Multiperiod Corporate Default Prediction
– A Forward Intensity Approach

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Research Question

- Default/bankruptcy prediction over different future periods

- Term structures of default probabilities
  - Short-term vs. long-term

- A forward intensity approach
  - Reduced form model
1 Literature Review

2 A forward intensity approach to multiperiod default prediction

3 Data and covariates

4 Empirical results

5 Conclusion
Outline

1. Literature Review
2. A forward intensity approach to multiperiod default prediction
3. Data and covariates
4. Empirical results
5. Conclusion
Literature Review

- Discriminat analysis
  - Beaver (1966, 1968), Altman (1968), etc.
  - Model output: credit scores

- Binary response models: logit/probit regressions
  - Ohlson (1980), Zmijewski (1984), etc.
  - Model output: default probability in the next one period

- Campbell, et al. (2008): multiple logit models
Recent development: duration analysis
- Shumway (2001), Chava and Jarrow (2004), etc.

- Two Poisson processes
  1. Default/bankruptcy
  2. Other exit: merger and acquisition, etc.
- Instantaneous intensity
  - Instantaneous rate of occurrence
  - Functions of the covariates
- Time-series dynamics of the covariates
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Spot combined exit intensity: "average" rate of exit occurrence

\[ \psi_{it}(\tau) \equiv - \frac{\ln(1 - F_{it}(\tau))}{\tau} = - \frac{\ln E_t \left[ \exp \left( - \int_t^{t+\tau} (\lambda_{is} + \phi_{is}) ds \right) \right]}{\tau} \]

\( F_{it}(\tau) \): the time-\( t \) conditional distribution function of the combined exit time evaluated at \( t + \tau \).
\( \lambda_{is} \): instantaneous intensity for default.
\( \phi_{is} \): instantaneous intensity for other exit.
Model Setup (Cont’d)

- **Forward exit intensity:** forward rate of exit occurrence
  \[
  g_{it}(\tau) \equiv \frac{F'_{it}(\tau)}{1 - F_{it}(\tau)} = \psi_{it}(\tau) + \psi'_{it}(\tau) \tau
  \]

- **Forward default intensity:** forward rate of default occurrence
  \[
  f_{it}(\tau) \equiv e^{\psi_{it}(\tau) \tau} \lim_{\Delta t \to \infty} \frac{P_t(t + \tau < \tau_{Di} = \tau_{Ci} \leq t + \tau + \Delta t)}{\Delta t}
  \]
  \[
  = e^{\psi_{it}(\tau) \tau} \lim_{\Delta t \to \infty} \frac{E_t \left[ \int_{t+\tau}^{t+\tau+\Delta t} \exp \left( - \int_t^s (\lambda_{iu} + \phi_{iu}) du \right) \lambda_{is} ds \right]}{\Delta t}
  \]

  \( \tau_{Di} \): default time of the \( i \)-th firm.

  \( \tau_{Ci} \): combined exit time of the \( i \)-th firm.

- **The default probability over** \([ t, t + \tau ] \) **becomes**
  \[
  \int_0^\tau e^{-\psi_{it}(s)s} f_{it}(s) ds
  \]
Model Setup (Cont’d)

- Model $f_{it}(\tau)$ and $g_{it}(\tau)$ directly as functions of state variables available at time $t$ and the horizon of interest, $\tau$.
- $g_{it}(\tau) \geq f_{it}(\tau)$
- $X_{it} = (x_{it,1}, x_{it,2}, \cdots, x_{it,k})$: the set of the state variables

\[
f_{it}(\tau) = \exp\left(\alpha_0(\tau) + \alpha_1(\tau)x_{it,1} + \alpha_2(\tau)x_{it,2} + \cdots + \alpha_k(\tau)x_{it,k}\right)\]
\[
g_{it}(\tau) = f_{it}(\tau) + \exp\left(\beta_0(\tau) + \beta_1(\tau)x_{it,1} + \beta_2(\tau)x_{it,2} + \cdots + \beta_k(\tau)x_{it,k}\right)\]

- Discretize the model for empirical implementation
Model Setup (Cont’d)

- **Forward default probability**

  ![Diagram showing forward default probability]

- **Cumulative default probability**

  ![Diagram showing cumulative default probability]
Estimating the Forward Intensity Model

- MLE
- Decomposable
- Non-sequential, parallelizable
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Data


- Database:
  - Compustat
  - CRSP
  - Bloomberg

- 12,196 U.S. public companies (both industrial and financial), 1,030,305 firm-month observations.
Covariates

- 3-month treasury rate
- Trailing 1-year S&P500 return
- Distance to default
- Cash and short-term investments/Total assets
- Net income/Total assets
- Relative size
- Market to book ratio
- Idiosyncratic volatility
- Level and trend
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Empirical results

Parameter Estimates

**D\textsubscript{T}D\textsubscript{AVG}**

**CASH/TA\textsubscript{AVG}**

**NI/TA\textsubscript{AVG}**

**SIGMA**
Parameter Estimates (Cont’d)

- $\text{DTD}_{\text{DIF}}$
- $\text{CASH/TA}_{\text{DIF}}$
- $\text{NI/TA}_{\text{DIF}}$
- $\text{SIZE}_{\text{DIF}}$
Empirical results

Aggregate Number of Defaults

1 month

3 month

6 month

12 month

24 month

36 month
Empirical results

In-Sample Accuracy

<table>
<thead>
<tr>
<th>Time Period</th>
<th>In-Sample Accuracy</th>
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<tbody>
<tr>
<td>1 month</td>
<td>93.41%</td>
</tr>
<tr>
<td>3 months</td>
<td>91.42%</td>
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<tr>
<td>6 months</td>
<td>88.37%</td>
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<tr>
<td>12 months</td>
<td>82.70%</td>
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<tr>
<td>24 months</td>
<td>72.09%</td>
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<tr>
<td>36 months</td>
<td>64.77%</td>
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Duan, Sun, and Wang (NUS)
Empirical results

Out-of-Sample (Over Time) Accuracy

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 month</td>
<td>91.99%</td>
</tr>
<tr>
<td>3 months</td>
<td>90.21%</td>
</tr>
<tr>
<td>6 months</td>
<td>87.36%</td>
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<tr>
<td>12 months</td>
<td>83.42%</td>
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<tr>
<td>24 months</td>
<td>74.35%</td>
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<tr>
<td>36 months</td>
<td>68.53%</td>
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Empirical results

Term Structures of Default Probabilities (Lehman Brothers)

Forward default probability (36 months before bankruptcy)

Cumulative default probability (36 months before bankruptcy)

Forward default probability (24 months before bankruptcy)

Cumulative default probability (24 months before bankruptcy)
Term Structures of Default Probabilities (Cont’d)

Forward default probability (12 months before bankruptcy)

Cumulative default probability (12 months before bankruptcy)

Forward default probability (3 months before bankruptcy)

Cumulative default probability (3 months before bankruptcy)
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A forward intensity approach for the prediction of corporate defaults over different future periods is proposed. Maximum likelihood analysis is conducted on a large sample of U.S. industrial and financial firms spanning the period 1991-2009. Several frequently used covariates are shown to be useful for prediction at both short and long horizons. The forward intensity model is amenable to aggregation, which allows analysts to assess default behavior at the portfolio and/or economy level.