Forward-Looking Market Risk Premium

Jin-Chuan Duan and Weiqi Zhang*

(First Draft: March 15, 2010; This Draft: September 28, 2010)

Abstract

A method for computing forward-looking market risk premium is developed in this paper. We first derive a theoretical expression that links forward-looking risk premium to investors' risk aversion and cumulative return's forward-looking volatility, skewness and kurtosis. In addition, investor's risk aversion is theoretically linked to volatility spread defined as the gap between the risk-neutral volatility deduced from option data and the physical return volatility exhibited by return data. The volatility spread formula serves as the basis for using the GMM method to estimate investor's risk aversion. We adopt the GARCH model for the physical return process, and estimate the model using the S&P500 daily index returns and then deduce the corresponding cumulative return's forward-looking variance, skewness and kurtosis. The forward-looking risk premiums are estimated monthly over the sample period of 2001-2009 and found to be all positive. The forward-looking risk premium was higher during volatile market periods (such as September 2001 and October 2008) and lower when the market was calm. Furthermore, two asset pricing tests are conducted. First, change in forward-looking risk premiums is negatively related to the S&P500 holding period return, reflecting that an increase in discount rate reduces current stock price. Second, market illiquidity positively affects forward-looking risk premium, indicating that forward-looking risk premium contains an illiquidity risk premium component.

Keywords: Risk premium, forward looking, GARCH, options, volatility spread, skewness, kurtosis.

JEL classification code: G12, G17, C51, C53

---

*Duan is with Risk Management Institute and Department of Finance, National University of Singapore. E-mail: bizdjc@nus.edu.sg. Zhang is with Department of Finance, National University of Singapore. E-mail: weiqizhang@nus.edu.sg. We would like to thank Yakov Amihud, Bruno Solnik and the seminar participants at ESSEC, University of Toronto and New York University for their helpful comments. The inputs received at Workshop on Econometric and Financial Studies after Crisis held at Academia Sinica in June 2010 are also gratefully acknowledged.

1 Introduction

Risk premium is the most important concept in finance, and its modeling has been a core of the modern finance theory. Risk premium is often used in financial research and applications, such as determining market efficiency, setting asset allocation (the decision to allocate assets between stocks and bonds), applying the Capital Asset Pricing Model (CAPM) to determine cost of capital, and forecasting growth of investment portfolios, among others.

Risk premium is obviously a forward-looking concept. In essence, it is compensation for holding an asset that will yield an uncertain return. In practice, however, the most commonly used method for estimating risk premium is average historical realized excess returns (Welch, 2000; Damodaran, 2008). The literature indicates that realized excess return and expected risk premium are fundamentally different (Elton, 1999; Arnott and Bernstein, 2002). Conceptually, using historical realized excess return relies on the belief that noises will cancel out in the long run. Thus, using historical risk premium is subject to tradeoff between reflecting recent market condition and estimation accuracy. Basically, the longer the estimation time period, the more statistically accurate the result becomes. But a longer time period also results in a higher likelihood of a regime shift in the estimation period so that the risk premium estimate becomes distorted by earlier regime(s).

Merton (1980) argued that historical risk premium fails to account for the effect of changes in the level of market risk. Simply put, when one moves into a volatile phase, forward-looking risk premium should become higher. But historical average of excess returns cannot be expected to reflect changing market conditions even if such a rise in market volatility is transient. Empirically, stock market can produce negative risk premium even for an estimation period longer than 10 years (e.g., from 1973 to 1984). Backward-looking is therefore a non-trivial problem facing the use of historical risk premium.

In order to get a sense on the magnitude of forward-looking risk premium, several studies surveyed academics, investors or business managers to get their views on risk premium (Welch, 2000; Graham and Harvey, 2007; Fernandez, 2009a). Welch (2000) surveyed 226 financial economists in 1997 and reported their forecast of long-term mean equity risk premium of 6% to 7%. Respondents claimed to revise their forecasts downwards when stock market rises. Graham and Harvey (2007) analyzed the results of surveys on Chief Financial Officers (CFO) and found a positive relation between market volatility and CFOs' risk premium expectations. Fernandez (2009a) surveyed professors and obtained an average risk premium of 6.3% in the USA. Although survey approaches may provide reasonable estimates of forward-looking risk premium, they are subject to limitations such as: 1) surveys are time consuming and thus cannot be updated frequently enough; 2) survey approaches usually prescribe a very long prediction horizon and not available for different horizons of interest;

and 3) surveys are expressions of subjective opinions and face unknown sample selection bias.

