How Does Credit Rating Management Behavior Impact Optimal Capital Structure Decision?

Chang-Chih Chen[a]

Department of Finance

National Sun Yat-sen University

Kaohsiung 804, Taiwan

886-7-525-2000 txt 4828

bra19810910@yahoo.com.tw

So-De Shyu[b]

Department of Finance

National Sun Yat-sen University

Kaohsiung 804, Taiwan

886-7-525-2000 txt 4814

dshyu@cm.nsysu.edu.tw

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[a] Chang-Chih Chen is a PhD candidate from Department of Finance, National Sun Yat-sen University at Kaohsiung in Taiwan. E-mail: bra19810910@yahoo.com.tw

[b] So-De Shyu is a professor from from Department of Finance, National Sun Yat-sen University at Kaohsiung in Taiwan. E-mail: dshyu@cm.nsysu.edu.tw

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Abstract

This paper examines the impact of credit rating management behavior in determining optimal capital structure with a system of rating transition multi-boundaries. We propose contingent claim models that capture firms' empirical behavior in threefold: the behavior as targeting initial rating, the behavior as linking firm's rating to the promised coupons, and the behavior as targeting minimum rating. We show that, if the policy targets a better initial rating, the firm is usually underleveraged. As long as the rating at issuing time is not too low, tax shields of rating-linked coupon debt are larger than those of standard debt with same par, and hence optimal leverage usage of firm having rating-linked coupon scheme is greater. Further, our result also indicates that the behavior as targeting minimum rating account for mean reversion in leverage dynamics. Following a downgrade from target minimum rating, managers appear to make over-repurchase choices for adjusting current rating back to initial target.

JEL Classification: G3, G32, G33

Keywords: optimal leverage, target initial credit rating, rating-linked coupon debt, target minimum rating.

I. Introduction

"Credit rating[1]" is always the most prevailing and significant measurement for corporate debt's default risk. A CFO survey by Graham and Harvey (2001) indicates that credit rating is referred to be second greatest concern for CFOs when considering debt issuance. Following Graham and Harvey, Molina (2005) also suggests that the leverage and credit rating shall be jointly considered in analysis, and thus uses debt's rating as a proxy for default risk in measuring leverage impact on default probability.

Motivated by the strong linkage between leverage and credit rating, there is a growing literature that makes effort to study the credit rating management associated with capital structure activities. For example, the evidence in Leary and Roberts (2005) discovers that, in order to manage credit rating, number of firms move their leverage toward a target by repurchase activities. Kisgen (2006a) proposes *Credit Rating-Capital Structure Hypothesis* (henceforth denoted by "CR-CS") that credit ratings can directly affect firm's capital structure decisions by managers. Different credit rating levels are associated with discrete benefits (costs) to the firm. When making capital structure decision, managers will balance these benefits (costs) against the benefits (costs) implied by traditional tradeoff theory, if the rating-dependent benefits (costs) are material. Kisgen (2006b) points out that the firms are observed to exhibit behavior consistent with targeting minimum ratings. The behavior following the receipt of downgrades and upgrades is asymmetric. Firms experiencing deterioration in credit rating will undertake leverage reducing activity to regain previous better rating, while upgraded firms do not significantly undertake such activity. Manso et al. (2008) extend Acharya et al. (2002) and Lando and Mortensen (2005) to concentrate on the issues of performance-sensitive debt.[2] They argue that, relative to fixed-coupon debt, default time of the issuer of performance-sensitive debt is earlier, and hence higher bankruptcy costs cause a smaller initial market value of the equity. Based on value-maximizing objective, the behavior as linking performance measure (e.g., credit rating) to debt's cashflow seems helpless and inefficient.

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[1] Standard and Poor's (2001) explicitly defines *issue* credit ratings to be an opinion of the creditworthiness of an obligor with respect to a specific financial obligation. A firm credit rating provides potential investors with its own information on credit quality beyond publicly available information. Hence credit ratings can release significant and sensitive information that is not public, if the firm may be unwilling to disclose private information that may compromise its strategic programs, in particular with regard to opponents.

[2] Performance-sensitive debt is the class of debt obligation whose interest payments is linked with some measure of borrower's performance, such as credit rating or financial ratios. In practice credit-sensitive note (see Acharya et al. (2002)) and step-up bond (see Lando and Mortensen (2005)) are both special cases of performance-sensitive debt. For notational convenience, in this study all these types of debt will be termed "rating-linked coupon debt". The idea of linking credit quality to debt's cashflow is not new. This type of credit sensitive derivatives has been openly traded since the late 1980s. For more details to their development, see Das and Tufano (1996), Acharya et al. (2002), Lando and Mortensen (2005), and Manso et al. (2008).

Unfortunately, most of above interesting sentiments on credit rating management are not explicitly explained by traditional capital structure theories.[3] Prior researches usually explore issues like tradeoff between tax benefits and bankruptcy costs, informational asymmetries, agency problem, dynamic restructuring, maturity structure, industry dynamics, or security design, but never restrict their attention on the effect of credit migration on optimization. Hence, a natural question that arises is how credit rating management behavior impacts firm's optimal capital structure decision.

To explore central issue of this paper, we shall construct a theoretic framework for capital structure that allows for examining those rating-related empirical behavior. Before modeling credit migration into capital structure problem, reviewing major rating-based pricing model of existing literatures is required. Jarrow et al. (1997), based on reduced-form approach, pioneer in treating credit migration as a Markov chain process and give an example of estimating term structure for credit spreads. Subsequent works that devote to extending this type of model in pricing credit derivatives include Kijima (1998), Kijima and Komoribayashi (1998), Kodera (2001), and Kang and Kim (2004). Nevertheless, there appear some restrictions on straightforwardly utilizing these models to examine optimal capital structure.

First and most importantly, the basic idea of reduced-form approach contravenes fundamentals of capital structure theory; The spirit of structural-form approach (firm-value based framework) always emphasizes that potential of bankruptcy is directly reacted by firm's asset value and capital structure. Reduced-form approach, however, holds that the bankruptcy is exogenous, and assumes that the default is irrelevant with capital structure.

Second, the estimations of probabilities with long rating-migration distance (rare events' probability) are too rough and almost omitted as zero; This may lead the potential of firm's default and the value of bankruptcy costs to be under estimated.

Third, it is unavailable to examine first hitting time of credit rating; In valuing a debt embedded with rating-dependent callable option, accurately computing survival probability of this random time is required.

To overcome those restrictions, this article attempts to build a firm-value based framework for credit migration. The new idea is implemented by expanding the design of single bankruptcy triggering threshold, originally proposed by Merton (1974) and Black and Cox (1976), to become a system of rating transition multi-boundaries (more details to this system is discussed in following section). Several advantages of this framework are numerated as below. First of all, the evolution of credit rating is

[3] This includes a group of studies, such as Modigliani and Miller (1963), Leland and Pyle (1977), Leland (1994), Leland and Toft (1996), Leland (1998), Collin-Dufrasne and Goldstein (2001), Goldstein et al. (2001), Ju et al. (2005), Miao (2005), and Demarzo and Sannikov (2006).

driven by the fluctuation of assets' value in the system of rating transition multi-boundaries; since the system is operated in continuous time, the accuracy of estimating probability with long migrating distance can be improved. Further, via rating transition boundaries and instantaneous asset value, investors can easily observe firm's current credit rating; this saves lots of time and cost of rechecking credit rating. Finally, the series of rating transition boundaries is uniquely depending on capital structure decision; this implies that not only the argument by Graham and Harvey (2001) and Molina (2005) that leverage and ratings shall be jointly determined is captured, but also estimated transition probabilities here can truly reflect firm-specific information (e.g. asset return volatility). Based on this framework, we rebuild contingent claim model for capital structure proposed by Ju et al. (2005) in the way of considering a target initial rating policy.[4] Close form solutions for varied contingent claims' value are derived by martingale method. Also, such way of doing is allowed for our extended models.

This paper develops two extended models for incorporating more rating-related empirical behavior into capital structure problem. One is the case of the firm selling a debt with rating-linked coupon scheme, which not only implement the idea of linking credit rating to debt's cashflow in practice, but also considers direct benefits (costs) to the firm experiencing rating upgrades (downgrades) that arise from a decrement (increment) in its coupon rate. The other is the debt with rating-dependent callable option derived from target minimum rating policy. In this case, there is only one chance leaves for managers to adjust firm's capital structure and credit rating. The repurchase activities are undertaken only when the rating fails to remain in target range during the debt's life. The size of repurchase is predetermined and depends on firm's credit rating policy. Via this repurchase, the firm will regain previous higher ratings temporarily after receiving a downgrade from target minimum rating.

Main findings of this study add to the literatures in threefold. Firstly in examining the behavior as targeting initial rating, we show that a firm intending better initial ratings (e.g., investment-grade rating) is generally underleveraged. Surprisingly, this provides a reasonable explanation for popular puzzle in corporate finance that why firms in reality under-use debt and forego valuable tax benefits[5]. While the behavior as linking firm's credit to promised coupon is considered, the result indicates that, as long as debt's rating at issuing time is not too low, the firm using rating-linked coupon debt always has greater tax shields, compared to the case of using standard debt with same par. Managers thus are motivated to issue more rating-linked coupon

[4] For practicability in estimating rating transition boundaries from market data, the model we choose for rebuilding shall be characterized by a finite maturity, such as the static model in Ju et al. (2005).
[5] This has motivated number of empirical studies, such as Leland (1998), Graham (2000), Graham et al. (2004), Ju et al. (2005), Kahle and Shastri (2005), and Molina (2005).

debt, implying that corresponded optimal leverage usage is greater also. Additionally, in the case that firm exhibits behavior consistent with target minimum rating, debt repurchases induced by the requirement of improving credit rating cause mean reversion in leverage dynamics. Following a downgrade from target minimum rating, managers will make over-repurchase choices for adjusting their current rating back to initial target.

The rest of this paper is structured as follows: In Section 2 below, the firm-value based framework for credit migration is constructed. Section 3 rebuilds contingent claim model to discuss the impact of target initial rating policy in determining optimal leverage. Section 4 and 5 consider extended models, "debt with rating-linked coupon scheme" and "debt with rating-dependent callable option", respectively, and further analyze the impact of firm's empirical behavior. The conclusion is drawn in Section 6.

II. A Firm-Value Based Framework of Credit Migration

Merton (1974) is the earliest example of serving total assets' value as firm's solvency, and defines that bankruptcy is triggered only when the solvency is insufficient to meet current obligation at debt maturity. The conception of single bankruptcy triggering threshold is the core of pricing theory for corporate debt. Black and Cox (1976) improve Merton (1974) to develop a model that can capture the rights of creditors to force a firm into bankruptcy once its asset value drop too low to violate the covenant. Such model is subsequently termed "first-passage-time model". Based on these two articles, this research expands the idea of single bankruptcy triggering boundary to become a system of rating transition multi-boundaries as in Figure 1.

Before interpreting this figure, some relevant notations need to be introduced. Without loss of generality, consider a circumstance in which an unlevered firm's manager may issue debt at time-0 with a maturity T. The finite interval $[0,T]$ is trading period for debt. Let $N=\{1,2,...,n\}$ be the space of all possible credit states, where state n denotes the highest credit rating state (i.e., AAA rate), state 2 denotes the lowest credit rating, and state 1 means for default rate. Given a debt rating system consisting of AAA, AA, A, BBB, BB, B, CCC, and D, total number of possible rating states n is then equal to 8. The notation $D_i^k, i,k \in N, k \neq 1$ symbolizes rating i's lower transition boundary under initial credit rating k. Each boundary here is predetermined by capital decision, and implicates a specific least required level of solvency (asset value) for holding corresponded credit rating. This means that, at the moment in which the path of asset value hits the boundary, firm will receive a rating downgrade or upgrade. Note that, no matter what initial rating is, transition boundary must be monotonically increasing with intended rating level. Such design tallies with

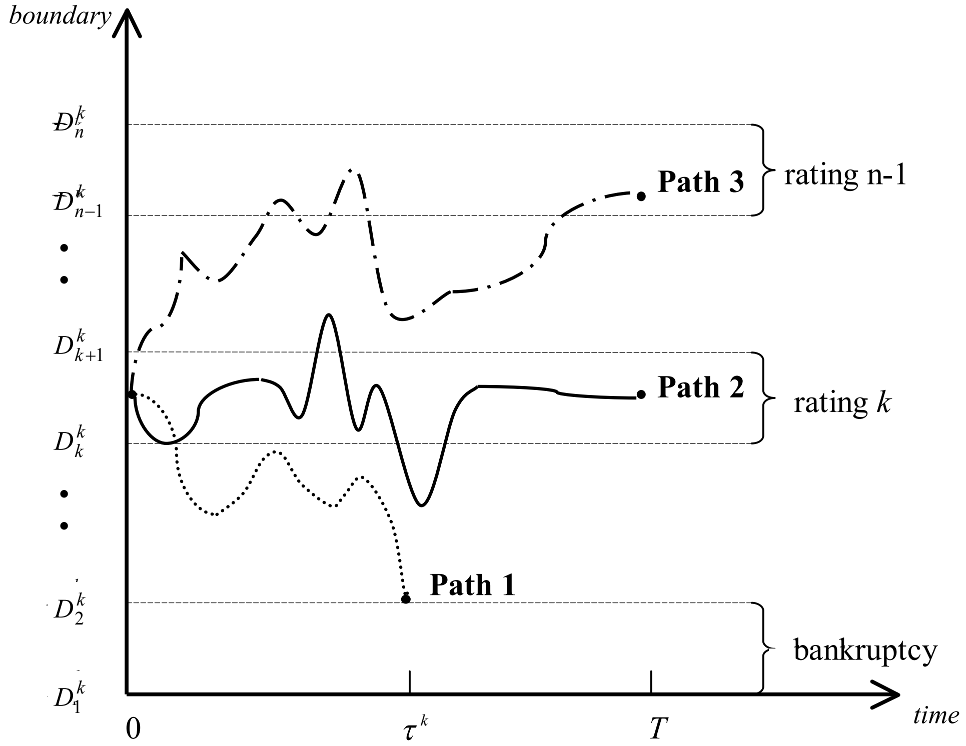

Figure 1. A system of rating transition multi-boundaries. The lines plot the path of assets' value in a system of rating transition multi-boundaries for firm with trading period [0,T], initial credit rating k, default time τ^k, and series of transition boundaries $D_1^k,\cdots,D_n^k$. Path 1 (*dotted line*) depicts the condition that rating falls to default state D from initial state during the period. Path 2 (*solid line*) portrays the condition that rating finally reenters into its initial state at maturity. Path 3 (*dash-dotted line*) draws the condition that rating finally transits from initial state to n-1 at maturity.

the reality that higher rating always requires stronger solvency. Also note that, for all k, the boundary notated with default rate D_1^k is usually assigned as zero, suggesting that there is no requirement for solvency in retaining this rating. The boundary D_2^k is gifted with the function of bankruptcy trigger, and hence can be set as par value of firm's debt. τ^k represents, conditioned on initial credit rating k, the random time of the occurrence of bankruptcy, which is defined as the moment that firm's asset value firstly hits boundary D_2^k over the trading period. The aim of defining τ^k in such way is to follow the absorbing property for default state D. Based on this property, τ^k is known to be equivalent to the time of credit rating firstly reaching default state.

