cash flow process, the model can endogenously determine the firm value and the claim value of firm’s derivative security under optimal debt level.
A firm can alter the variables of the model based on industry and firm specific properties to determine if doing a recapitalization at any point in time. Since the objective of the management is to maximize shareholder’s wealth, a firm would recapitalize only if the benefits outweigh the adjustment costs. Finally, the numerical results of simulation analysis of our model are consistent with those expected in literature and intuition.
Reference


Figure 1  Simulated leverage dynamics. The asset value under different leverage levels and periods displayed in upper and left figures. And right figure showed the firm keep the leverage (black line) close to optimal leverage (gray line) after dynamic adjustment; the original leverage without recapitalization (dotted line) deviate from the optimal leverage over time.
Figure 2  The leverage under different volatility of cash flow. The vertical dotted line represented the recapitalization at that time node. The firm with larger the volatility has the lower leverage and frequency of adjustment.

Figure 3  The dynamics leverage and the size of adjustment coupon. \((\sigma=0.05)\) The gap between the original leverage and the leverage after adjustment is significant. The firm chooses to recapitalize more frequency because it has stable cash flow and higher debt capacity.
Figure 4  The dynamics leverage and the size of adjustment coupon. \((\sigma=0.10)\) It is showed that the leverage becomes lower and the size of adjustment coupon becomes larger when the volatility of cash flow increases.

Figure 5  The dynamics leverage and the size of adjustment coupon. \((\sigma=0.15)\) Under large volatility of cash flow, the firm becomes conservative and tends to not recapitalize too frequently. But if it is necessary, the firm still chooses to adjust its leverage, results in large size of adjustment coupon.
Figure 6  The leverage under different risk-free rate environment. The vertical dotted line represented the recapitalization at that time node. Under the high risk-free rate environment, the firm has the higher leverage and adjustment more frequently. But under our framework, the effect of risk-free rate on capital structure is not significant.

Figure 7  The dynamics leverage and the size of adjustment coupon. (r=6.5%) The firm under the high risk-free rate environment has low firm value, which result in low optimal coupon level, but the optimal coupon level also related directly to risk-free rate, so two forces cancel out.
Figure 8  The dynamics leverage and the size of adjustment coupon. (r=5.5%)

Figure 9  The dynamics leverage and the size of adjustment coupon. (r=4.5%)
Figure 10  The leverage under different bankruptcy cost scenarios. The good firm with lower bankruptcy cost has higher leverage ratio and it recapitalizes frequently; on the other hand, the bad firm with high bankruptcy cost has lower leverage and does like to recapitalize.

Figure 11  The dynamics leverage and the size of adjustment coupon. (α=0.15)  The good firm with lower bankruptcy cost has higher leverage ratio, and it has larger debt capacity so that can recapitalizes frequently.
Figure 12  The dynamics leverage and the size of adjustment coupon. \((\alpha=0.30)\)

Figure 13  The dynamics leverage and the size of adjustment coupon. \((\alpha=0.50)\)
Figure 14  The leverage under different structure of fixed cost. There still exists the proportional cost when comparing different structure of fixed cost. The frequency of adjustment is affected significantly by the fixed cost. The larger the fixed cost is, the less frequency of adjustment.

Figure 15  The leverage under different structure of proportional cost. There still exists the fixed cost when comparing different structure of proportional cost. The frequency of adjustment is also affected by the proportional cost, but not significant as fixed cost. If there is not an existence of proportional cost, the firm would like to adjust the capital structure more frequently.
Figure 16  The leverage and size of adjustment coupon under the fixed cost of adjustment. The firm recapitalizes with large size of adjustment coupon at once because there are the same adjustment cost at the time.

Figure 17  The leverage and size of adjustment coupon under the proportional cost of adjustment. The firm recapitalizes frequently but the size of adjustment coupon is small because the proportional cost of adjustment exists.
Figure 18  The leverage and size of adjustment coupon under the large fixed cost of adjustment. The firm do not like to recapitalization because large fixed of adjustment, but it would adjust large size when it is necessary.

Figure 19  The leverage and size of adjustment coupon under the large proportional cost of adjustment. The firm do not like to recapitalization because large proportional cost of adjustment, and it would adjust small size when it is necessary.
Table 5: The base case parameter values