In this paper, we propose a practical method to compute forward-looking market risk premium that is based on combining estimated investor's risk aversion with cumulative return's physical moments (variance, skewness and kurtosis) on a forward-looking basis. Assuming a particular form of stochastic discount factor, our method uses the volatility spread formula (risk-neutral return volatility minus physical return volatility) developed in Bakshi and Madan (2006) to estimate investor's risk aversion. The volatility spread is expressed in terms of the cumulative return's physical skewness and kurtosis, and the risk aversion parameter is estimated by applying the GMM estimation method to the volatility spread relationship. Instead of using the return time series realized over the period covered by options' maturity to estimate physical return moments as in Bakshi and Madan (2006), we deduce forward-looking physical return moments from the GARCH model estimated with daily returns. The GARCH model offers a practical way of reflecting prevailing market condition and provides us with forward-looking physical return moments for any horizon of interest.[1] An important part of our method is the forward-looking risk premium formula that links risk premium to forward-looking physical volatility, skewness and kurtosis. With a risk aversion parameter estimate in place, we can combine it with forward-looking physical moments to produce our estimate of forward-looking risk premium for any horizon of interest.

Option prices have previously been used in the literature to estimate risk premium. Bhar, Chiarella and Runggaldier (2004) adopted a parametric system by assuming the Black-Scholes model and imposing the risk premium as the product of volatility and a latent variable following a mean-reverting process. The Kalman filter was then run on the parametric system to obtain the smoothed estimate of the latent variable. Since the Black-Scholes option pricing formula is independent of the asset risk premium, options in their study can only marginally help in pinning down risk premium through the information generated from the mean returns on options over time. In a more recent paper, Santa-Clara and Yan (2010) used a different parametric model and derived the risk premium as a function of two latent variables (volatility and jump intensity). Their implementation avoids filtering by assuming two option prices are observed without error at any time point so as to enable them to back out the two latent variables for different points of time. In contrast to these methods, our approach relies on a generic moment expansion which does not need any parametric option pricing model. In our empirical analysis, we utilize the result developed in the model-free risk-neutral literature to extract risk-neutral volatility from option portfolios without having to deal with individual options.

[1] To understand this point, assume that the daily return time series is governed by the GARCH(1,1) model. The historical volatility based on, say 90 daily returns, will be different from the 90-day return volatility implied by the GARCH model on a forward-looking basis.

Our estimates of forward-looking risk premium (annualized), based on the S&P500 return and option data and being repeatedly estimated on a monthly basis, range from 1.6% (June 2005) to 539.3% (Oct 2008) with higher premiums during extreme market periods. This result is consistent with the common belief that investors require higher compensation for taking higher risk, and thus risk premium should be high when market is uncertain. In contrast, the estimates of risk premium using historical excess returns, the CAPM and the Fama-French three-factor model (Fama and French, 1996) are unable to adequately reflect market conditions. It is worth noting that forward-looking risk premiums are positive throughout the entire sample period, whereas the risk premium estimated from other methods (historical average excess returns, the CAPM and the Fama-French three-factor model) are often negative.

Similarly, the Fama and French (2002) approach to estimating equity risk premium using fundamentals (dividend and earnings growth rates) can be quite volatile. Our empirical analysis shows that it can yield substantially negative risk premiums during bad times. Since all asset pricing theories suggest that market risk premium should be positive if investors are risk averse, negative risk premiums are rather difficult to interpret and can create difficulties in applications. In summary, comparing different risk premium measures suggests that forward-looking risk premium is a more reasonable way of gauging the appropriate level of compensation for bearing risk in a fast-moving equity market.

Asset pricing theories suggest numerous implications for market risk premium. Two specific implications are tested in this paper using forward-looking risk premium. The first one is the relationship between change in risk premiums and change in current stock prices. Using forward-looking risk premium, we confirm the theoretical relationship that an increase in discount rate (risk premium) decreases current stock price while controlling for expected future cash flows. Second, we show that forward-looking risk premium is positively related to illiquidity, i.e., the presence of an illiquidity premium. The relationship is not simply a manifestation of the liquidity-volatility relationship.