Now illustrate the economic implication of figure. Path 1 (dotted line) depicts the condition that credit rating falls to state D from initial state k during the trading period; After making capital structure decision, observe that the established series of boundaries and initial solvency (asset value) jointly determine starting rating at k

level. [6] Over time, the path of asset value knocks default-triggered threshold D_2^k at a certain time prior to T. This especial time is the so-called τ^k. Once the bankruptcy is declared, creditors may force managers to liquidate debt obligation with assets and the firm will be reorganized. The dotted line is therefore terminated at τ^k, reflecting that the rating enters into an absorbing state. Path 2 (solid line) portrays the condition that credit rating finally reenters into its initial state k at maturity; It is notable that, besides asset value being located in the interval $[D_k^k, D_{k+1}^k]$ at time-T, its path still keeps larger than threshold D_2^k during the whole period. This implies the twofold requirement for firm solvency-not only its level at maturity shall be sufficient for holding rating k, but also remaining in survival range is to prevent from the violation of default covenant. Path 3 (dash-dotted line) draws the case in which credit rating transits from initial state k to terminal state *n-1*; Similar to the significance in Path 2, the asset value remains in the range $[D_2^k, D_n^k]$ during debt's life, but further stays in an interval corresponded to rating's migrating distance $[D_{n-1}^k, D_n^k]$ at expiring date. Summing up above statements, there are two points that should be stressed here: (i) the dynamics of credit rating is driven by the fluctuation in asset value (solvency) only, and (ii) the series of transition boundaries (capital structure decision) governs the transiting likelihood of credit ratings. Through this system, instantaneous credit state can be easily observed at continuous time. The possibility of rating transition thus is capable to be examined also. Following we develop a model to implement the idea of Figure 1.

2.1 The Model

Consider an economy with continuous trading time. A reference firm's assets are assumed to have total value unaffected by capital structure, in which can be denoted, at time-t, by $V(t), t \in \mathcal{R}_+$ following a diffusion process as

$$\frac{dV(t)}{V(t)} = (\mu - \delta)dt + \sigma\, dW_{\mathrm{P}}(t) \tag{1}$$

where μ represents appreciation rate, δ stands for payout ratio of total assets' value, and σ plays constant volatility. $W_{\mathrm{P}}(t)$ denotes a single dimensional Weiner process defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathrm{P})$. Ω is the sample space which also contains subspace N. Sigma algebra, $\mathcal{F}_t$, collects the information generated by the observation of asset value up to time-t, and is available to all agents in the economy. P is so-called real measure for historical probability. To examine this

[6] Since credit rating system is not applicable to a totally equity-financed firm, it is improper to exogenously assign any state to initial rating in this framework. Starting credit state here depends on the relation between total asset values (solvency) and debt's par (leverage), corresponding with the view suggested by Graham and Harvey (2001) and Molina (2005) that leverage and ratings are jointly determined. Given firm-specific information on capital structure, investors can straightforwardly observe debt's initial credit state and then estimate other corresponded rating transition boundaries.

framework under risk neutral world, we still suppose that default-free bonds are allowed for trading in this economy and pay a constant interest rate r.

Now consider the definition of rating process. Let $\{\eta^k(s); s \in [0,T], k = 2,..,n\}$ be a Markov process on finite state space N with initial state $\eta^k(0) = k$. Following the idea in Figure 1, firm-value based representation for rating process is shown as, when $s = 0$,

$$\begin{cases} \{\eta^k(s) = k\} \equiv \{V(s) \in [D_k^k, \infty]\}, \ k = n \\ \{\eta^k(s) = k\} \equiv \{V(s) \in [D_k^k, D_{k+1}^k]\}, \ k = 2,..., n-1 \end{cases} \quad (2),$$

and when $0 < s \le T$,

$$\begin{cases} \{\eta^k(s) = j\} \equiv \left\{ \inf_{0 \le u \le s} V(u) \in [D_2^k, \infty], V(s) \in [D_j^k, \infty] \right\}, \ j = n \\ \{\eta^k(s) = j\} \equiv \left\{ \inf_{0 \le u \le s} V(u) \in [D_2^k, \infty], V(s) \in [D_j^k, D_{j+1}^k] \right\}, \ j = 2,..., n-1 \\ \{\eta^k(s) = j\} \equiv \left\{ \inf_{0 \le u \le s} V(u) \in [0, D_2^k] \right\}, \ j = 1 \end{cases} \quad (3)$$

where $\inf_{0 \le u \le s} V(u)$ implicates minimum of the path of asset value over the period $[0,s]$.

The default-triggering time derived from (2) and (3) has corresponding definition

$$\tau^k \equiv \inf\{\eta^k(s) = 1 \mid s \in [0,T]\} \equiv \inf\{V(s) \le D_2^k \mid s \in [0,T]\} \quad (4).$$

Applying the probability theory with above expressions (2)-(4), rating transition at any time during the trading period can be formed as a $n \times n$ continuous time probability matrix[7]

$$\mathrm{P}_{0,s} = \begin{pmatrix} p_{\mathrm{n}}^{n}(0,s) & p_{\mathrm{n-1}}^{n}(0,s) & \cdots & p_{1}^{n}(0,s) \\ p_{\mathrm{n}}^{n-1}(0,s) & p_{\mathrm{n-1}}^{n-1}(0,s) & \cdots & p_{1}^{n-1}(0,s) \\ \vdots & \vdots & \vdots & \vdots \\ p_{\mathrm{n}}^{2}(0,s) & p_{\mathrm{n-1}}^{2}(0,s) & \cdots & p_{1}^{2}(0,s) \\ 0 & 0 & \cdots & 1 \end{pmatrix} \quad (5).$$

Note that each entry of (5),

$$p_j^k(0,s) \equiv \mathrm{P}\left(\eta^k(s) = j \mid \eta^k(0) = k\right) \quad \text{for } j,k \in N,$$

which symbolizes the transition likelihood of going from rating k at time-0 to j at

[7] To verify the validity of this matrix, one can do so via *Chapman-Kolmogorov equation.*

time-s under historical measure P**.** The sum of each entry in same row must equals to one for all k. Based on the property that default state is absorbing, the last row in transition matrix has been explained.

To solve the transition matrix, assume that there is a particular equivalent martingale measure Q on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0})$, which allows for examining transition probabilities and default time. The corresponding Radon-Nikodym derivatives $d\mathrm{Q}/d\mathrm{P} := \xi(s)$, $s \in [0, T]$ satisfies following integral equation

$$\xi(T) = \xi(0) + \int_{[0,T]} \xi(s)\gamma(s)dW_{\mathrm{P}}(s)$$

where $\gamma(s) \equiv (r-\mu)/\sigma$ is a $\mathcal{F}$-predictable process, meaning that the information on $\gamma(s)$ is available before time-s. Utilizing Girsanov's theorem[8], we change the underlying measure from P to Q for Weiner process

$$W_{\mathrm{Q}}(t) = W_{\mathrm{P}}(t) - \int_0^t \gamma(s)ds,$$

and rewrite the dynamics of assets' value, equation (1), to have

$$\frac{dV(t)}{V(t)} = (r-\delta)dt + \sigma\, dW_{\mathrm{Q}}(t). \qquad (6)^{9}$$

By the use of equation (6), reflection principle, and martingale method, closed form solution for risk neutral transition probabilities is derived and given in following theorem.

THEOREM 1: Conditioned on a non-default initial rating k, risk neutral rating transition probabilities of going from initial rating to j at time-s are

$$q_j^k(0,s) = \begin{cases} \mathrm{N}(\theta_2) - \left(\dfrac{D_2^k}{V(0)}\right)^{\chi} \mathrm{N}(\theta_1) & , j = \mathrm{n} \\ \mathrm{N}(\theta_4) - \mathrm{N}(\theta_6) - \left(\dfrac{D_2^k}{V(0)}\right)^{\chi} [\mathrm{N}(\theta_3) - \mathrm{N}(\theta_5)] & , j = 2,\ldots,\mathrm{n}-1 \\ \left(\dfrac{D_2^k}{V(0)}\right)^{\chi} \mathrm{N}(\theta_7) + 1 - \mathrm{N}(\theta_8) & , j = 1 \end{cases}$$

where $s \in [0, T]$, $\chi = 2\lambda/\sigma^2$, and $\lambda = r - \delta - (1/2)\sigma^2$. The arguments to $\mathrm{N}(\cdot)$, the standard normal cumulative distribution, are given by

8 For more details to Girsanov's theorem, see the page 190-200 in Karatzas and Shreve (1991).

9 Jarrow et al. (1997) utilize risk premia adjustment to transform the measure from real world to risk neutral world. The risk neutral transition probability in that article is a product of an actual transition probability and corresponded risk premia adjustment, which can be estimated from the observable market data (e.g., the price of zero-coupon bond). Differing with them, our approach is to straight-forwardly change the measure by Girsanov's theorem, and seems simpler and effortless.

$$\theta_1 = f(2,-1,0,-1) \text{，} \theta_2 = f(0,1,0,-1)$$
$$\theta_3 = f(2,-1,0,-1) \text{，} \theta_4 = f(0,1,0,-1)$$
$$\theta_5 = f(2,-1,-1,0) \text{，} \theta_6 = f(0,1,-1,0)$$
$$\theta_7 = f(1,-1,0,0) \text{，} \theta_8 = f(-1,1,0,0).$$

and compound function is expressed as

$$f(a,b,c,d) = \frac{1}{\sigma\sqrt{s}}\left[a\ln\left(D_2^k\right) + b\ln\left(V(0)\right) + c\ln\left(D_{j+1}^k\right) + d\ln\left(D_j^k\right) + \lambda s\right].$$

Via the formulas in Theorem 1, the evolution of credit rating during the debt's lifespan can be examined. In following sections these will help us value defaultable contingent claims derived from firm's capital decision, such as corporate debt, tax benefits, and bankruptcy costs. For detailed derivation and proof of Theorem 1, one can refer to the appendix in Liao and Wang (2002).[10] That article provides a clear example for the application of reflection principle in first-passage-time model.

2.2 A Numerical Example

A numerical example of estimating rating transition boundaries and calculating transition probabilities is given as below. For tractability, let $r = 5\%$, $\delta = 3.75\%$, $\sigma = 38.02\%$[11], $V(0) = \$100$, and $T = 5$. The estimation of rating transition boundaries is roughly achieved in five steps. In first step we take an economy-wide five-year rating transition probability matrix, which can be estimated from market data (e.g., S&P's special report). Given this probability matrix and the combination of above parameters, we figure out a rating transition boundary matrix based on economic-wide credit migration in second step. Following, in third step we divide this boundary matrix into its last column to obtain a new matrix, termed economic-wide rating transition structure. Each element in this matrix denotes a specific multiplier of the lowest required solvency that corresponds to its own migration distance. Each series composed of multipliers in same row represents the rating transition structure over estimated period under a certain initial credit state. Via firm assets' value and the diagonal of transition structure matrix, the implied relations between initial credit states and debt issuing amount (as measured by its par) are clarified in fourth step. Importantly, these help us establish the rule of determining starting credit state. In final step, multiplying the chosen debt's face value by corresponded rating transition structure yields the series of rating transition boundaries. Note that such doing not only makes sense that the series of boundaries is unique for firm and depending on its

[10] Liao and Wang (2002) make the use of reflection principle to value reset options. This paper follows the technique in that article to achieve main derivation.

[11] The value of the assets' volatility is taken from Ju et al. (2005).

Table I

Economy-Wide 5-Year Transition probabilities for Credit Rating

This table shows the economy-wide 5-year transition probabilities for credit rating, and is taken from Christensen et al. (2004). We make some adjustments to the numbers to ensure that ranking order obeys following properties: (i) each element in the matrix is strictly non-negative and the sum of elements in same row always equals to one, (ii) better ratings should never have greater chance of bankruptcy, (iii) as migration distance is longer, the chance of migration should become less, and (iv) the possibility of migrating to a given rating will be larger for more closely adjacent rating categories.

	Transited Ratings							
ICR	AAA	AA	A	BBB	BB	B	CCC	D
AAA	9.61E-01	3.64E-02	2.11E-03	1.38E-04	3.14E-06	1.39E-07	1.06E-08	1.13E-08
AA	7.76E-03	8.86E-01	1.02E-01	4.06E-03	2.32E-04	1.25E-05	1.44E-06	1.06E-06
A	1.50E-03	1.97E-02	8.78E-01	7.97E-02	1.52E-02	5.37E-03	4.33E-04	2.84E-05
BBB	1.34E-03	1.96E-03	7.72E-02	8.58E-01	5.44E-02	6.24E-03	8.97E-04	3.55E-04
BB	1.19E-04	2.62E-04	1.44E-02	1.49E-01	7.54E-01	7.43E-02	6.28E-03	1.61E-03
B	1.65E-05	2.33E-04	4.59E-03	2.23E-02	3.57E-02	6.96E-01	1.80E-01	6.04E-02
CCC	2.80E-07	1.72E-06	1.41E-04	9.11E-04	3.96E-03	8.17E-02	5.87E-01	3.26E-01
D	0	0	0	0	0	0	0	1

Note: ***ICR*** denotes initial credit rating.

capital structure decision, but also captures the argument in Graham and Harvey (2001) and Molina (2005) that rating and leverage shall be jointly determined.

Recall the restriction on computing the probabilities with long rating migration distance in existing literatures. Such type of probability is often omitted as zero and thus is under estimated. Further, this will lead to the failure in solving rating transition boundaries by market data. Fortunately, a continuous time hidden Markov chain model, introduced by Christensen et al. (2004), can address this issue. Main advantage of using their model in estimating rating transition probabilities is to improve the estimated accuracy for those with special focus on rare events. However, since the practice of estimation is beyond the scope of this study, here we directly take a five-year transition probability matrix from that article and compile it as Table I after making some adjustments. The adjustments to probability matrix are made to ensure that ranking order obeys following properties: (i) each element in the matrix is strictly non-negative and the sum of elements in same row always equals to one, (ii) better ratings should never have greater chance of bankruptcy, (iii) as migration distance is longer, the chance of migration should become less, and (iv) the possibility of migrating to a given rating will be larger for more closely adjacent rating categories.

Economic-wide rating transition boundary matrix is compiled in Table II. By weighted operation, we show economic-wide rating transition structure in Table III. The size of multiplier in transition structure matrix determines the required intensity for solvency. The fact in which the multiplier in same row is monotonically increasing with intended rating implies that, if a firm tends to hold a better credit rating, the

Table II

Economy-Wide Rating Transition Boundaries

This table shows the rating transition boundaries implied by economy-wide credit migration. It is assumed that the risk-free interest rate $r = 5\%$, the firm's payout ratio $\delta = 3.75\%$, the maturity of debt $T = 5$ years, the volatility of firm's assets $\sigma = 38.02\%$, and initial value of firm's underlying asset $V(0)$=$100. Economy-wide rating transition probabilities are given as Table I.

	Transited Ratings							
ICR	AAA	AA	A	BBB	BB	B	CCC	D
AAA	16.51	6.63	3.39	1.61	0.96	0.70	0.58	0
AA	580.47	25.71	7.95	3.84	2.13	1.52	1.18	0
A	924.08	416.56	25.03	13.17	8.69	4.44	2.14	0
BBB	951.90	746.61	244.16	20.04	9.37	5.66	3.62	0
BB	1686.59	1297.39	471.98	170.66	22.75	9.51	5.18	0
B	2529.76	1430.70	669.03	380.78	272.56	40.69	15.71	0
CCC	5223.39	3739.36	1620.28	1012.47	662.21	235.71	35.23	0
D	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Note: ***ICR*** denotes initial credit rating.

minimum required level for its solvency becomes greater. Table IV displays implied relations between initial credit ratings and debt issuing amount. As the amount of issuance is larger, initial credit rating will be determined at a lower state. Such relation is consistent with much of empirical evidences including Huang and Huang (2003), Klock et al. (2005), Molina (2005), Kisgen (2006a), and Guttler and Wahrenburg (2007) that average leverage levels appear to have negative-correlation with credit ratings. Based on the result exhibited in Table IV, managers can optimally make the decision of leverage usage for each target credit rating.