<table>
<thead>
<tr>
<th>Parameters and Variables</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$, the volatility of cash flow</td>
<td>0.10</td>
</tr>
<tr>
<td>$\mu$, the growth rate of cash flow</td>
<td>3.5%</td>
</tr>
<tr>
<td>$\delta$, the initial cash flow at period 0</td>
<td>100</td>
</tr>
<tr>
<td>$r$, risk-free rate</td>
<td>4.5%</td>
</tr>
<tr>
<td>$q$, the risk premium</td>
<td>3.0%</td>
</tr>
<tr>
<td>$\tau_c$, the corporate profits tax rate</td>
<td>0.35</td>
</tr>
<tr>
<td>$\tau_d$, the dividends tax rate</td>
<td>0.20</td>
</tr>
<tr>
<td>$\tau_i$, the personal tax rate</td>
<td>0.35</td>
</tr>
<tr>
<td>$\alpha$, the bankruptcy cost</td>
<td>0.5</td>
</tr>
<tr>
<td>$\varepsilon_F$, the fixed cost of debt</td>
<td>0.1%</td>
</tr>
<tr>
<td>$\omega_F$, the fixed cost of equity</td>
<td>0.2%</td>
</tr>
<tr>
<td>$\varepsilon_P$, the proportional cost of debt</td>
<td>1.0%</td>
</tr>
<tr>
<td>$\omega_P$, the proportional cost of equity</td>
<td>5.0%</td>
</tr>
<tr>
<td>$k$, the adjustment exponent</td>
<td>1</td>
</tr>
</tbody>
</table>
Appendix A

Following Goldstein, Ju, and Leland (2001), we derive the default level, $V_B$, by invoking the smoothing-pasting condition:

$$\left. \frac{\partial E}{\partial V} \right|_{V'=V_B} = 0$$

According to equation (19), we know that

$$E = (1 - \tau_{\text{eff}}) (V_{\text{solv}} - V_{\text{int}})$$

where $V_{\text{solv}} = V - V_B P_B$, $V_{\text{int}} = \frac{C}{r} \left(1 - P_B\right)$, and $P_B = \left(\frac{V}{V_B}\right)^x$

Then,

$$\left. \frac{\partial E}{\partial V} \right|_{V'=V_B} = (1 - \tau_{\text{eff}}) \left( \frac{\partial V_{\text{solv}}}{\partial V} - \frac{\partial V_{\text{int}}}{\partial V} \right) \bigg|_{V'=V_B}$$

where

$$\frac{\partial V_{\text{solv}}}{\partial V} = 1 - V_B (\frac{x}{V_B}) (\frac{V}{V_B})^{-x-1}$$

$$\frac{\partial V_{\text{int}}}{\partial V} = -\frac{C}{r} (\frac{x}{V}) (\frac{V}{V_B})^{-x-1}$$

So,

$$\left. \frac{\partial E}{\partial V} \right|_{V'=V_B} = 0 \Rightarrow \left. \frac{\partial V_{\text{solv}}}{\partial V} \right|_{V'=V_B} - \left. \frac{\partial V_{\text{int}}}{\partial V} \right|_{V'=V_B} = 0$$

$$\Rightarrow V_B = \left( \frac{x}{1+x} \right) \frac{C}{r}$$

$$\Rightarrow V_B = \lambda \frac{C}{r} \quad \text{where} \quad \lambda = \frac{x}{x+1}$$
Appendix B

Assuming the current firm value is $V_0$, we derive the optimal coupon level by differentiating the value of debt plus equity with respect to $C$ and setting the equation to zero.

$$\frac{\partial V}{\partial C}_{V-V_0} = 0$$

where

$$V = D + E = \left[ (1 - \tau_i) - (1 - \tau_{eff}) \right] V_{int} + (1 - \alpha)(1 - \tau_{eff}) V_{def} + (1 - \tau_{eff}) V_{solv}$$

Solving, we get

$$\Rightarrow 0 = \left[ (1 - \tau_i) - (1 - \tau_{eff}) \right] \frac{\partial V_{int}}{\partial C}_{V-V_0} + (1 - \alpha)(1 - \tau_{eff}) \frac{\partial V_{def}}{\partial C}_{V-V_0} + (1 - \tau_{eff}) \frac{\partial V_{solv}}{\partial C}_{V-V_0}$$

$$\Rightarrow 0 = \left[ (1 - \tau_i) - (1 - \tau_{eff}) \right] \left[ \frac{1}{r} - \frac{1}{r} (1 + x) \left( \frac{\lambda}{r} \right)^{\gamma} V_0^{-\gamma} \right]$$

$$+ (1 - \alpha)(1 - \tau_{eff}) \left[ \frac{\lambda}{r} (x + 1) \left( \frac{\lambda}{r} \right)^{\gamma} V_0^{-\gamma} \right]$$

$$+ (1 - \tau_{eff}) \left[ - \frac{\lambda}{r} (1 + x) \left( \frac{\lambda}{r} \right)^{\gamma} V_0^{-\gamma} \right]$$

$$\Rightarrow C^* = \left( \frac{r}{\lambda} \right) V_0 \left[ \left( \frac{1}{1 + x} \right) \left( \frac{A}{A + B} \right) \right]^{\gamma}$$

where $B = \lambda \alpha (1 - \tau_{eff})$, $A = \tau_{eff} - \tau_i$