The remainder of the paper is organized as follows. Section 2 presents the theory of forward-looking risk premium. Section 3 presents the econometrics for estimating investors' risk aversion and for deducing cumulative return's variance, skewness and kurtosis from the GARCH model. Section 4 describes the data, the estimates of forward-looking risk premium, and the comparisons with other risk premium measures. Section 5 presents the analysis of two asset pricing implications of forward-looking risk premium, and Section 6 concludes.

2 The Theory of Forward-Looking Risk Premium

In this section, we first show that the risk-free interest rate can be expressed by the risk neutral moments of the market portfolio. Then, we derive the expression of forward-looking market risk premium under the standard assumption of stochastic discount factor that can be justified by a power utility. It is a common practice in the asset pricing literature to derive an asset pricing model by combining a particular form of stochastic discount factor with lognormal asset returns (Hansen and Singleton, 1983; Grossman and Shiller, 1981; Campbell and Cochrane, 2000). Instead of assuming lognormal asset returns, we allow for higher moments in the derivation of market risk premium in order to reflect the well-known empirical regularities.

Denote the market portfolio's value by S_t and its cumulative return over the time period t to $t+\tau$ (continuously compounded) by $R_t(\tau) = \ln(S_{t+\tau}/S_t)$. At time t, $R_t(\tau)$ is a random return to be realized later at time $t+\tau$. Let $r_t(\tau)$ and $\delta_t(\tau)$ denote the continuously compounded risk-free interest rate and dividend yield of the market portfolio over the period from t to $t+\tau$, respectively. In order to characterize the distributions implied by return and option data, we need to specify two probability measures. Let $\mu_{Pt}(\tau), \sigma_{Pt}(\tau), \theta_{Pt}(\tau)$ and $\kappa_{Pt}(\tau)$ be the mean, standard deviation, skewness and kurtosis of market portfolio under the physical measure P. The use of t and τ is to make it clear that these moments can be time-varying and depend on the length of the period over which cumulative return is defined. Their equivalents under the risk-neutral measure Q are denoted with the subscript Q.

We first derive an approximate relationship for the equilibrium risk-free interest rate using the devise of risk-neutral measure Q, knowing that the expected asset return inclusive of cash dividends should equal the risk-free interest rate when the expectation is performed with the risk-neutral measure. By a simple expansion argument (see Appendix A for details), we have the following result:

$$r_t(\tau) \approx \delta_t(\tau) + \mu_{Qt}(\tau) + \frac{1}{2}\sigma^2_{Qt}(\tau) + \frac{1}{6}\theta_{Qt}(\tau)\sigma^3_{Qt}(\tau) + \frac{1}{24}\sigma^4_{Qt}(\tau)[\kappa_{Qt}(\tau)) - 3]. \tag{1}$$

In the above, the risk-free interest rate is expressed as a function of risk-neutral moments, and the approximate relationship is generic in the sense that it does not depend on the form of the stochastic discount factor.

To obtain a useful expression for the physical market risk premium, we later need to express the risk-free rate in terms of physical return moments. Our derivations are based on the following assumption.

Assumption 1. The stochastic discount factor over time t to $t+\tau$ is $e^{-\gamma R_t(\tau)}$, and the moment generating function of $R_t(\tau)$ exists under either measure P and Q.[2]

[2]Note that the stochastic discount factor as in Assumption 1 can be deduced from the power utility

Under the above assumption, Bakshi and Madan (2006) derived an expression for volatility spread:

$$\frac{\sigma_{Qt}^2(\tau) - \sigma_{Pt}^2(\tau)}{\sigma_{Pt}^2(\tau)} \approx -\gamma\sigma_{Pt}(\tau)\theta_{Pt}(\tau) + \frac{\gamma^2}{2}\sigma_{Pt}^2(\tau)[\kappa_{Pt}(\tau) - 3]. \tag{2}$$

Similar to their study, the above expression also serves as the basis for our empirical estimation of the risk aversion parameter γ.

One can similarly derive analytical expressions for risk-neutral expected return, variance, skewness, kurtosis in terms of physical return moments. Substitute these expressions for risk-neutral moments into the risk-free rate equation in (1), the following new expression for market risk premium can be derived.