The numbers of firm's rating transition probabilities are calculated and compiled as a matrix in Table V.[12] Each row of this matrix represents, under a certain initial rating, the probability distribution of rating transition at debt's maturity. The value of each element measures the chance of credit rating finally entering into a corresponded state from a given initial state. Intuitively, if initial credit rating is lower, the series of transition boundaries becomes greater to reflect more difficulty in standing on better ratings but higher potential for going to bankrupt. Due to this fact, the firm with lower initial rating finally has higher chance of obtaining worse credit ratings and greater default frequencies. Also, this explains why in practice it is easier for a high-levered firm to receive downgrades and to be bankrupt.

[12] Because main intention in this subsection is to demonstrate the procedures of estimating transition boundaries, values of debt's par here are chosen for tractability in computation, rather than those that reach optimization.

Table III

Economy-Wide Rating Transition Structure

This table shows economy-wide rating transition structure. Each element here denotes a specific multiplier of lowest required solvency that corresponds to its own migration distance. The size of multiplier determines the required intensity for solvency. Thus numbers in same row are monotonically increasing with transited ratings, implying that, if a firm intends to hold a better rating, the minimum required level for its solvency must be raised up.

	Transited Ratings							
ICR	AAA	AA	A	BBB	BB	B	CCC	D
AAA	28.3446	11.3811	5.8115	2.7617	1.6522	1.2014	1	0
AA	490.8550	21.7397	6.7188	3.2436	1.8048	1.2860	1	0
A	431.7437	194.6243	11.6963	6.1519	4.0587	2.0723	1	0
BBB	262.7213	206.0619	67.3879	5.5299	2.5871	1.5624	1	0
BB	325.4149	250.3215	91.0655	32.9285	4.3885	1.8356	1	0
B	161.0033	91.0553	42.5798	24.2344	17.3466	2.5895	1	0
CCC	148.2484	106.1293	45.9863	28.7354	18.7945	6.6899	1	0
D	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Note: ***ICR*** denotes initial credit rating.

III. Capital Structure Model with Target Initial Rating Policy: The base case

As pointed out by Kisgen's articles, benefits and costs associated with different rating levels play the great concern for managers in making firm's capital structure decision and credit rating management. Credit rating levels affect whether certain investor groups (e.g., banks and pension funds) are permitted to invest in firm's bonds. Additionally, bond covenants could require a change in coupon rate at different rating levels, and some contracts are signed conditional on a minimum-rating requirement. These benefits and costs to firms, however, are not explicitly considered by traditional tradeoff theory. In this section we thus attempt rebuilding the model proposed by Ju et al. (2005)[13] in the way of considering a simplest credit rating policy-"target initial rating policy". The rebuilt model not only implements CR-CS into capital structure problem, but also allows us to investigate the impact of target credit rating policy on

[13] Several aspects of difference between Ju et al. (2005)'s and our model are summarized as below. Firstly, they consider two alternative models. The one is a "static" model, in which a firm does not replace the expiring debt and hence is entirely equity-financed after maturity date. The other is a "dynamic" model, in which new debt will be reissued while old debt obligation is liquidated for maintaining a long-term target debt to total capital ratio. For simplicity, only the static mode is studied in this paper due to no significant divergence in main features between them. Further, bankruptcy-triggering boundary in their model is supposed to be an exponential function of time, while such assumption is released here to let rating transition multi-boundaries be horizon. The setting of horizon rating transition boundaries is not unusual and is consistent with the spirit of positive net worth-protected debt case in Leland (1994) and constant default boundary case of Longstaff and Schwartz (1995). Finally, the determination of coupon rate in Ju et al. (2005) is endogenous and irrelevant with firm's rating policy. This rate in our model, however, is exogenously determined by market observation of average spreads on defaultable bonds whose current ratings are same with firm's target. The purpose of our doing is to capture credit rating effect on tradeoff theory.

Table IV

Relation between Initial Credit Rating and Range of Debt Issuing Amount

This table shows implied relations between initial credit ratings (including AAA, AA, A, BBB, BB, B, and CCC) and the range of debt issuing amount (as measured by its face value). The numbers are jointly determined by the diagonal of transition structure matrix (as reported by Table III) and initial value of firm's underlying asset *V*(0)=$100.

	Initial Credit Ratings (Investment-Grade)			
Variable	AAA	AA	A	BBB
FV ($)	0~3.5280	3.5280~4.5998	4.5998~8.5497	8.5497~18.0834
	Initial Credit Ratings (Speculative-Grade)			
Variable	BB	B	CCC	D
FV ($)	18.0834~22.7867	22.7867~38.6175	38.6175~99.9999	N/A

Note: ***FV*** denotes face value.

optimal decision. To combine rating-dependent benefits and costs with tradeoff theory, the rate of coupon here is assumed to be discrete and to rely upon target initial rating. The determination of this rate is exogenous via observing average spreads on market defaultable bonds whose current ratings are same with firm's target.[14] The effect of credit rating levels on the allowance of investing in corporate debts would not be considered. In Section 5 more discussion to this effect will be given and linked to firm's motivation behind targeting a minimum rating.

3.1 The Model

Without loss of generality, consider a circumstance in which a reference firm is unlevered at initial time-0 and tends to issue a debt with maturity T at the value $D_L^k(0)$. The dynamics of its total assets value satisfies equation (1). At initial time,

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[14] The notion of exogenously determining coupon rate is inconsistent with most traditional models. Such doing, however, can not only implements rating-dependent benefits and costs, documented by Kisgen's articles, but also captures some useful empirical viewpoints. For instance, Holthausen and Leftwich (1986), Hand et al. (1992), and Goh and Ederington (1993) find statistically significant negative stock returns upon the announcement of downgrades. Ederington and Goh (1998) and Kim and Nabar (2007) show that rating downgrades result in negative equity returns. Huang and Huang (2003) concludes that credit risk accounts for only a small fraction of the observed yield spreads for investment grade bonds of all maturities, but accounts for a much higher fraction of yield spreads for junk bonds.

To further clarify this point, we use the capital asset payout principle

$$\delta V(t)dt = \left[\delta^E(t) + (1-\beta)C\right]dt .$$

This equation has an interpretation that the levered firm liquidates asset at the ratio δ of the total asset value, hence $\delta V(t)dt$ equals the sum of a time-varying dividend $\delta^E(t)dt$ paid to equity holders and after-tax coupon paid to debt holders $(1-\beta)C\,dt$ over the time dt. In case where initial rating targeted by firm is lower, the possibility of receiving downgrades in future becomes higher to lead the reduction in equity's return and in dividend $\delta^E(t)$ paid to stockholders. Since asset's payout ratio δ is fixed, the promised coupon payment shall be raised up to reflect the increment in credit spreads. Due to this fact, the empirical relation between credit migration and coupon rate has been implied.

Table V

Firm's Rating Transition Probability over Debt's Lifespan

This table shows the computations of rating transition probability for a levered firm issuing debt with face value \$2, \$4, \$6, \$13, \$20, \$30, and \$60 respectively. It is assumed that the risk-free interest rate $r = 5\%$, the firm's payout ratio $\delta = 3.75\%$, the maturity of debt $T = 5$ years, the volatility of firm's assets $\sigma = 38.02\%$, and initial value of firm's underlying asset $V(0)$=\$100.

	Ratings at the Maturity							
FV(ICR)	AAA	AA	A	BBB	BB	B	CCC	D
\$2(AAA)	6.24E-01	2.94E-01	6.77E-02	1.35E-02	9.98E-04	9.64E-05	1.04E-05	2.00E-05
\$4(AA)	5.82E-05	4.26E-01	4.58E-01	9.61E-02	1.71E-02	2.17E-03	3.67E-04	5.48E-04
\$6(A)	1.46E-05	5.78E-04	5.25E-01	2.68E-01	1.11E-01	7.72E-02	1.50E-02	2.84E-03
\$13(BBB)	3.32E-06	8.94E-06	1.83E-03	5.13E-01	3.09E-01	1.10E-01	3.04E-02	3.63E-02
\$20(BB)	7.08E-08	2.92E-07	8.28E-05	5.02E-03	4.16E-01	3.69E-01	1.01E-01	1.09E-01
\$30(B)	4.50E-07	1.06E-05	3.96E-04	3.22E-03	7.33E-03	4.62E-01	2.79E-01	2.48E-01
\$60(CCC)	8.97E-09	7.23E-08	1.04E-05	9.63E-05	5.69E-04	2.17E-02	3.16E-01	6.62E-01
\$N/A(D)	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E+00

Note: ***FV*** denotes face value.
ICR denotes initial credit rating.

managers will determine debt's rating at a certain non-default level k consistent with long-term target to implement target initial rating policy.[15] Based on the decision regarding target rating, firm's leverage usage is bounded in a corresponded range.[16] The optimization of capital structure decision is reached by maximizing the wealth of equity-holders. The debt has a unique series of rating transition boundaries $D_1^k, \cdots, D_8^k$, determined by its face value F_L^k. The debt continuously pays the coupon at a constant annualized rate C_L^k depended on target initial rating. The coupon payment shields income from taxes at effective rate β, and tax benefits enjoyed by debt have the value $TB_L^k(0)$ at initial time.

The debt requires a protective minimum rating covenant that, if credit rating falls to state D at any time during the life of debt, the firm is forced into bankruptcy. The default-triggering time thus can be expressed as

$$\tau_L^k \equiv \inf\left\{ s \in [0,T],\ V(s) \le F_L^k \right\} \equiv \inf\left\{ s \in [0,T], \eta^k(s) = 1 \right\}.$$

Once the bankruptcy is declared, equity (stock) becomes valueless and only a portion $1-\alpha$ of levered value of assets can be redeemed by debt holders. The fraction loss of assets' value is supposed to be expended in bankruptcy process. Bankruptcy costs

[15] The consideration for firm determining its long-term target rating is quite complicated and probably irrelevant with capital structure decision, such as managers' preference or industrial dynamics. To restrict the attention on our central issue, here we do not attempt to address the issue of optimal long-term credit rating, since it is beyond the scope of this study.

[16] Implied relation between initial ratings and the amount of debt issuance is illustrated in Table IV.

therefore are the present value of expected losses in bankruptcy and can be denoted by $BC_L^k(0)$.

Following Leland and Toft (1996) and Ju et al. (2005), the value of debt at initial time sums a contribution from coupon, a contribution from recovered payment to debt holders if the firm bankrupts, and the repayment of par if bankruptcy does not occur until to maturity T, that is,

$$D_L^k(0) = \mathrm{E}_{\mathrm{Q}}\left(\int_0^T C_L^k \mathrm{e}^{-rs} 1_{\{\tau_L^k > s\}} ds + \frac{TV_L^k(0)}{V(0)}(1-\alpha)V\left(\tau_L^k\right)\mathrm{e}^{-r\tau_L^k} 1_{\{\tau_L^k \le T\}} + F_L^k \mathrm{e}^{-rT} 1_{\{\tau_L^k > T\}} \middle| \mathcal{F}_0 \right) \tag{7}$$

where $TV_L^k(0) = V(0) + TB_L^k(0) - BC_L^k(0)$ is total levered value of firm at time zero and $1(\cdot)$ denotes the indicator function. The factor $TV_L^k(0)/V(0)$[17] implements modeling decision that, upon bankruptcy, the firm reorganizes and debt holders are allowed to takeover it and to become new shareholders. After bankruptcy process, new shareholders optimally raise the new debt and receive $TV_L^k(0)/V(0)$ of remaining asset value $(1-\alpha)V(\tau_L^k)$. By the use of Theorem 1, explicit solution for $D_L^k(0)$ is as follows.

$$D_L^k(0) = \int_0^T C_L^k \mathrm{e}^{-rs} \sum_{j=2}^{n} q_j^k(0,s)\, ds + \int_0^T \frac{TV_L^k(0)}{V(0)}(1-\alpha)F_L^k \mathrm{e}^{-rs} f_{\tau_L^k}(s) ds + F_L^k \mathrm{e}^{-rT} \sum_{j=2}^{n} q_j^k(0,T) \tag{8}$$

where

$$f_{\tau_L^k}(s) = \frac{\partial q_1^k(0,s)}{\partial s} = \left(\frac{D_2^k}{V(0)}\right)^{\chi} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(\theta_7(s))^2}{2}} \frac{\partial \theta_7(s)}{\partial s} - \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(\theta_8(s))^2}{2}} \frac{\partial \theta_8(s)}{\partial s} \tag{9}$$

denotes the probability density for firstly entering default state at time-s.

It is known that bankruptcy process consumes $\alpha V(\tau_L^k)$, and hence the present value of bankruptcy costs is

$$BC_L^k(0) = \mathrm{E}_{\mathrm{Q}}\left(\alpha\, V\left(\tau_L^k\right)\mathrm{e}^{-r\tau_L^k} 1_{\{\tau_l^k \le T\}} \middle| \mathcal{F}_0\right) \tag{10}$$

or

$$BC_L^k(0) = \int_0^T \alpha\, F_L^k \mathrm{e}^{-rs} f_{\tau_L^k}(s) ds \tag{11}$$

Notice that, in (8), the value of remaining asset we use is levered, but the value of lost assets used for computing bankruptcy cost is unlevered. This is because the value in (11) corresponds to the cost to the original stockholders before the firm is levered.

The interest tax shields of debt accrue to the firm as long as it does not go to bankruptcy. Thus the current value of tax benefits can be computed by

[17] For more details of refinancing, see Ju et al. (2005).

$$TB_L^k(0) = \mathrm{E}_{\mathrm{Q}}\left(\int_0^T \beta C_L^k \mathrm{e}^{-rs} 1_{\{\tau_L^k > s\}} ds \middle| \mathcal{F}_0 \right) \tag{12}$$

or

$$TB_L^k(0) = \int_0^T \beta C_L^k \mathrm{e}^{-rs} \sum_{j=2}^{n} q_j^k(0,s)\, ds$$

The equity's value equals total levered value of the firm less the value of debt, that is

$$E_L^k(0) = V(0) + TB_L^k(0) - BC_L^k(0) - D_L^k(0) \tag{13}$$

For the objective of maximizing equity value, one can differentiate equation (13) with respect to debt's face value, set this expression equal to zero, and then solve optimal debt ratio. In following subsection the impact on firm's optimal capital structure decision of target initial rating policy will be studied and compared with traditional tradeoff theory.

3.2 Impact of Targeting Initial Rating on Optimal Capital Structure Decision

The well-known tradeoff theory states that a value-maximizing firm will balance the benefits of debts against the costs of debts to determine optimal capital structure. Based on this spirit, here we combine the effect of target initial rating policy on traditional tradeoff benefits (i.e., the value equaling tax benefits minus bankruptcy costs) with the investigation into capital structure decision. At the beginning of analysis, some relevant parameters shall be selected. For simplicity, let economic -wide rating transition structure follow Table III. Assume that total assets' value of the unlevered reference firm is normalized to equal \$100 at initial time, which is divided among one hundred shares, each worth \$1. The dynamics of total assets' value has constant payout ratio $\delta = 3.75\%$ and volatility $\sigma = 38.02\%$. The maturity of debt sold by firm $T = 5$, the effective tax rate $\beta = 40\%$, the bankruptcy cost parameter $\alpha = 30\%$, the risk-free interest rate $r = 5\%$, and five-year rating-dependent average spread on corporate debt's coupons

$$\begin{aligned}\frac{C_L^k}{F_L^k} - r = {} & 1_{\{k=\mathrm{AAA}\}} 40\mathrm{bps} + 1_{\{k=\mathrm{AA}\}} 55\mathrm{bps} + 1_{\{k=\mathrm{A}\}} 120\mathrm{bps} + 1_{\{k=\mathrm{BBB}\}} 210\mathrm{bps} \\ & + 1_{\{k=\mathrm{BB}\}} 330\mathrm{bps} + 1_{\{k=\mathrm{B}\}} 470\mathrm{bps} + 1_{\{k=\mathrm{CCC}\}} 620\mathrm{bps}\end{aligned} \tag{14}$$ [18].