Proposition 1 *Under Assumption 1, the τ-period market risk premium can be expressed as a function of investors' risk aversion, physical return variance, skewness and kurtosis:*

$$\begin{aligned}\mu_{Pt}(\tau) + \delta_t(\tau) - r_t(\tau) \approx & \left(\gamma - \frac{1}{2}\right)\sigma_{Pt}^2(\tau) - \frac{3\gamma^2 - 3\gamma + 1}{6}\sigma_{Pt}^3(\tau)\theta_{Pt}(\tau) \\ & + \frac{4\gamma^3 - 6\gamma^2 + 4\gamma - 1}{24}\sigma_{Pt}^4(\tau)[\kappa_{Pt}(\tau) - 3]. \end{aligned} \tag{3}$$

Proof: See Appendix B.

Suppose that there is no physical return skewness or excess kurtosis; that is, $\theta_{Pt}(\tau) = 0$ and $\kappa_{Pt}(\tau) = 3$. The above result implies that risk premium for the market portfolio is $\mu_{Pt}(\tau) + \delta_t(\tau) - r_t(\tau) = \left(\gamma - \frac{1}{2}\right)\sigma_{Pt}^2(\tau)$, a well-known result under lognormality. The equity premium expression in equation (3) makes it easier to understand the role played by return skewness and kurtosis. It suggests that the presence of skewness and excess kurtosis will alter risk premium. One can show that $3\gamma^2 - 3\gamma + 1$ is always positive. That implies that negative skewness will increase risk premium. The importance of negative skewness in pricing assets has been previously documented in, for example, Kraus and Litzenberger (1976) and Harvey and Siddique (2000). Similarly, one can show that $4\gamma^3 - 6\gamma^2 + 4\gamma - 1 > 0$ when $\gamma > \frac{1}{2}$. Therefore, when investors' risk aversion exceeds one-half, leptokurtosis (fat tails) will also increase risk premium.

If we can find a practical way to estimate γ and physical return moments for different horizons of interest on a forward-looking basis, equation (3) will provide a way of generating forward-looking market risk premiums for different horizons of interest. Indeed, that is what we will be doing next.

---

function: $U(W) = W^{1-\gamma}/(1-\gamma)$ when the economic agent maximizes the expected utility of the end-of-the-period wealth.

3 Econometric Formulation

Similar to Bakshi and Madan (2006), we use the volatility spread equation in (2) to estimate γ. Let I_t be some set of instruments whose values are known at time t. A GMM estimation can be performed using the following orthogonality condition:

$$E\left\{\frac{\sigma^2_{Qt}(\tau)-\sigma^2_{Pt}(\tau)}{\sigma^2_{Pt}(\tau)}+\gamma\sigma_{Pt}(\tau)\theta_{Pt}(\tau)-\frac{\gamma^2}{2}\sigma^2_{Pt}(\tau)[\kappa_{Pt}(\tau)-3]\middle| I_t\right\} = 0 \tag{4}$$

In order to implement the above expression, we need a time series of risk-neutral return variance and three time series of physical return moments (variance, skewness and kurtosis).

A model-free risk-neutral return variance $\sigma^2_{Qt}(\tau)$ can be computed by forming appropriate portfolios of broad-based market index options. Such an approach was established in Britten-Jones and Neuberger (2000), Carr and Madan (2001), and Jiang and Tian (2005). The theory linking risk-neutral return variance to an option portfolio is presented in Appendix C. Its exact empirical implementation will be elaborated in the next section.

In this paper, physical return variance, skewness, and kurtosis are obtained using the GARCH model. Our approach differs from the ex-post sample moments approach of Bakshi and Madan (2006). Recall that our objective is to come up with forward-looking physical risk premium. Using a popular GARCH model with the feature of asymmetric volatility response (i.e., leverage effect), we are able to deduce forward-looking higher return moments for various horizons of interest through a combination of analytical formulas and bootstrap sampling.

We adopt the nonlinear asymmetric GARCH(1,1) model of Engle and Ng (1993), hereafter NGARCH(1,1), for the market portfolio's return dynamic under the physical probability P:

$$\ln\frac{S_{t+1}}{S_t}=\mu+\sigma_{t+1}\varepsilon_{t+1} \qquad \text{for} \quad t=0,1,\cdots \tag{5}$$

where

$$\sigma^2_{t+1}=\beta_0+\beta_1\sigma^2_t+\beta_2\sigma^2_t(\varepsilon_t-\eta)^2 \tag{6}$$

and ε_{t+1} are *i.i.d.* random variables with $E^P(\varepsilon_{t+1})=0$ and $E^P(\varepsilon^2_{t+1})=1$. Note that we need to impose the restrictions: $\beta_0>0$, $\beta_1\geq 0$, $\beta_2\geq 0$ to ensure conditional variances staying positive. Parameter η reflects the so-called leverage effect. When $\eta>0$, a negative return shock generates a larger future volatility than does a positive shock of the same magnitude. According to Duan (1997), the NGARCH(1,1) model is strictly stationary if

$\beta_1 + \beta_2(1+\eta^2) \leq 1$. When $\beta_1 + \beta_2(1+\eta^2) < 1$, the NGARCH(1,1) model will have a finite stationary variance and hence it is also variance-stationary.

Because we need to allow for skewness and fat-tails, we do not commit to a particular conditional distribution. The parameters in the NGARCH(1,1) model are estimated by the quasi-maximum likelihood method. For both the estimation and the forward-looking risk premium applications, we need to know physical return moments for multi-period horizons, say, 20 trading days (corresponding to 28 calender days). It is possible to derive a simple formula for the conditional variance of τ-period cumulative return. The derivation of the following formula is given in Appendix D:

$$\sigma^2_{Pt}(\tau) = \frac{1-\lambda^\tau}{1-\lambda}\sigma^2_{t+1} + \frac{(\tau-1)\beta_0}{1-\lambda} - \frac{\lambda(1-\lambda^{\tau-1})\beta_0}{(1-\lambda)^2} \tag{7}$$

where $\lambda = \beta_1 + \beta_2(1+\eta^2)$.

However, conditional skewness and kurtosis of the cumulative return under the GARCH model do not lead to workable analytical formulas. Thus, we resort to bootstrapping as a way of obtaining these required quantities. Basically, we use the data set available at time t to obtain an estimated NGARCH(1,1) model. The model is then applied to the data set to generate a time series of standardized residuals (mean 0 and variance 1, but not necessarily normally distributed). When simulating the NGARCH(1,1) model to obtain cumulative returns, we start from σ_{t+1} and randomly sample from the set of standardized residuals to move the system forward to time $t+\tau$. The smooth stratified bootstrap method as in Pitt (2002) is applied to sampling from the standardized residuals. After sampling many times, conditional skewness and kurtosis of the cumulative return of interest can be approximated by their sample equivalents.

4 Empirical Analysis

4.1 Data

The S&P500 index returns and option prices are used in the empirical study. The S&P500 index values, their option prices, and the risk-free interest rates over the period of January 1996 to October 2009 are taken from OptionMetrics. Monthly sampling frequency is implemented, and τ in our estimation implementation equals 28 calender days. At each option expiration date, we move backwards 28 calender days and refer to this point as the observation date. This procedure gives us non-overlapping call and put options with a maturity of 28 calender days. The corresponding 28-day risk-free rate is obtained by interpolating zero-rate curve.

Risk-neutral variance of the 28-calendar day cumulative return is calculated for each observation date using the prices of the S&P500 index options with the remaining maturity of 28 calendar days. The formula used to calculate risk-neutral variance is shown in Appendix C. We follow CBOE to set K_t (determining at time t which calls and puts are considered out-of-the-money in the algorithm) as the first available strike price below the forward index level where the forward index level is determined by the call-put pair with the smallest price differential. Numerical integration is performed over the available strike prices. As argued in Jiang and Tian (2005), the discretization error is unlikely to have material impact on the calculation of risk-neutral variance.

The NGARCH(1,1) model is used to compute the 28-calender day conditional physical variance, skewness and kurtosis. The daily S&P500 closing index values over the five years immediately before each observation date are used in the quasi-maximum likelihood estimation of the NGARCH(1,1) model to obtain the parameter estimates for $\mu, \beta_0, \beta_1, \beta_2$, and η. In addition, the conditional physical next trading day return variance, i.e., σ_{t+1}^2, corresponding to the observation date is obtained as a by-product of the estimation. For each observation date, we then calculate the 28-calender day conditional variance analytically using equation (7). Because the GARCH parameters are obtained on the trading-day basis, we apply the actual number of trading days in the next 28-calendar period. The conditional 28-calendar day skewness and kurtosis are obtained by smoothed bootstrap simulations using the pool of 5-year worth of standardized residuals corresponding to the observation date in an attempt to preserve skewness and kurtosis in the data. A bootstrapped sample size of 100,000 is used to advance the system one trading day at a time until reaching the 28-calendar day maturity. Again, the actual number of trading days in the next 28 calendar days is used in simulation. We then compute the averages of simulated cumulative return raised to various powers that are need for obtaining their sample equivalents.