Examine the ratio of debt to total capital that maximizes total firm value for alternative choices of target initial credit ratings. Figure 2 and 3 respectively plot this relationship for firm issuing debt with investment-grade and speculative-grade initial ratings. The peak of each line segment in both figures implicates optimal choice of debt ratio under a corresponded target initial rating. When initially targeting an invest-

[18] The choices of average credit spreads are broadly referred to Huang and Huang (2003) and Moody's special report. The negative relation between average credit spreads and target ratings is consistent with empirical findings of most existing literatures, including Longstaff and Schwartz (1995), Huang and Huang (2003), Lando and Mortensen (2005), and Kisgen (2006b).

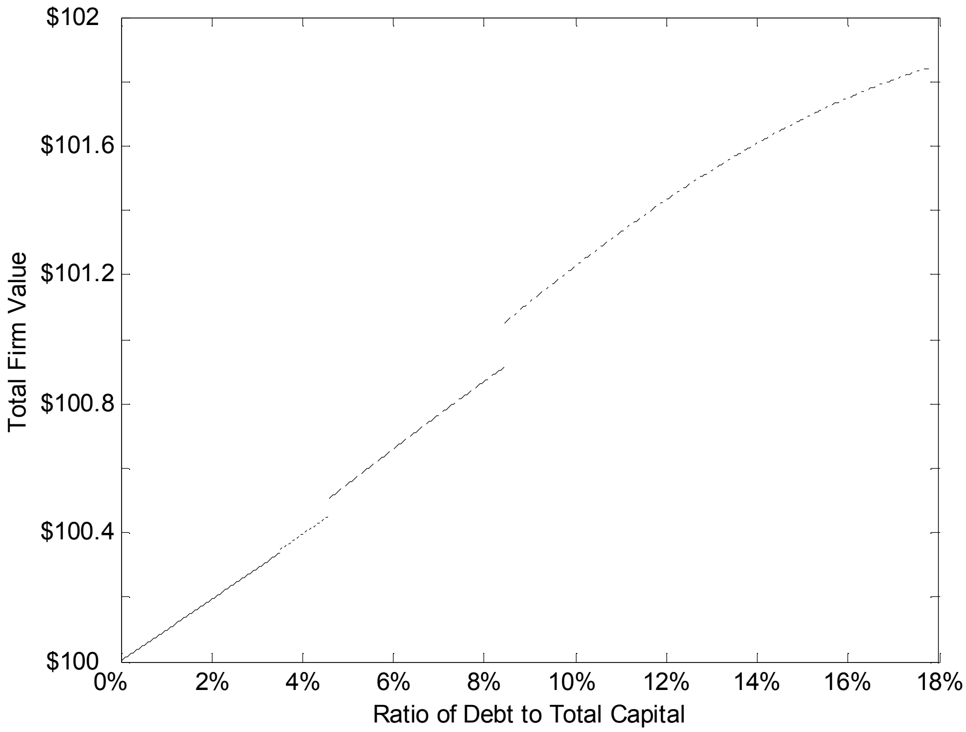

Figure 2. Total firm value as a function of ratio of debt to total capital with investment-grade target initial ratings. The lines plot total firm value at varying debt ratios for four different levels of target initial rating: AAA (*solid line*), AA (*dotted line*), A (*dashed line*), and BBB (*dash-dot line*). It is assumed that the risk-free interest rate $r = 5\%$, the firm's payout ratio $\delta = 3.75\%$, the effective tax rate $\beta = 40\%$, the bankruptcy costs $\alpha = 30\%$, the maturity of debt $T = 5$ years, the volatility of firm's assets $\sigma = 38.02\%$, and initial value of firm's underlying asset $V(0)$=$100. The average spread on debt's coupons is dependent on target initial rating, equaling 40 basis points for AAA, 55 basis points for AA, 120 basis points for A, and 210 basis points for BBB. Debt's par value and rating transition boundaries are jointly determined by target initial rating policy.

ment-grade or BB rating, the optimal debt ratio equals to the ratio of debt capacity to total capital, but such fact is reversed if managers target the rating at CCC. Among all choices of leverage in figures, the optimal choice under target rating B has maximal firm value, suggesting that this choice is also an optimum in case where the target rating policy is annulled. Compared to the case in the absence of policy, it is notable that all choices of leverage made by firms with policy of initially targeting at higher levels (i.e., investment grade or BB) are underleveraged, but those are over-leveraged when initially targeting at CCC level. Surprisingly, such result explains an important puzzle in corporate finance that why firms fairly use less leverage, despite larger tax benefits enjoyed by the debt (this has motivated much of researches including Leland (1998), Graham (2000), Graham et al. (2004), Ju et al. (2005), Kahle and Shastri (2005), and Molina (2005)). Briefly speaking, the reason that the firms appear to use debt so conservatively in the reality is to possess a better credit rating and to earn associated benefits.

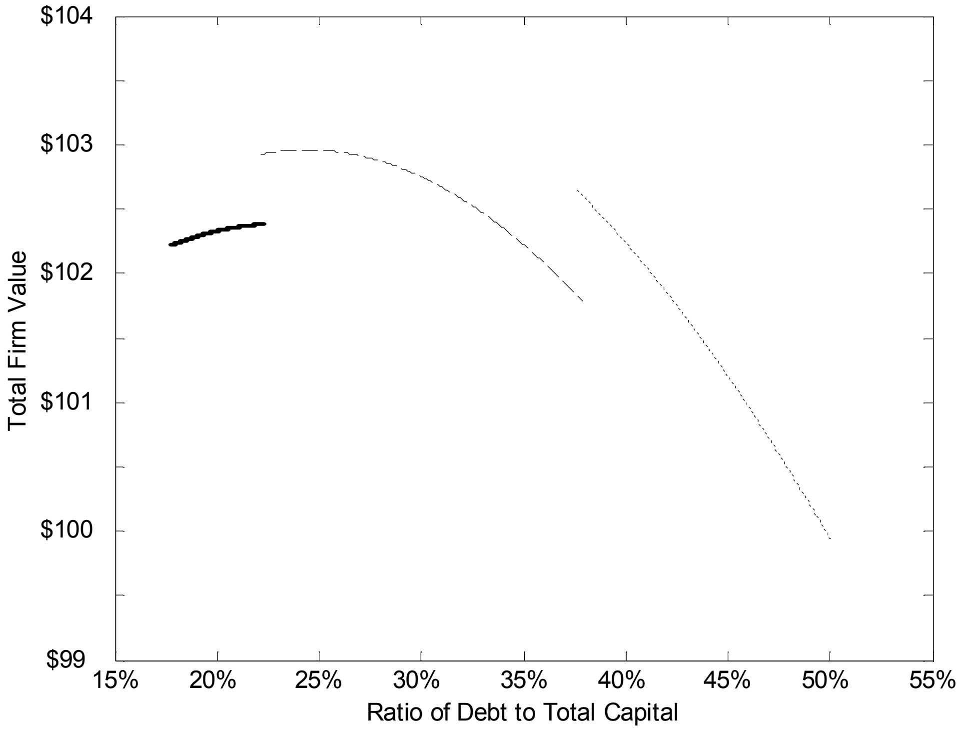

Figure 3. Total firm value as a function of ratio of debt to total capital with speculative-grade target initial ratings. The lines plot total firm value at varying debt ratios for three different levels of target initial rating: BB (*solid line*), B (*dashed line*), and CCC (*dotted line*). It is assumed that the risk-free interest rate $r = 5\%$, the firm's payout ratio $\delta = 3.75\%$, the effective tax rate $\beta = 40\%$, the bankruptcy costs $\alpha = 30\%$, the maturity of debt $T = 5$, the volatility of firm's assets $\sigma = 38.02\%$, and initial value of firm's underlying asset $V(0)$=$100. The average spread on debt's coupons is dependent on target initial rating, equaling 330 basis points for BB, 470 basis points for B, and 620 basis points for CCC. Debt's par value and rating transition boundaries are jointly determined by target initial rating policy.

In Figure 2 and 3 it is still noted that several jumps are appeared near the changes in target initial rating. The jumping sizes induced by the changes in target rating around investment-grade are smaller than those around junk. To make this clear, recall Kisgen's CR-CS. The hypothesis holds that different credit rating levels are associated with discrete benefits (costs) to the firm. If the benefits (costs) induced by the change in credit rating are material, managers will balance these benefits (costs) against traditional benefits (costs) when making capital decision. For incorporating CR-CS, here the effect of the changes in target rating is considered as discrete rating -dependent spreads on coupon rate. Thus these discrete effects explain the jumps observed in figures. The fact in which the increment in credit spread is increasing as target rating declines demonstrates the behavior of jumping sizes.

Table VI presents model outputs under different levels of target initial rating where the objective is to maximize the share value. The number of optimal capital structure is shown in Row 1. Optimal debt ratios are clearly very sensitive to initial target credit rating, equaling 3.5162% while targeting rating at AAA, and 37.6206%

Table VI

Model Outputs for Firm with/without Target Initial Rating Policy Where Objective Is to Maximize Share Value

This table shows model outputs for optimally levered firm issuing debt with target initial credit rating k ranging from AAA to CCC. Row I and II report the ratio of debt to total capital and of debt capacity to total capital respectively. The items associated with equity include market value of equity, changes in per share price, and numbers of outstanding share, which are reported by Row III, IV, and V respectively. The items associated with debt include market value of debt, bankruptcy costs, and tax benefits, which are reported by Row VI, VII, and VIII respectively. It is assumed that the risk-free interest rate $r=5\%$, the firm's payout ratio $\delta=3.75\%$, the effective tax rate $\beta=40\%$, the bankruptcy costs $\alpha=30\%$, the maturity of debt $T=5$ years, the volatility of firm's assets $\sigma=38.02\%$, and initial value of firm's underlying asset $V(0)$=\$100. The average spread on debt's coupons is dependent on target initial rating, equaling 40 basis points for AAA, 55 basis points for AA, 120 basis points for A, 210 basis points for BBB, 330 basis points for BB, 470 basis points for B, and 620 basis points for CCC. Rating transition boundaries are jointly determined by target initial rating policy and optimal capital structure choice.

	Target Initial Credit Ratings							
Variables	AAA	AA	A	BBB	BB	B	CCC	No Target
Debt/Total Capital	3.5162%	4.5792%	8.4721%	17.7566%	22.2560%	24.4334%	37.6206%	24.4334%
Debt Capacity/ Total Capital	3.5162%	4.5792%	8.4721%	17.7566%	22.2560%	37.9399%	49.7935%	49.7935%
Equity:								
Value of Equity w Debt (\$)	96.7467	95.7400	91.9333	82.4802	77.1868	73.9007	58.6708	73.9007
Change in Share Price (\$)	0.0034	0.0045	0.0092	0.0184	0.0238	0.0296	0.0265	0.0296
No. of Shares w Debt	96.4219	95.3105	91.0990	80.9899	75.3902	71.7760	57.1562	71.7760
Debt:								
Value of Debt (\$)	3.5902	4.7106	8.9825	19.3599	25.1963	29.0596	43.9792	29.0596
Bankruptcy Cost (\$)	0.0003	0.0011	0.0208	0.3863	0.8300	1.1303	3.8009	1.1303
Tax Benefits (\$)	0.3371	0.4517	0.9367	2.2264	3.2132	4.0905	6.4509	4.0905

while targeting rating at CCC. The negative relation between debt ratio and initial target rating is consistent with most empirical evidence, including, Huang and Huang (2003), Klock et al. (2005), Molina (2005), Kisgen (2006a), and Guttler and Wahrenburg (2007). Row 4 in Table VI reports the effect of leverage usage on the price of per share for each target rating level. Except for initially targeting the rating at B, the numbers of the changes in share price are always smaller than that without target rating policy. From the perspective of traditional tradeoff theory, this implicates that the capital decisions made with target initial rating policy seem inefficient, since such policy hampers the benefits to shareholders derived from leverage. If so, however, an interesting question is raised that why the mangers forego additional levered benefits to implement target rating policy. The answer may well lie in the idea behind CR-CS. According to the hypothesis, managers here will balance the advantages of higher credit ratings against the loss in abandoning traditional tradeoff benefits when making capital choice. Thus the firm makes under-levered choices to implement target initial rating policy only when the benefits from possessing better ratings dominate the costs of abandoning additional levered benefits. Unfortunately, our model does not allow for addressing the problem of jointly making optimal rating and leverage decision, and hence the tradeoff between the benefits associated with higher rating levels and the loss in abandonment of additional levered benefits is incapable to be examined.[19] Despite this lack, the account of firm forgoing a tradeoff-benefits-maximizing choice can still be inferred as the cognition that possessing better ratings may outweigh deriving more levered benefits in determining capital structure. Additionally, the change in per share price, equaling \$0.0265 when initially targeting at rating CCC, is less than that equaling \$0.0296 in the case of target initial rating B, which makes sense that firms in reality are reluctant to obey the capital policy of targeting an extremely low rating. We also discover that, under lower target ratings, the value of bankruptcy costs and tax benefits all become greater (see the Row 7 and 8). This is because larger leverage usage leads higher chance of bankruptcy, and tax shields are increasing with the spreads on coupon rate.

IV. Extended Model: Debt with Rating-Linked Coupon Scheme

Due to a strong linkage between the coupon and debt-holders' return, the level of coupon rate in intuitive sense shall reflect the extent of risk exposed by underlying debt (e.g., credit risk) as truly as possible. The spread between coupon and risk-less interest rate symbolizes the compensation for the risk debt-holders bear. Usually, the changes in risk-bearing are observed by tracking the transitions in credit rating. If so, the assumption in base case that the firm determines its coupon at a constant rate depended on initial rating is insufficient. Such doing considers current rating only but ignores the possibility of future rating's transition, since firm's credit can always improve or deteriorate during the life of debt. A larger number of examples of firms in practice issuing rating-linked coupon debts have been introduced by limited studies, including Das and Tufano (1996), Acharya et al. (2002), and Lando and Mortensen (2005). From the arguments of Manso et al. (2008), however, the idea of linking credit rating to debt's cash flow is inefficient in capital structure implication. Relative to fixed coupon debt with same market value, bankruptcy costs from rating-linked coupon debt are higher, and hence equity's initial market value is lower. This raises

[19] Due to this lack, the advantages from possessing better ratings cannot be adequately captured also when computing the changes in per share price. In this study shareholders' levered benefits hence depend on the tradeoff between tax benefits and bankruptcy costs only. However, since the consideration behind value-maximizing choice is more complicated in real world, traditional tradeoff benefit merely can explain a part of determinant of the benefits derived from leverage to shareholders. Thus the intuition behind optimal choice here does not reflect the entire reality.

another important twofold-issue related to our study that how the behavior as linking firms' rating to promised coupons impacts optimal capital structure decision, and why firms in practice issue rating-linked coupon debts. To address this issue, in Section 4 we extend the base model replacing the scheme of constant coupon with rating-linked coupon. Following, the joint impact on optimal decision of target initial rating policy and rating-linked coupon scheme will be examined also.

4.1 The model

Replicate the situation in base case to omit repeatedly introducing main economic setup. Thus the subscript f can be substituted for L in referring to quantities. Firstly restrict the attention on the most significant feature of this model- "rating-linked coupon scheme". While a firm exhibits the behavior as linking its credit to the promised coupons, interest payment carried by debt is no longer fixing, and would jump with the changes in credit rating during debt's lifespan. That is, for $0 \le s \le T$,

$$\begin{aligned} C_f^k(s) &= C^{\mathrm{AAA}} 1_{\{\eta^k(s)=\mathrm{AAA}\}} + C^{\mathrm{AA}} 1_{\{\eta^k(s)=\mathrm{AA}\}} + C^{\mathrm{A}} 1_{\{\eta^k(s)=\mathrm{A}\}} + C^{\mathrm{BBB}} 1_{\{\eta^k(s)=\mathrm{BBB}\}} \\ &\quad + C^{\mathrm{BB}} 1_{\{\eta^k(s)=\mathrm{BB}\}} + C^{\mathrm{B}} 1_{\{\eta^k(s)=\mathrm{B}\}} + C^{\mathrm{CCC}} 1_{\{\eta^k(s)=\mathrm{CCC}\}} \\ &= \sum_{j=2}^{n} C^j 1_{\{\eta^k(s)=j\}} \end{aligned} \tag{15}$$

Notice that there leaves no role for C^{D} to play in expression (15). This is because that debt's protective covenant may force the firm to go to bankruptcy once the rating falls to D level. For comparability, the spreads between coupon and risk-less interest rate here are assumed to be consistent with those in the base case

$$\begin{aligned} \frac{C_f^k(s)}{F_L^k} - r &= 1_{\{\eta^k(s)=\mathrm{AAA}\}} 40\mathrm{bps} + 1_{\{\eta^k(s)=\mathrm{AA}\}} 55\mathrm{bps} + 1_{\{\eta^k(s)=\mathrm{A}\}} 120\mathrm{bps} + 1_{\{\eta^k(s)=\mathrm{BBB}\}} 210\mathrm{bps} \\ &\quad + 1_{\{\eta^k(s)=\mathrm{BB}\}} 330\mathrm{bps} + 1_{\{\eta^k(s)=\mathrm{B}\}} 470\mathrm{bps} + 1_{\{\eta^k(s)=\mathrm{CCC}\}} 620\mathrm{bps}. \end{aligned} \tag{16}$$

As long as the firm receives an upgrade (downgrade) at any time during debt's lifespan, required spreads on coupon rate will instantly decrease (increase) to bring direct benefits (costs) associated with credit rating levels.

To show initial value of the debt paying rating-linked coupon, rewrite (7) by replacing C_L^k with (15)

$$\begin{aligned} D_f^k(0) = \mathrm{E_Q}\Bigg(&\sum_{j=2}^{n} \int_0^T C^j \mathrm{e}^{-rs} 1_{\{\tau_f^k > s\} \cap \{\eta^k(s)=j\}} ds + \frac{TV_f^k(0)}{V(0)} (1-\alpha) V\left(\tau_f^k\right) \mathrm{e}^{-r\tau_f^k} 1_{\{\tau_f^k \le T\}} \\ &+ F_f^k \mathrm{e}^{-rT} 1_{\{\tau_f^k > T\}} \Big| \mathcal{F}_0 \Bigg) \end{aligned} \tag{17}.$$

Comparing with base case, the mere discrepancy between (7) and (17) is appeared on the contribution from coupon payment. While the coupon rate no longer depends on initial rating only, the structure of coupon value becomes more complicated. Given different instantaneous ratings, coupon paid by the firm has individual present value. In pricing total contribution to the debt from coupon payment, these values shall be separately computed with corresponded coupon rate first, and then be summed up to yield the total value of rating-linked coupon. To further study the difference in the value of rating-linked and constant coupon, a closer inspection will be given in the following subsection. Making the use of Theorem 1 and (9), (17) can be solved as

$$\begin{aligned} D_f^k(0) &= \sum_{j=2}^{n}\int_0^T C^j \mathrm{e}^{-rs} q_j^k(0,s)\,ds + \int_0^T \frac{TV_f^k(0)}{V(0)}(1-\alpha)F_f^k \mathrm{e}^{-rs} f_{\tau_f^k}(s)\,ds \\ &\quad + F_f^k \mathrm{e}^{-rT}\sum_{j=2}^{n} q_j^k(0,T) \end{aligned}$$

Following the spirit of (10) and (12), the bankruptcy costs and the tax benefits in this model have respective current value

$$BC_f^k(0) = \mathrm{E}_{\mathrm{Q}}\left(\alpha V\left(\tau_f^k\right)\mathrm{e}^{-r\tau_f^k}\, 1_{\{\tau_f^k \le T\}} \,\middle|\, \mathcal{F}_0\right) \tag{18}$$

and

$$\begin{aligned} TB_f^k(0) &= \mathrm{E}_{\mathrm{Q}}\left(\int_0^T \beta\, C_f^k(\mathrm{s})\mathrm{e}^{-rs}\, 1_{\{\tau_f^k > s\}}\, ds \,\middle|\, \mathcal{F}_0\right) \\ &= \mathrm{E}_{\mathrm{Q}}\left(\sum_{j=2}^{n}\int_0^T \beta\, C^j\, \mathrm{e}^{-rs}\, 1_{\{\tau_f^k > s\}\cap\{\eta^k(s)=j\}}\, ds \,\middle|\, \mathcal{F}_0\right) \end{aligned} \tag{19}$$

Note that (18) is almost equivalent to (10). This is because there is not any influence on default-triggered protective covenant embedded with debt, no matter how rating transitions drive coupon rate. The value of interest tax shield becomes more sensitive to rating's migration, compared to (12). This reflects the fact that coupon in (19) is depending on instantaneous rating level, rather than fixing. Explicit solutions for (18) and (19) are given respectively as

$$BC_f^k(0) = \int_0^T \alpha\, F_f^k \mathrm{e}^{-rs} f_{\tau_f^k}(s)\,ds$$

and

$$TB_f^k(0) = \sum_{j=2}^{n}\int_0^T \beta\, C^j \mathrm{e}^{-rs} q_j^k(0,s)\,ds\,.$$

The equity has initial value equaling the unlevered value of the assets plus the value of tax benefits from the debt minus the bankruptcy costs minus the value of the debt,

$$E_f^k(0) = V(0) + TB_f^k(0) - BC_f^k(0) - D_f^k(0).$$

Table VII

Comparison between Outputs of Model with Rating-Linked/Constant Coupon scheme Where Objective Is to Maximize Share Value

This table shows the comparison between value-maximizing outputs of model with rating-linked/constant coupon scheme for firm targeting initial rating at a level *k* ranging from AAA to CCC. Column I and VI report the ratio of debt to total capital. Column II and VII report tax benefits. Column III and VIII report bankruptcy costs. Column IV and IX report the market value of debt. Column V and X report changes in per share price. It is assumed that the risk-free interest rate $r = 5\%$, the firm's payout ratio $\delta = 3.75\%$, the effective tax rate $\beta = 40\%$, the bankruptcy costs $\alpha = 30\%$, the maturity of debt $T = 5$ years, the volatility of firm's assets $\sigma = 38.02\%$, and initial value of firm's underlying asset *V*(0)=$100. The average spread on debt's coupons in the case of rating-linked/constant coupon scheme is dependent on instantaneous/initial rating, equaling 40 basis points for AAA, 55 basis points for AA, 120 basis points for A, 210 basis points for BBB, 330 basis points for BB, 470 basis points for B, and 620 basis points for CCC. Rating transition boundaries are jointly determined by target initial rating policy and optimal capital structure choice.

	The Case of Rating-Linked Coupon Scheme					The Case of Constant Coupon Scheme				
TICR	Debt/Total Capital	Tax Benefits ($)	Bankruptcy Costs ($)	Value of Debt ($)	Change in Share Price ($)	Debt/Total Capital	Tax Benefits ($)	Bankruptcy Costs ($)	Value of Debt ($)	Change in Share Price ($)
AAA	3.5158%	0.3476	0.0003	3.6165	0.0035	3.5162%	0.3371	0.0003	3.5902	0.0034
AA	4.5775%	0.4873	0.0011	4.7997	0.0049	4.5792%	0.4517	0.0011	4.7106	0.0045
A	8.4613%	1.0652	0.0208	9.3040	0.0104	8.4721%	0.9367	0.0208	8.9825	0.0092
BBB	17.7075%	2.5089	0.3863	20.0687	0.0212	17.7566%	2.2264	0.3863	19.3599	0.0184
BB	22.1800%	3.5653	0.8300	26.0836	0.0274	22.2560%	3.2132	0.8300	25.1963	0.0238
B	25.7608%	4.4500	1.3331	30.9361	0.0312	24.4334%	4.0905	1.1303	29.0596	0.0296
CCC	37.6325%	6.4182	3.8009	43.8945	0.0262	37.6206%	6.4509	3.8009	43.9792	0.0265
No Target	25.7608%	4.4500	1.3331	30.9361	0.0312	24.4334%	4.0905	1.1303	29.0596	0.0296

Note: ***TICR*** denotes target initial credit rating.

4.2 Impact of Linking Credit Rating to Coupons on Optimal Capital Structure Decision

In this subsection we jointly explore the issues that how the behavior as linking firm's rating to promised coupons impacts optimal capital structure decision, and why a firm in practice issues rating-linked coupon debt. Given the combination of model's parameters same with base case, comprehensive outputs of model with rating-linked /constant coupon scheme are compiled in Table VII. Observe that, along different target initial credit ratings, the relation of optimal debt rations between both cases is asymmetric. When the firm initially targets at investment-grade or BB level, optimal ratios of debt to total capital in the case of rating-linked coupon scheme are smaller than those with constant coupon scheme (see the contrast between numbers in Column 1 and 6). This will be reversed, however, if target initial rating is lower than BB. To explain this fact, we shall move the attention on the comparison of tax benefits and bankruptcy costs between both cases. Since the present values of coupon payment and tax benefits are strongly positive-correlated, an in-depth inspection to the comparison of the value of rating-linked/constant coupon is required and helpful.

Table VIII reports the structure of present value of rating-linked/constant coupon. It is apparent that the values in both cases are identical only when instantaneous rating is consistent with initial target. Otherwise, the values of rating-linked coupon are always larger (smaller), if instantaneous rating is inferior (superior) to its initial target when paying the coupon. Except for the case of targeting initial rating at CCC, rating-linked coupon payments have greater total present values usually. There are two seasons for explaining such results. First is the nonlinear negative relation between credit rating and required credit spreads; more clearly, the increment in spreads is increasing as the rating falls. Second is the asymmetry in the estimation of rating transition probability. Figure 4 and 5 provide clear checks to this asymmetry. In a joint view of these two figures, the probabilities of firm's credit remaining in ratings lower than initial target during debt's life are significantly greater, while those of remaining in ratings higher than initial target usually approximate to zero. This has an interesting implication that a levered firm is always more likely to receive downgrades than upgrades no matter what initial rating is targeted.[20] Summing up the above statements helps us easily clear why tax benefits of rating-linked coupon debt in Table VII (see Column 2) are commonly larger than the case of constant coupon scheme (see Column 7), excluding initially targeting at rating CCC.[21]

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[20] The result also makes a response to the argument in Kim and Nabar (2007) that rating agencies in practice may have asymmetric loss functions, and hence upgrades are not as timely as downgrades.

[21] The finding in Lando and Mortensen (2005) concludes that tax benefits of step-up bonds are lower than those of standard bonds, and is inconsistent with ours. Such discrepancy can be attributed to the conflict between assumption of reduced- and structural-form approach. The former, used by Lando and Mortensen, usually assumes that the default is irrelevant with capital structure. In the spirit of our models, however, the default is jointly determined by asset value and capital structure.

Table VIII

Comparison between Structure of Present Value of Rating-Linked/Constant Coupon Payment

This table shows comparison between the structure of present value of rating-linked/constant coupon payment. It is assumed that the risk-free interest rate $r = 5\%$, the firm's payout ratio $\delta = 3.75\%$, the maturity of debt $T = 5$ years, the volatility of firm's assets $\sigma = 38.02\%$, and initial value of firm's underlying asset $V(0)$=$100. The average spread on debt's coupons in the case of rating-linked/constant coupon scheme is dependent on instantaneous/initial rating, equaling 40 basis points for AAA, 55 basis points for AA, 120 basis points for A, 210 basis points for BBB, 330 basis points for BB, 470 basis points for B, and 620 basis points for CCC. Rating transition boundaries are jointly determined by target initial rating policy and optimal capital structure choice.

	Ratings Kept by a Firm When Paying the Coupon							
	AAA		AA		A		BBB	
TICR	Value of Rating-Linked Coupon ($)	Value of Constant Coupon ($)	Value of Rating-Linked Coupon ($)	Value of Constant Coupon ($)	Value of Rating-Linked Coupon ($)	Value of Constant Coupon ($)	Value of Rating-Linked Coupon ($)	Value of Constant Coupon ($)
AAA	3.4566E-01	3.4566E-01	4.2622E-01	4.1470E-01	7.8865E-02	6.8689E-02	1.6653E-02	1.2665E-02
AA	3.2953E-06	3.3869E-06	4.6320E-01	4.6320E-01	6.7948E-01	6.0824E-01	6.5483E-02	5.1187E-02
A	3.5326E-07	4.0559E-07	3.1569E-05	3.5266E-05	9.6173E-01	9.6173E-01	1.1047E+00	9.6464E-01
BBB	1.5382E-07	2.0225E-07	5.6677E-07	7.2505E-07	3.5236E-04	4.0351E-04	2.3291E+00	2.3291E+00
BB	9.8163E-09	1.5088E-08	5.0225E-08	7.5111E-08	2.9631E-05	3.9667E-05	4.4749E-03	5.2312E-03
B	4.4068E-07	1.0537E-06	1.2898E-05	2.8844E-05	7.2314E-04	1.3789E-03	9.0634E-03	1.4551E-02
CCC	9.7054E-08	2.0130E-07	7.6006E-07	1.5338E-06	1.1407E-04	2.0607E-04	1.2116E-03	1.9113E-03
	BB		B		CCC		Total	
TICR	Value of Rating-Linked Coupon ($)	Value of Constant Coupon ($)	Value of Rating-Linked Coupon ($)	Value of Constant Coupon ($)	Value of Rating-Linked Coupon ($)	Value of Constant Coupon ($)	Value of Rating-Linked Coupon ($)	Value of Constant Coupon ($)
AAA	1.4932E-03	9.7146E-04	1.8106E-04	1.0080E-04	2.3455E-05	1.1309E-05	8.6910E-01	8.4280E-01
AA	8.9184E-03	5.9635E-03	1.0340E-03	5.9164E-04	1.8435E-04	9.1350E-05	1.2183E+00	1.1293E+00
A	3.4445E-01	2.5730E-01	2.1410E-01	1.3685E-01	3.8130E-02	2.1108E-02	2.6631E+00	2.3417E+00
BBB	2.9956E+00	2.5625E+00	7.5121E-01	5.4985E-01	1.9594E-01	1.2421E-01	6.2722E+00	5.5661E+00
BB	3.4260E+00	3.4260E+00	4.7007E+00	4.0222E+00	7.8197E-01	5.7950E-01	8.9131E+00	8.0330E+00
B	3.0739E-02	4.1237E-02	7.8595E+00	7.7610E+00	3.2250E+00	2.4082E+00	1.1125E+01	1.0226E+01
CCC	8.7016E-03	1.1742E-02	5.0383E-01	5.8174E-01	1.5532E+01	1.5532E+01	1.6045E+01	1.6127E+01

Note: ***TICR*** denotes target initial credit rating.