Table 1 presents the summary statistics for risk-neutral volatility and physical forward-looking volatility, skewness and kurtosis. Qualitatively consistent with the prior findings in the literature, risk-neutral volatility has a higher mean value vis-a-vis physical forward-looking volatility. Also revealed by the summary statistics, the 28-calendar day forward-looking returns are negative skewed ($\theta_P(\tau) < 0$) and leptokurtic ($\kappa_P(\tau) > 3$).

Figure 1 plots the 28-calender day risk-neutral volatility versus physical forward-looking volatility. The curve representing risk-neutral volatility generally lies above the one for physical forward-looking volatility, especially for the period of 1996-1999. The average volatility spread between risk-neutral volatility and physical forward-looking volatility over the whole sample period is 3.9%. The average volatility spread for the period from January 1996 to December 1999 is 7.2% and for the period from January 2000 to October 2009 is 2.6%. Although we compute physical return volatility differently, our results are qualitatively consistent with the literature that has documented volatility spread (Bakshi and Madan, 2006; Christensen and Prabhala, 1998). Figure 2 plots the 28-calender day physical forward-looking skewness and kurtosis computed from the NGARCH(1,1) model. Panel A shows that returns are generally negative skewed, and negative skewness is more pronounced from 1998 to 2003, and again from 2007 to 2009. Panel B shows that returns are more leptokurtic from 1998 to 2003, and again from 2007 to 2009.

4.2 Investors' Risk Aversion

To obtain the most recent risk aversion over different estimation periods, we use a 5-year moving window of data (updated monthly) to estimate γ. Specifically, γ is estimated for every observation date (one per month) using the 5-year data prior to and including the observation date to generate 60 monthly volatility spreads for the GMM estimation. The GMM method adopted here is the one with the Newey-West adjusted covariance matrix. Three sets of instruments are used and they are same as Bakshi and Madan (2006). Set 1 contains a constant plus $\sigma^2_{Q,t-1}(\tau)$. Set 2 contains a constant, $\sigma^2_{Q,t-1}(\tau)$, and $\sigma^2_{Q,t-2}(\tau)$. Finally, Set 3 contains a constant, $\sigma^2_{Q,t-1}(\tau)$, $\sigma^2_{Q,t-2}(\tau)$, and $\sigma^2_{Q,t-3}(\tau)$. The results from the three sets are qualitatively similar. Therefore, we only report in Table 2 those from using Set 3.

Although we have 166 monthly results for all return moments of interest as shown in Table 1, we can only conduct the GMM estimation and test on a moving-window basis for 106 times, because the first test needs return moments for five years (60 months). None of the 106 rolling tests of the model is rejected, based on testing the over-identifying restrictions at the 5% significance level. The estimated γ's are all significant with the mean being 4.25, and the smallest t-statistic equals 2.62. The estimated γ's range from 1.8 to 7.1. These estimates for risk aversion seem intuitively sensible and are comparable to the ones obtained in some previous studies such as Bliss and Panigirtzoglou (2004), which reported a risk aversion estimate of 4.08 (power utility) or 6.33 (exponential utility) using the risk-neutral and physical density functions of the S&P500 return and option data.

For comparison, we also estimate γ by applying the approach of using ex-post sample moments as in Bakshi and Madan (2006) to our data. The results show that the volatility spread model still passes the test on the overidentifying restrictions, but the estimated γ is around 93, a unreasonably large risk aversion parameter value. In contrast, the full sample γ estimated using forward-looking physical moments is 4.4. The cause for the huge difference in the parameter estimate can be attributed to the fact that the sample moments based on ex-post realized returns are available only at the end of the period of interest which are incompatible with the spirit of the theoretical relationship. Consequently, it forces the risk aversion parameter to accommodate the gap between forward-looking risk-neutral volatility and ex-post physical volatility, and causes a distorted estimate of risk aversion. Another possible reason is that the use of a relatively small sample of returns (daily returns over one month) may have under-estimated the magnitude of higher return moments (Jackwerth and Rubinstein, 1996), and the under-estimated skewness and kurtosis in turn need a much larger risk aversion in order to match the volatility spread.