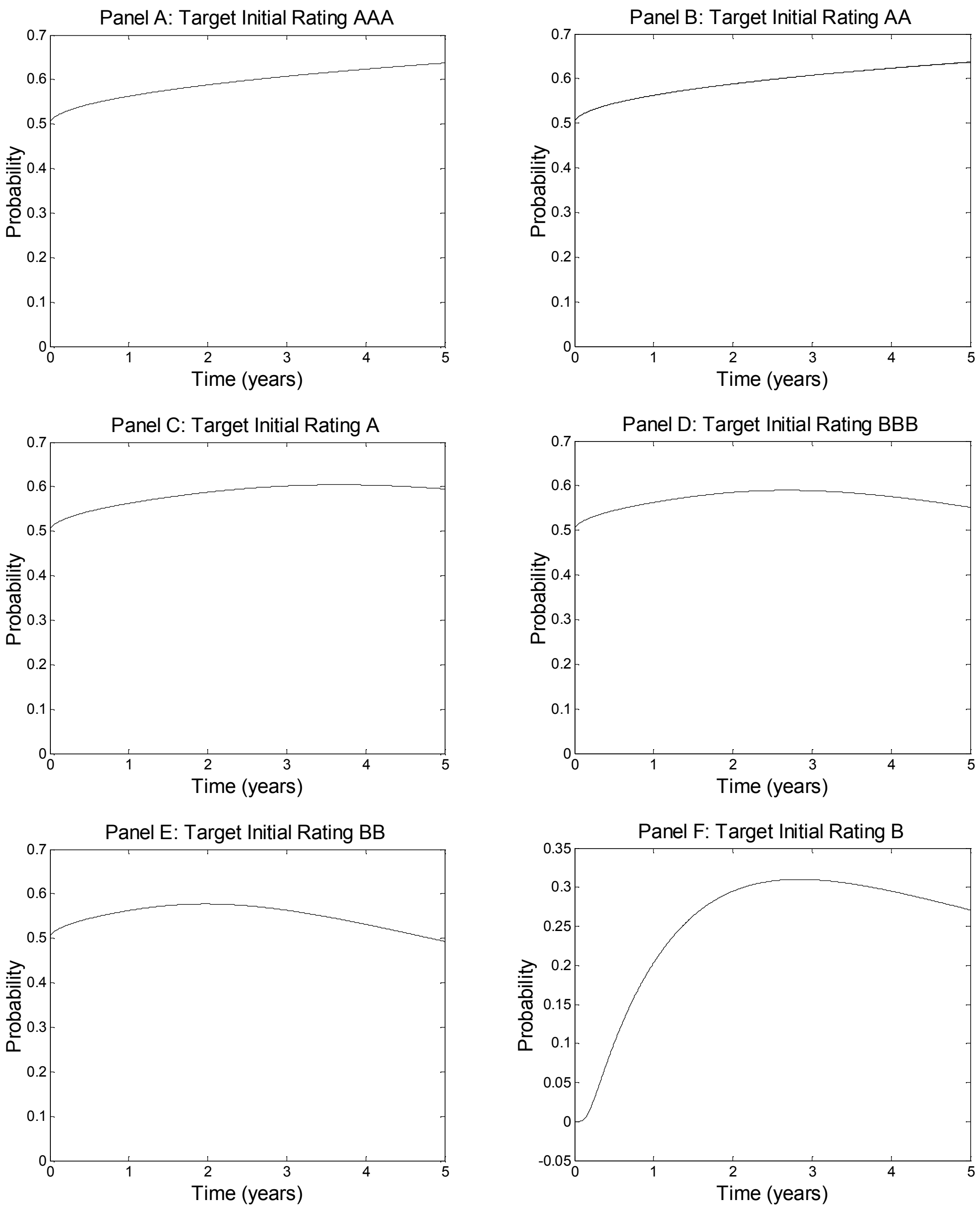

Figure 4. Probability of firm's credit remaining in the ratings lower than initial target as a function of time. The plots examine the probabilities of firm's credit remaining in the ratings lower than initial target as a function of time with target initial rating AAA (panel A), AA (panel B), A (panel C), BBB (panel D), BB (panel E), and B (panel F) respectively. It is assumed that the risk-free interest rate $r = 5\%$, the firm's payout ratio $\delta = 3.75\%$, the effective tax rate $\beta = 40\%$, the bankruptcy costs $\alpha = 30\%$, the maturity of debt $T = 5$, the volatility of firm's assets $\sigma = 38.02\%$, and initial value of firm's underlying asset *V*(0)=$100. Rating transition boundaries are jointly determined by optimal capital structure and target initial rating policy.

The value of bankruptcy costs in the case with rating-linked/constant coupon scheme are shown in Column 3 and 8 of Table VII respectively, and appeared to be almost alike. This phenomenon, which interlinks preceding result reported by Table VI, implies that in both cases not only optimal debt issuing amount managers choose are identical, but also the firm may exhibit underleveraged or overleveraged behavior.

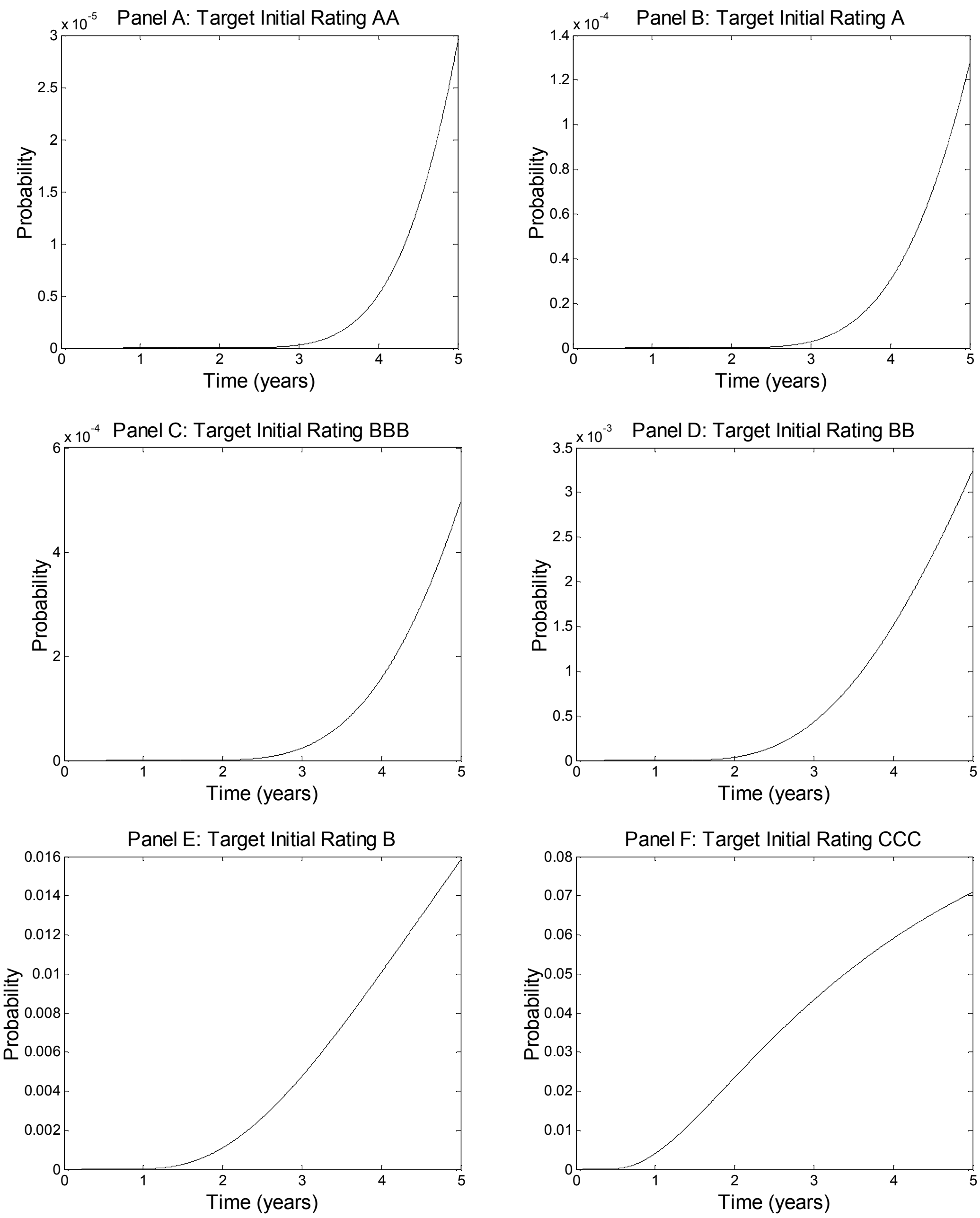

Figure 5. Probability of firm's credit remaining in the ratings higher than initial target as a function of time. The plots examine the probabilities of firm's credit remaining in the ratings higher than initial target as a function of time with target initial rating AA (panel A), A (panel B), BBB (panel C), BB (panel D), B (panel E), and CCC (panel F) respectively. It is assumed that the risk-free interest rate $r = 5\%$, the firm's payout ratio $\delta = 3.75\%$, the effective tax rate $\beta = 40\%$, the bankruptcy costs $\alpha = 30\%$, the maturity of debt $T = 5$, the volatility of firm's assets $\sigma = 38.02\%$, and initial value of firm's underlying asset $V(0)$=\$100. Rating transition boundaries are jointly determined by optimal capital structure and target initial rating policy.

Surprisingly, the outlook of Manso et al. (2008) on bankruptcy costs is inconsistent with ours. That article shows that the default time of the issuer of performance-sensitive debt is earlier than that of fixed-coupon debt with same face value, and hence the former has greater bankruptcy costs. The cause to this difference comes from two sources. (i) They treat bankruptcy as an endogenous decision of the firm, following unprotected debt case in Leland (1994). The bankruptcy-triggered boundary in that article is endogenously chosen for the objective of maximizing share value, rather than exogenously set as debt's face value (protected debt case in Leland (1994)). (ii) They assume that the principal of debt sold by the firm will not be repaid forever (i.e., consol bond). Thus there leaves no role for debt's face value to play in the valuation of bankruptcy costs. Altogether, in endogenous bankruptcy case, the firm chooses a higher default-triggered boundary for rating-linked coupon debt, compared to constant coupon debt with same par. But this boundary is indifferent for both debts in exogenous bankruptcy case. Because the value of bankruptcy costs is an increasing function of default-triggered boundary, this has clarified the inconsistency.

In addition, a notable exception arises in the case where target initial rating policy is annulled (or the case that the firm targets initial rating at B). Under this special situation, the value of bankruptcy costs of debt with rating-linked coupon scheme equaling \$1.3331 is larger than that with constant coupon scheme equaling \$1.1303, suggesting that the optimal usage of debt is no longer indifferent in both cases. This is due to the fact that constraints on debt usage are released in the absence of target initial rating policy. Hence optimal choice of debt issuing amount depends on the tradeoff between tax benefits and bankruptcy costs. Based on tradeoff theory, the firm in such case may use more rating-linked coupon debt relative to standard debt, because of the cognition that the former commonly yields bigger tax benefits under same face value and bankruptcy loss.

Lump the analysis to tax benefits and bankruptcy costs together to have the explanation for the pattern of optimal debt ratios. When making underleveraged choices to target initial rating at higher levels, the behavior as linking credit rating to the promised coupons aids the firm in obtaining a greater total levered value and a smaller optimal debt ratio via increasing tax benefits. On the other hand, if managers target an extremely low rating and makes overleveraged choices, tax benefits from rating-linked coupon debt are smaller than the base case, since the firm paying rating-linked coupon may experience a downward jump in coupon payment. Thus this yields a larger optimal ratio of debt to total capital.

To clear why a firm in practice issue rating-linked coupon debt, we have the co-focus on the market value of debt (numbers in Column 4 and 9) and the changes in per share price due to leverage (numbers in Column 5 and 10) reported by Table VII. Observe that the debt with rating-linked coupon scheme is more valuable than standard debt with same face value, except for the case of target initial rating CCC. Similarly, the levered benefits to shareholders derived from issuing rating-linked coupon debt are usually greater than those from issuing constant coupon debt. The intuition behind these facts implicates that, unless the firm is overleveraged and holds a too low credit rating, rating-linked coupon scheme can not only allow debt-holders to earn more coupons but also benefit shareholders by additional tax shields. Loosely speaking, as long as underlying credit quality at the time of issuance is not too bad, using rating-linked coupon debt in general is truly more beneficial to the firm.

V. Extended Model: Debt with Rating-Dependent Callable Option

Start from the point to credit rating effect on the allowance of investing in corporate debts. Regulations based on ratings in practice determine whether certain investor groups can purchase risky bonds (see Cantor and Packer (1994) and SEC (2003)). For example, banks have already been restricted from holding the bonds with speculative-grade ratings since 1936. In 1989, savings and loans were prohibited from investing in junk bonds also. Due to these facts, Kisgen (2006b) argues that, in reality, maintaining a particular rating level provides benefits to firms. Thus managers are motivated to target minimum credit ratings at which the regulations affect investments in a firm's bonds, such as targeting the investment-grade level or minimum B ratings. Following rating downgrades, firms attempt reducing capital market activity and leverage ratio to regain their previous better rating. A similar argument by Leary and Roberts (2005) also holds that firms will adjust their leverage ratio closer to a target in managing credit ratings. To study the impact of these interesting behaviors on capital decision, this section builds an extended model that allows for considering a rating-dependent callable option on the debt. In particular, this option is restricted to be effective only when firm's credit has already fallen below target minimum rating. Through repurchase activities induced by the exercise of callable option, the firm can reduce the leverage and regain previous higher rating temporarily.

5.1 The model

Following the economic setup in section 3, replace subscript L with R to refer to quantities of this model. Consider now the case in which the reference firm has a policy of jointly targeting initial rating at k, consistent with long-term target, and minimum rating at m where $m \le k$. Under this policy, managers are gifted with a right to restructure firm's capital; namely, one-time callable option on the debt. The option is rating-dependent and can be exercised only if firm's rating fails to remain in the target range $[m, n]$ during the life of debt. To manage credit rating, managers will undertake a debt repurchase activity instantly at the time, which debt's rating firstly enters into a state lower than target minimum rating (e.g., state $m-1$),

$$\begin{aligned} \psi_m^k &\equiv \inf\left\{ \eta^k(s) = m-1 \;\middle|\; n \ge k \ge m > 2 \,,\; s \in [0,T] \right\} \\ &\equiv \inf\left\{ V(s) \le D_m^k \;\middle|\; n \ge k \ge m > 2 \,,\; s \in [0,T] \right\}. \end{aligned} \tag{20}$$

In making repurchase activity, managers will retire the outstanding debt at par value.[22] The size of repurchase, $\rho \in [0,1]$, is predetermined on the goal of regaining a rating consistent with their initial target.[23] After repurchase process, firm's default-triggered threshold drops from F_R^k to $(1-\rho)F_R^k$, equaling the par value of remaining debt. The corresponding rating transition boundaries fall by the same factor to become $(1-\rho)D_1^k,\cdots,(1-\rho)D_8^k$. Since $(1-\rho)F_R^k = (1-\rho)D_2^k < D_m^k$, firm's default time should never be earlier than the repurchase time, and hence has following expression

$$\begin{aligned}\tau_R^{k,m} &\equiv \inf\left\{ \eta^k(s)=1 \;\middle|\; n \ge k \ge m > 2\,,\; s \in (\psi_m^k, T\,] \right\} \\ &\equiv \inf\left\{ V(s) \le (1-\rho)F_R^k \;\middle|\; n \ge k \ge m > 2\,,\; s \in [\,0, T\,] \right\}.\end{aligned} \tag{21}$$

To examine (20) and (21), Theorem 1 can be simplified as below.