4.3 Forward-Looking Risk Premium

Using the risk aversion estimate along with physical forward-looking variance, skewness and kurtosis, we can compute forward-looking risk premium for each observation date. Table 3 presents the results. The estimated forward-looking risk premiums (annualized) are all positive and vary from 1.6% (June 2005) to 539% (October 2008). Furthermore, the results reveal very high risk premium during the internet bubble bursting period (2001 to early 2003) and the sub-prime mortgage crisis (late 2007 to 2009). There are several particularly large risk premiums during these two periods. In September 2001, the 9-11 terrorist attack resulted in the closure of NYSE from September 11 to 17, and during the first re-opening day, the S&P500 index fell 4.9% and the Dow Jones Industry Average fell 7.1%, which was the single biggest day drop over our sample period. Therefore, it is not surprising that the forward-looking risk premium for that month reaches 17.4% (=2.088/12). In July 2002, WorldCom, which was the second largest long distance phone company, filed for bankruptcy. It was the largest bankruptcy up to that time, and investors' confidence was severely shattered.[3] Our estimated forward-looking risk premium is 14% (=1.672/12) for that month.

More recently, January 2008 was an especially volatile month for stock markets around the world for fears of the sub-prime mortgage crisis.[4] Two months later, Bear Stearns collapsed and was merged with JPMorgan Chase in a distress sale. The sub-prime mortgage crisis reached its peak in September and October of 2008. Several major institutions (e.g., Lehman Brothers and Merrill Lynch) were either failed or acquired with government assistance. Interestingly, our forward-looking risk premium hit its highest point of 45% (=5.393/12) in October 2008 and its second highest point of 29.5% (=3.545/12) in November 2008. In summary, these aforementioned events are coupled with extremely high forward-looking risk premiums that could not possibly be captured by any backward-looking risk premium measure.

To further explore the relationship between forward-looking risk premiums and economic conditions, we plot in Figure 3 the time series of monthly forward-looking risk premiums along with the NBER recessions (the shaded area).[5] Along with recessions, we also indicate in the plot the internet bubble bursting period and the sub-prime mortgage crisis period.

[3] The Economist, July 23, on WorldCom.
[4] BBC News: http://news.bbc.co.uk/2/hi/business/7199552.stm
[5] Obtained from http://wwwdev.nber.org/cycles/cyclesmain.html

This figure shows that during a recession or a crisis period, the forward-looking risk premiums are usually higher. This result is consistent with the common belief that during bad times, investors usually demand higher returns.

The forward-looking risk premium estimation can be extended to longer horizons by setting a larger τ using the physical forward-looking return variance, skewness and kurtosis specific to the horizon. Since physical forward-looking skewness and kurtosis need to be computed with a bootstrapping method, we need to address the simulation errors arising from simulating over a longer horizon. Since the estimated GARCH model has a vary high volatility persistence, simulation noises cannot be attenuated very quickly, which causes simulated skewness and kurtosis to exhibit larger swings when the horizon is initially lengthened up to some point. Nevertheless, the pattern of forward-looking skewness (or kurtosis) as a function of horizon clearly presents itself, and a spline smoothing can be applied to obtain the smoothed values for forward-looking skewness (or kurtosis). Figure 4 presents these smoothed higher moments along with the smoothed forward-looking risk premiums for different horizons up to one year (252 trading days). These plots are presented for two specific time points: a relatively volatile time (September 2001) and a relatively quiet time (September 2003). The smoothing is done by a cubic polynomial spline with one knot at 125 trading days using the least-square estimation on 252 data points.

It is evident from the plots in Figure 4 that the forward-looking risk premium term structure can have various shapes. Its pattern has a great deal to do with the term structures of physical forward-looking skewness and kurtosis. Basically, the physical return distribution becomes more negatively skewed and with fatter tails as the horizon is lengthened. This has the effect of increasing forward-looking risk premium. Once the horizon passes a certain point, the behavior of skewness and kurtosis begin to reverse. As expected, the volatility behaves in a typical mean-reverting manner. But the behavior of skewness and kurtosis are more complex. This interesting feature of the GARCH model is not generally understood and rarely explored in the literature. In essence, cumulative return moves further away from normality due to stochastic mixture effect of the time-varying volatility. But once the horizon is long enough, the effect of the Central Limit Theorem will kick in and moves the cumulative return back towards normality.