COROLLARY 1: Conditioned on initial credit rating k, the total probability of repurchase from time zero to s is, for $0 \le s \le T$,

$$\begin{aligned}\mathrm{Q}\left(\psi_m^k \le s \mid \mathcal{F}_0\right) &\equiv \mathrm{Q}\left(\min_{0 \le u \le s} \eta^k(u) \le m-1 \;\middle|\; \eta^k(0)\right) \equiv 1 - q_\psi^{k,m}(0,s) \\ &= 1 + \left(\frac{D_m^k}{V(0)}\right)^{\chi} \mathrm{N}(\theta_9) - \mathrm{N}(\theta_{10})\end{aligned} \tag{22}$$

where

$$\theta_9 = \frac{1}{\sigma\sqrt{s}}\left[\ln\left(D_m^k\right) - \ln\left(V(0)\right) + \lambda s\right]$$

$$\theta_{10} = \frac{1}{\sigma\sqrt{s}}\left[\ln\left(V(0)\right) - \ln\left(D_m^k\right) + \lambda s\,\right].$$

Differentiating (22) with respect to s gives the probability density of the occurrence of repurchase at time-s

$$f_{\psi_m^k}(s) = -\frac{\partial q_\psi^{k,m}(0,s)}{\partial s} = \left(\frac{D_m^k}{V(0)}\right)^{\chi} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(\theta_9(s))^2}{2}} \frac{\partial \theta_9(s)}{\partial s} - \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(\theta_{10}(s))^2}{2}} \frac{\partial \theta_{10}(s)}{\partial s}\,.$$

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[22] The idea of restructuring the capital through debt repurchase has received some attentions from Leland (1994, 1998) and Goldstein et al. (2001). In the light of Leland (1998), "capital restructuring point" is considers as the time, which the debt is fully retired and replaced by newly issued debt due to firm assets' value hitting a predetermined threshold. His model allows an unlimited number of the occurrences of capital restructuring. Subsequent debt issuance at each restructuring points incurs a fractional cost of the principal issued. In conjunction with Leland's notion, here the role of repurchase-triggered time (as expression (20)) can be served as a single capital restructuring point. Since only one chance of adjusting capital structure is left for managers during trading period, the costs of capital restructuring and debt repurchase are ignored in this study.

[23] Making such assumption helps us capture an important idea from empirical evidence that firms in reality will have a "strict" target debt ratio, and appear to return their leverage to this target in long-run (see Graham and Harvey (2001), Fama and French (2002), and Leary and Roberts (2005)).

COROLLARY 2: Conditioned on initial credit rating k, the total probability of default from time zero to s is, for $0 \le s \le T$,

$$Q\left(\tau_R^{k,m} \le s \mid \mathcal{F}_0\right) \equiv 1 - q_{\tau_R}^{k,m}(0,s) = 1 + \left\{\frac{(1-\rho)F_R^k}{V(0)}\right\}^{\chi} \mathrm{N}(\theta_{11}) - \mathrm{N}(\theta_{12}) \tag{23}$$

where

$$\theta_{11} = \frac{1}{\sigma\sqrt{s}}\left[\ln\left((1-\rho)F_R^k\right) - \ln(V(0)) + \lambda s\right],$$

$$\theta_{12} = \frac{1}{\sigma\sqrt{s}}\left[\ln(V(0)) - \ln\left((1-\rho)F_R^k\right) + \lambda s\right].$$

The probability density of the occurrence of bankruptcy at time-s can be yielded via differentiating (23) with respect to s, that is

$$f_{\tau_R^{k,m}}(s) = -\frac{\partial q_{\tau_R}^{k,m}(0,s)}{\partial s} = \left\{\frac{(1-\rho)F_R^k}{V(0)}\right\}^{\chi} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(\theta_{11}(s))^2}{2}} \frac{\partial\theta_{11}(s)}{\partial s} - \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(\theta_{12}(s))^2}{2}} \frac{\partial\theta_{12}(s)}{\partial s}$$

Now incorporate rating-dependent callable option into the pricing of contingent claims associated with this model. Firstly, the current value of debt is shown as

$$\begin{aligned} D_R^{k,m}(0) = \mathrm{E}_Q\Big(& \int_0^T (1-\rho)C_R^k \mathrm{e}^{-rs} 1_{\{\tau_R^{k,m} > s \ge \psi_m^k\}} ds + \int_0^T C_R^k \mathrm{e}^{-rs} 1_{\{\tau_R^{k,m} > s,\, \psi_m^k > s\}} ds \\ & + F_R^k \mathrm{e}^{-rT} 1_{\{\tau_R^{k,m} > T,\, \psi_m^k > T\}} + \rho F_R^k \mathrm{e}^{-r\psi_m^k} 1_{\{\psi_m^k \le T\}} + (1-\rho)F_R^k \mathrm{e}^{-rT} 1_{\{\tau_R^{k,m} > T \ge \psi_m^k\}} \\ & + \frac{TV_R^k(0)}{V(0)}(1-\alpha)V\left(\tau_R^{k,m}\right)\mathrm{e}^{-r\tau_R^{k,m}} 1_{\{\tau_R^{k,m} \le T\}} \Big| \mathcal{F}_0\Big) \end{aligned}$$

(24).

The implication behind the structure of (24) is sketched as follows. Due to repurchase effect on debt's market value, the total contribution from coupon payment consists of two parts:

(i) $\int_0^T (1-\rho)C_R^k \mathrm{e}^{-rs} 1_{\{\tau_R^{k,m} > s \ge \psi_m^k\}} ds$,

which indicates the contribution from remaining coupon after repurchase process, and

(ii) $\int_0^T C_R^k \mathrm{e}^{-rs} 1_{\{\tau_R^{k,m} > s,\, \psi_m^k > s\}} ds$,

which denotes the contribution from full coupon conditioned on the callable option not being exercised yet. Similarly, the principal is repaid to debt-holders under three different conditions; (i) If credit rating remains in target range during the whole period, debt-holders will receive full par F_R^k at debt's maturity. (ii) During the repurchase process, managers raise fund ρF_R^k to repay debt-holders. (iii) If the callable option has been exercised but bankruptcy does not occur until to maturity, the par value of remaining debt $(1-\rho)F_R^k$ will be refunded at expiring date. Lastly, upon bankruptcy, the amount $\left[TV_R^k(0)/V(0)\right](1-\alpha)V\left(\tau_R^{k,m}\right)$ represents the contribution from recovery payment to debt-holders. Using Theorem 1, Corollary 1 and Corollary 2, we have the explicit solution of (24)

$$\begin{aligned} D_R^{k,m}(0) =& \int_0^T C_R^k \mathrm{e}^{-rs} q_\psi^{k,m}(0,s)ds + F_R^k \mathrm{e}^{-rT} q_\psi^{k,m}(0,T) + \int_0^T (1-\rho) C_R^k \mathrm{e}^{-rs}\left[1-q_\psi^{k,m}(0,s)\right]ds \\ & -\int_0^T (1-\rho) C_R^k \mathrm{e}^{-rs}\left[1-q_{\tau_R}^{k,m}(0,s)\right]ds + (1-\rho)F_R^k \mathrm{e}^{-rT}\left[1-q_\psi^{k,m}(0,T)\right] \\ & -(1-\rho)F_R^k \mathrm{e}^{-rT}\left[1-q_{\tau_R}^{k,m}(0,T)\right] + \int_0^T \rho F_R^k \mathrm{e}^{-rs} f_{\psi_m^k}(s)ds \\ & + \int_0^T \frac{TV_R^k(0)}{V(0)}(1-\alpha)(1-\rho)F_R^k \mathrm{e}^{-rs} f_{\tau_R^{k,m}}(s)ds \end{aligned} \tag{25}$$

Following consider the value of bankruptcy costs and tax benefits. From the implication of (21), it is obviously that the repurchase activity is always prior to firm's default. Hence, after debt repurchase, bankruptcy-triggered boundary shrinks to equal the par of remaining debt, meaning that $V\left(\tau_R^{k,m}\right) = (1-\rho)F_R^k$. Similar to (10), here the present value of bankruptcy costs is

$$\begin{aligned} BC_R^{k,m}(0) &= \mathrm{E_Q}\left(\alpha V\left(\tau_R^{k,m}\right)\mathrm{e}^{-r\tau_R^{k,m}} 1_{\{\tau_R^{k,m} \le T\}} \mid \mathcal{F}_0 \right) \\ &= \int_0^T \alpha (1-\rho) F_R^k \mathrm{e}^{-rs} f_{\tau_R^{k,m}}(s)ds \end{aligned}$$

On the other hand, in computing the value of tax benefits, it should be stressed that the coupon paid by the firm before/after debt repurchase is different, since a part of payment will be cut after repurchase activity. In this model initial total value of tax shields contains two parts: the part accrues to the firm before debt repurchase, and the part accrues after repurchase. That means,

$$\begin{aligned} TB_R^{k,m}(0) &= \mathrm{E_Q}\left(\beta \int_0^T C_R^k \mathrm{e}^{-rs} 1_{\{\tau_R^{k,m} > s,\, \psi_m^k > s\}} ds + \beta \int_0^T (1-\rho) C_R^k \mathrm{e}^{-rs} 1_{\{\tau_R^{k,m} > s \ge \psi_m^k\}} ds \mid \mathcal{F}_0 \right) \\ &= \beta \left\{ \int_0^T C_R^k \mathrm{e}^{-rs} q_\psi^{k,m}(0,s)ds + \int_0^T (1-\rho) C_R^k \mathrm{e}^{-rs}\left[1-q_\psi^{k,m}(0,s)\right]ds \right. \\ & \left. \quad - \int_0^T (1-\rho) C_R^k \mathrm{e}^{-rs}\left[1-q_{\tau_R}^{k,m}(0,s)\right]ds \right\} \end{aligned}$$

Similar to (13), the equity's value reflects four terms: the firm's asset value, plus the value of tax benefits, less the value of bankruptcy costs, less the value of debt

$$E_R^{k,m}(0) = V(0) + TB_R^{k,m}(0) - BC_R^{k,m}(0) - D_R^{k,m}(0).$$

5.2 Impact of Targeting Minimum Rating on Optimal Capital Structure Decision

This subsection focuses on investigating the impact of the behavior as targeting minimum rating on optimal capital structure decision. To simplify our analysis, let the reference firm target minimum rating at BBB (minimum investment-grade policy),[24] and also suppose that the given combination of model parameters is identical with that in the base case.

In the setting of this section, it is known that debt repurchase induced by the exercise of rating-dependent callable option determines the impact of target minimum rating policy. For example, the degree of improvements in firm's credit rating and movements in leverage are jointly relied on the size of repurchase activity. To have a closer inspection to this impact, we compile the outputs of model with/without minimum investment-grade policy as Table IX.[25] The numbers of repurchase ratio are shown in Column 2. The repurchase ratios both are decreasing with the distance between initial rating and minimum rating targeted by firm in Panel B and C. Interestingly, the intuition behind them is different. In Panel B (repurchase ratios are determined on the objective of maximizing share value), the result makes sense that, as the size of the departure from firm's target minimum rating is bigger, additional tradeoff benefits from restructuring the capital become higher, and hence managers are motivated to undertake a repurchase activity with larger scale. The repurchase ratios, however, are determined on the objective of regaining target initial rating in Panel C. The longer the distance between target initial rating and minimum rating is, the more the improvement in firm's credit is required via repurchase. Since the size of repurchase is positively related to the rating level regained by firm, this clarifies the pattern of repurchase ratios in Panel C.

Column 6 in Table IX reports the effect of leverage on per share price for varying distance between target initial rating and minimum rating. Relative to numbers shown in the base case (Panel A), numbers with repurchase ratios derived from traditional tradeoff theory (Panel B) seem larger slightly, while those with repurchase ratios derived from target initial rating policy (Panel C) are smaller. To further explore this point, the depiction of tradeoff theory combined with repurchase effect is exhibited in Figure 6. The peak of each line in this figure implicates the choice of repurchase ratio that maximizes percentage change in tradeoff benefits (i.e., maximizes share value). Hence the values of optimal repurchase ratio here are coincident with the numbers reported in Column 2 of panel B of Table IX, equaling 13.4165% in Panel A, 4.5829% in Panel B, and 0.0000% in Panel C and D. Still notice that the improvements in firm's credit quality via repurchase are usually very slight in case where the ratios of repurchase are chosen for maximizing share value (see Column 3 of Panel B of Table

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[24] Johnson (2003) documents that firms receiving a downgrade from BBB will "travel" more categories than other rating levels, implying that firms downgraded from this rating can often be downgraded to BB or lower level. Kisgen (2006a) also finds that managers appear to be most concerned with ratings changes from investment-grade to junk.

[25] To compare with the choices implied by traditional tradeoff theory, we still consider a case in which managers determine repurchase ratio on the objective of maximizing tradeoff benefits (see Panel B).

Table IX

Outputs of Model with/without Minimum Investment-Grade Policy Where the Objective Is to Maximize Share Value

This table shows value-maximizing outputs of model with/without minimum investment-grade policy for firm targeting initial rating at a level ranging from AAA to BBB. The repurchase ratio is respectively determined on the objective of maximizing share value and of regaining target initial rating in panel B and C. Column I reports the ratio of debt to total capital. Column II reports the repurchase ratio. Column III reports the rating regained by firm via repurchase. Column IV reports tax benefits. Column V reports bankruptcy costs. Column VI reports the change in share price. It is assumed that the risk-free interest rate $r = 5\%$, the firm's payout ratio $\delta = 3.75\%$, the effective tax rate $\beta = 40\%$, the bankruptcy costs $\alpha = 30\%$, the maturity of debt $T = 5$ years, the volatility of firm's assets $\sigma = 38.02\%$, and initial value of firm's underlying asset $V(0)$=$100. The average spread on debt's coupons is dependent on target initial rating, equaling 40 basis points for AAA, 55 basis points for AA, 120 basis points for A, and 210 basis points for BBB. Rating transition boundaries are jointly determined by target initial rating policy and optimal capital structure choice.

Panel A: The base case

TICR	Debt/Total Capital	Repurchase Ratio	Regained Rating via Repurchase	Tax Benefits ($)	Bankruptcy Costs ($)	Change in Share Price ($)
AAA	3.516156%	N/A	N/A	0.337116	2.6646E-04	3.3685E-03
AA	4.579165%	N/A	N/A	0.451711	1.0943E-03	4.5062E-03
A	8.472111%	N/A	N/A	0.936665	2.0848E-02	9.1582E-03
BBB	17.756551%	N/A	N/A	2.226419	3.8626E-01	1.8402E-02

Panel B: The case where repurchase ratio is determined on the objective of maximizing share value

TICR	Debt/Total Capital	Repurchase Ratio	Regained Rating via Repurchase	Tax Benefits ($)	Bankruptcy Costs ($)	Change in Share Price ($)
AAA	3.516155%	13.4165%	BBB	0.337016	1.1906E-04	3.3690E-03
AA	4.579164%	4.5829%	BBB	0.451498	8.5810E-04	4.5064E-03
A	8.472111%	0.0000%	N/A	0.936665	2.0848E-02	9.1582E-03
BBB	17.756551%	0.0000%	N/A	2.226419	3.8626E-01	1.8402E-02

Panel C: The case where repurchase ratio is determined on the objective of regaining ***TICR***

TICR	Debt/Total Capital	Repurchase Ratio	Regained Rating via Repurchase	Tax Benefits ($)	Bankruptcy Costs ($)	Change in Share Price ($)
AAA	3.516172%	90.2567%	AAA	0.336397	2.0021E-11	3.3640E-03
AA	4.579304%	85.0798%	AA	0.447589	5.5743E-09	4.4759E-03
A	8.481949%	47.4028%	A	0.799737	9.7357E-04	7.9876E-03
BBB	17.756551%	>0.0000%*	BBB	2.226419	3.8626E-01	1.8402E-02

Note: ***TICR*** denotes target initial credit rating.

* When a firm initially targets rating at BBB level, there is no distance between initial and minimum target for credit rating. Since our system is operated in continuous time, managers can immediately regain BBB by the exercise of callable option following a rating downgrade, even if repurchase ratio is fairly small.

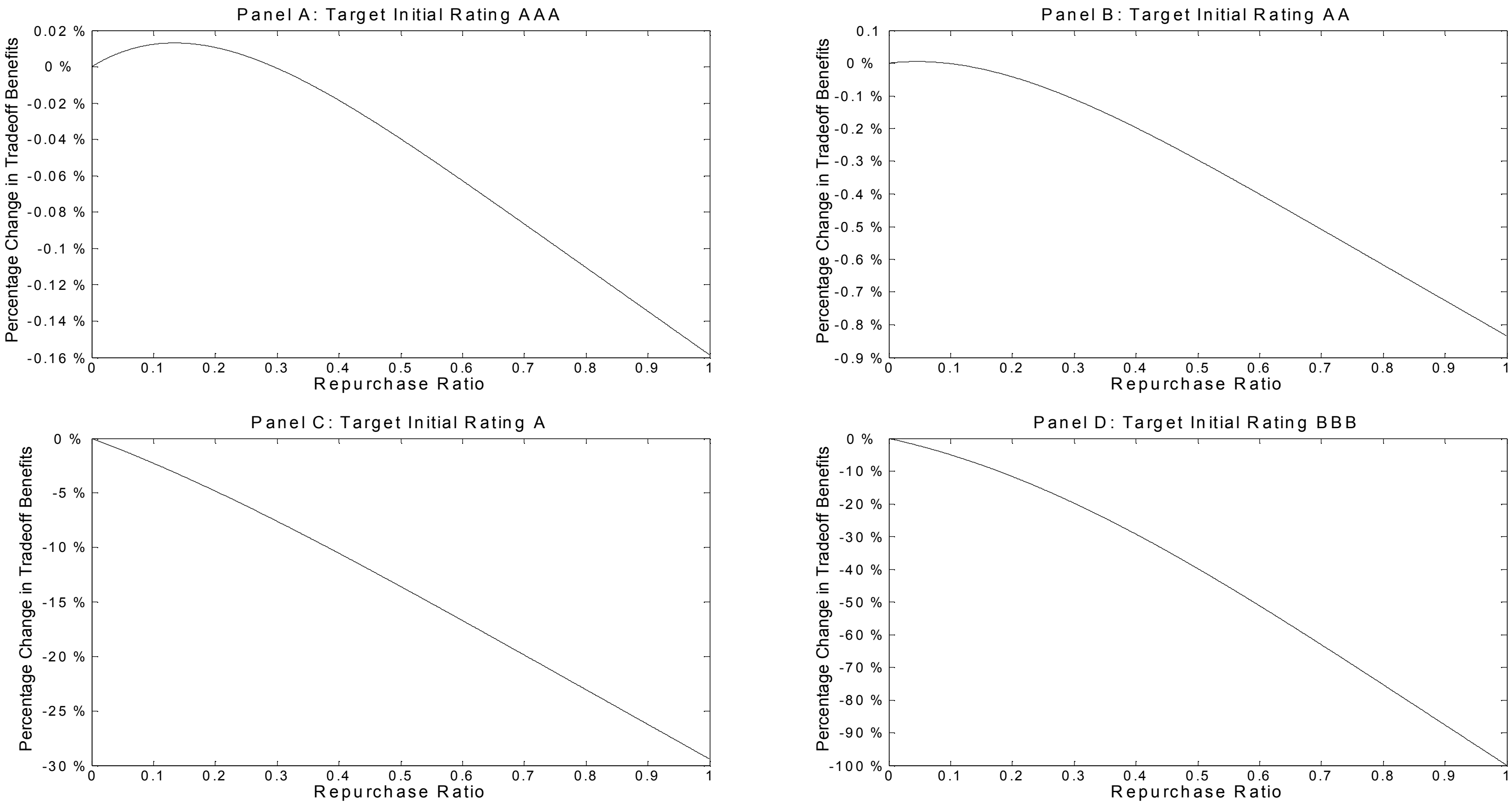

Figure 6. Percentage change in tradeoff benefits as a function of repurchase ratio when the debt has minimum investment-grade policy. The plots examine percentage changes in tradeoff benefits as a function of repurchase ratio for firm with minimum investment-grade policy and target initial rating AAA (panel A), AA (panel B), A (panel C), and BBB (panel D) respectively. Percentage changes in tradeoff benefits are computed by $X(0,\rho)/x$ -1 where $x \equiv X(0,\rho=0)$ and

$$X(0,\rho) \equiv \beta\left\{\int_0^T C_R^k e^{-rs} q_\psi^{k,m}(0,s)\,ds + \int_0^T (1-\rho) C_R^k e^{-rs}\left[1-q_\psi^{k,m}(0,s)\right]ds - \int_0^T (1-\rho) C_R^k e^{-rs}\left[1-q_{\tau_R}^{k,m}(0,s)\right]ds\right\} - \int_0^T \alpha(1-\rho) F_R^k e^{-rs} f_{\tau_R^{k,m}}(s)\,ds \cdot$$

It is assumed that the risk-free interest rate $r=5\%$, the firm's payout ratio $\delta=3.75\%$, the effective tax rate $\beta=40\%$, the bankruptcy costs $\alpha=30\%$, the maturity of debt $T=5$, the volatility of firm's assets $\sigma=38.02\%$, and initial value of firm's underlying asset V(0)=\$100. Rating transition boundaries are jointly determined by optimal capital structure and target initial rating policy.

IX). This suggests that, if firm's policy requires more improvements in credit quality, such as regaining target initial rating after repurchase, managers necessarily make over-repurchase choices (see the numbers in Column 2 of Panel C of Table IX). In the light of Figure 6, however, firm undertaking large-scale repurchase activities will experience fractional loss in the tradeoff benefits (that's why the numbers in Column 6 of Panel C of Table IX are smaller than those in Panel A and B). The outcome is not surprising. Recall the argument in CR-CS that, when making capital decision, managers shall balance the benefits associated with higher rating levels against the loss in abandoning traditional tradeoff benefits. Obviously, as long as benefits from regaining previous target ratings dominate, managers have incentive to make over-repurchase choices in adjusting leverage and credit rating, despite of sacrificing a portion of tradeoff benefits.[26]

Now consider the leverage dynamics behind the behavior as targeting minimum rating. Table X reports the distance between initial target debt ratio and adjusted/ unadjusted debt ratio[27] in case where firm's credit has already fallen below target minimum rate BBB. From this table, observe that the distances between unadjusted debt ratio and initial target in both panels are very sensitive to the target initial rating, equaling 32.69 percentage points while target rating is AAA, and 7.7 percentage points while target rating is A. Such fact establishes a linkage between Figure 6 and the argument in Kisgen (2006b), showing that the benefit of moving back to target leverage is an increasing function of both the extent of departure from firm's optimal leverage and the distance between target initial rating and minimum rating.

The numbers of the distances between adjusted ratio and initial target ratio of debt to total capital are shown in Column 4 of Table X. The size of these numbers measures the degree of moving current leverage ratio toward initial target. Note that the numbers reported in Panel B almost approximate to zero, and are significantly small than those in Panel A. More exactly, in the case that firm's policy requires the credit to retrieve initial target rating after repurchase, leverage ratio will be adjusted back to a level so close to initial target.[28] On the other hand, if the improvements in credit rating required by the policy are slight, the movements in firm's leverage become small due to a little-scale repurchase activity. For these results, we have the economical implication in twofold. (i) The degree of moving leverage ratio toward

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[26] This also responds to the preceding result reported by Table VI that, to possess a better credit rating, managers will forgo a portion of tradeoff benefits in making capital structure choice.

[27] Following the spirit of our models, adjusted/unadjusted debt ratios are respectively computed by

$$\mathrm{E_Q}\left((1-\rho)F_R^k / TV_R^k(\psi_m^k)\middle| \mathcal{F}_0\right) \text{ and } \mathrm{E_Q}\left((1-\rho)F_R^k / TV_R^k(\psi_m^k)\middle| \mathcal{F}_0 \vee \{\rho=0\}\right)$$

[28] Such movement in leverage is corresponding with the empirical finding by Graham and Harvey (2001), Fama and French (2002), and Leary and Roberts (2005), implying that firms in the reality may adjust their current rating back to long-term target (as like initial target rating in our model) for managing credit rating.

Table X

Distance between Initial Target Debt Ratio and Adjusted/Unadjusted Debt Ratio In Case Where Firm's Credit Has Fallen Below BBB

This table shows the distance between initial target debt ratio and adjusted/unadjusted debt ratio with target initial credit rating *k* ranging from AAA to A, in case where firm's credit has fallen below target minimum rating BBB. The repurchase ratio is respectively determined on the objective of maximizing share value and of regaining target initial rating in panel A and B. Column I reports adjusted debt ratio via repurchase activities. Column II reports unadjusted debt ratio. Column III reports initial target debt ratio. Column IV reports the distance between adjusted debt ratio and its initial target. Column V reports the distance between unadjusted debt ratio and its initial target. It is assumed that the risk-free interest rate $r = 5\%$, the firm's payout ratio $\delta = 3.75\%$, the effective tax rate $\beta = 40\%$, the bankruptcy costs $\alpha = 30\%$, the maturity of debt $T = 5$ years, the volatility of firm's assets $\sigma = 38.02\%$, and initial value of firm's underlying asset $V(0)$=\$100. The average spread on debt's coupons is dependent on target initial rating, equaling 40 basis points for AAA, 55 basis points for AA, and 120 basis points for A. Rating transition boundaries are jointly determined by target initial rating policy and optimal capital structure.

Panel A: The case where repurchase raio is determined on the objective of maximizing share value

ITCR	Adjusted Debt Ratio via Repurchase-(1) (%)	Unadjusted Debt Ratio-(2) (%)	Initial Target Debt Ratio-(3) (%)	Distance between (1) and (3) (%)	Distance between (2) and (3) (%)
AAA	31.349567	36.207487	3.516155	27.833412	32.691332
AA	29.413562	30.826549	4.579164	24.834398	26.247384
A	16.250780	16.250780	8.472111	7.778669	7.778669

Panel B: The case where repurchase ratio is determined on the objective of regaining ***ITCR***

ITCR	Adjusted Debt Ratio via Repurchase-(1) (%)	Unadjusted Debt Ratio-(2) (%)	Initial Target Debt Ratio-(3) (%)	Distance between (1) and (3) (%)	Distance between (2) and (3) (%)
AAA	3.527981	36.207487	3.516172	0.011809	32.691316
AA	4.599747	30.826549	4.579304	0.020444	26.247245
A	8.546820	16.250780	8.481949	0.064870	7.768831

Note: *ITCR* denotes initial target credit rating.

initial target is strongly positive-correlated to the required improvements in firm's credit under target minimum rating policy. (ii) The leverage dynamics implied by the behavior as targeting minimum rating follows a mean- reverting process.[29] Now put the focus on clarifying mean-reverting leverage dynamics. Due to the requirement of credit rating policy here, it is known that the firm targets its initial rating at a level consistent with long-term target. Initial optimal leverage thus can be treated as long-term mean in dynamics. Once firm's credit falls below target minimum rating during the life of debt, managers instantly exercise the callable option on the debt to adjust credit rating. No matter what repurchase ratio is, leverage ratio can always be moved toward initial target. If so, numbers reported in Column 4 of Table X has demon

[29] Mean-reversion in leverage dynamics has been found by number of empirical works, including Jalilvand and Harris (1984), Roberts (2001), Fama and French (2002), and Leary and Roberts (2005). A theoretic work by Collin-Dufrasne and Goldstein (2001) argues that the adjustments of outstanding debt level in response to changes in firm value will cause mean-reverting leverage ratio. Via debt repurchase activities induced by the intention of adjusting credit ratings, our results make an interesting link between Kisgen's contention and those existing literatures.

-strated that the behavior as trying to regain previous better ratings may account for mean reversion in leverage dynamics. Certainly, this will also hold in more a generalized case with unlimited numbers of adjusting credit rating, which gives the conclusion that mean-reverting behavior empirically exhibited by firms' leverage can be imputed to the administration of target minimum rating policy.

VI. Conclusion

By building the system of rating transition multi-boundaries, we propose a new firm-value based framework for credit migration, and rebuilds the contingent claim model for capital structure. Departing from traditional approaches of existing literatures (such as ODE approach in Leland's articles), we use martingale method in deriving close form solutions of contingent claim's value. To explore the central issue of this study, the models we develop capture three types of empirical behavior regarding credit rating management- the behavior as targeting initial rating (the base model), the behavior as linking firm's credit to debt's coupons (the extended model: debt with rating-linked coupon scheme), and the behavior as targeting minimum rating (the extended model: debt with rating-dependent callable option).

Our results elucidate the impact of credit rating management behavior on optimal capital structure decision. Firstly, the behavior as targeting initial rating at a better level (e.g. investment-grade or BB) explains the important puzzle in corporate finance that why firms fairly abandon additional tax benefits enjoyed by debt to have low leverage ratios (see Leland (1998), Graham (2000), Graham et al. (2004), Ju et al. (2005), Kahle and Shastri (2005), and Molina (2005)). Due to the cognition that benefits from possessing a better rating may outweigh traditional tax benefits, managers in the reality use debts conservatively and make under-levered choices, rather than the choice of maximizing traditional tradeoff benefits.

In addition, as long as firm's rating at the time of debt issuance is not too low, the behavior as linking credit rating to debt's interest can not only allow the debtholders for earning more coupons but also benefit shareholders by additional tax shields. When making capital decision, managers in general may prefer issuing rating-linked coupon debt relative to standard debt, and hence corresponded optimal leverage ratio is higher also. Further, we show that levered benefits to shareholders in the case of firm applying rating-linked coupon scheme are commonly greater than those in the case of constant coupon scheme, which clarifies the motivation behind the use of rating-linked coupon debts in practice (see Lando and Mortensen (2005)).

Lastly, the behavior as targeting minimum rating is verified to be the cause to mean reversion in leverage dynamics. This helps us link the argument in Kisgen (2006b) to several interesting literatures that find leverage mean-reverting behavior, such as Jalilvand and Harris (1984), Roberts (2001), Fama and French (2002), and Leary and Roberts (2005). Beyond the linkage above, our analysis indicates that debt repurchase activities induced by the intention of regaining initial target rating can generate another leverage ratio so close to initial optimal level (as the long-term mean in our model), following a downgrade from target minimum rating. The size of repurchase is strongly positive-correlated with both the degree of moving leverage ratio toward initial target and the improvements in firm's credit required by target minimum rating policy. To adjust their current rating back to initial target, managers appear to make over-repurchase choices, despite fractional loss in tradeoff benefits.

This paper can be applied in several further dimensions. The problem of jointly making optimal capital structure and credit rating decision can be studied. Other interesting extensions include dynamic repurchasing or a stochastic term structure of risk-less interest rate. We expect that more following studies and applications will be devoted pertaining to this subject.

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