4.4 Comparison with Other Measures of Risk Premium

The prior literature points out that expected risk premium and realized risk premium are fundamentally different concepts, and confusions arise for not properly distinguishing the two concepts (Elton, 1999; Arnott and Bernstein, 2002; Fernandez, 2009a,b). Perhaps due to the lack of a better alternative, many measures of expected risk premium continue to rely on some form of ex-post market risk premium as an input to obtain the estimate for expected risk premium. In this section, we present five measures of risk premium, and compare them to forward-looking risk premium.

4.4.1 Historical Measures

Historical average of realized excess returns is the most commonly used estimate of expected risk premium. In this section, two measures using average historical excess return (3 years and 5 years) are considered. In addition, two measures using average historical cross-sectionally estimated risk premium (applying the Fama-Macbeth (1973) regression for the CAPM and the Fama-French three-factor model (Fama and French, 1996)) are constructed. When risk premiums are computed by the Fama-Macbeth method, we measure returns over regular calender months so as to be consistent with the standard practice.

Table 4 presents the S&P500 risk premium based on historical average of daily excess returns over three years (Panel A) and over five years (Panel B). For each observation date, we estimate its historical risk premium by averaging daily excess returns over three or five years immediately before the observation date. The numbers reported in this table are annualized but not in percentage. Panel A shows that the three-year historical risk premiums are mostly negative for the period of 2001-2004. Similarly, Panel B shows that the five-year historical risk premiums are mostly negative for the period of 2002-2005. In contrast, the forward-looking risk premiums reported in Table 3 are all positive.

Table 5 presents the S&P500 historical risk premiums estimated from the CAPM and the Fama-French three-factor model by applying the Fama-Macbeth two-stage estimation approach.[6] In the first stage, for every stock and for each month, we use the monthly returns from the preceding five-year period to estimate beta(s) from a time series regression of stock returns on factor returns. In the second stage and for each month, we regress stock returns cross-sectionally against their betas to determine the monthly risk premium for each factor. After obtaining monthly risk premium(s), we estimate the beta(s) for the S&P500 index (against the CRSP value-weighted index and the factor portfolios when appropriate). The S&P500 index's beta(s) time the corresponding factor risk premium(s) gives rise to the S&P500 index monthly risk premium. Repeat it for each month in the five-year estimation period. The risk premiums reported in the table are the five-year average of the monthly risk premiums. The estimation is performed for both the CAPM (Panel A) and the Fama-French three-factor model (Panel B). Table 5 shows that the estimated risk premiums from both asset pricing models are often negative. For the CAPM, the risk premiums are mostly negative in 2002-2005, whereas for the Fama-French three-factor model, the risk premiums are generally negative in 2001-2005.

[6]Monthly stock returns are taken from CRSP. The Fama-French three factors are obtained from Kenneth R. French's website, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

The correlation coefficients reported in Table 6 shows that the five-year historical risk premium is not significantly correlated with the forward-looking risk premium. However, the three-year historical risk premium is negative correlated (-0.25) with the forward-looking risk premium. A shorter time span such as three years makes the historical risk premium more reflective of recent returns. In a down market, the historical risk premium becomes negative, but the bad news pushes up the forward-looking risk premium to result in a negative correlation. The correlation between the risk premiums from the CAPM and the Fama-French three-factor model are strongly positive (0.75). Moreover, both risk premiums are positively correlated with the five-year historical risk premium (0.43 for the CAPM and 0.71 for the Fama-French three-factor model). In contrast, neither of the two risk premiums from applying the Fama-Macbeth estimation approach is significantly correlated with the forward-looking risk premium (-0.15 for the CAPM and -0.04 for the Fama-French three-factor model). The message from the empirical analysis is clear; that is, forward-looking premium differs from historical risk premium (or premiums deduced from the CAPM and the Fama-French three-factor model) not just in concept but also in reality.

Earlier in Figure 3, we examined forward-looking risk premiums in relation to the NBER recessions and other crisis periods. To better appreciate how the historical risk premium and the one from the Fama-French three-factor model behave over different market conditions, we add them to the same graph. It is evident from Figure 3 that these two risk premium measures are hardly reflective of the NBER recessions or crises, a behavior that sharply differs from that of forward-looking risk premium.

4.4.2 Fama and French (2002) Equity Premium

Fama and French (2002) estimated expected stock returns using average dividend yield plus the average rate of capital gain estimated by dividend and earnings growth rate. Their approach provide us an alternative measure of expected risk premium, which can be compared with forward-looking risk premium. By their approach, the real S&P500 risk premium based on earnings growth is

$$RXY_t = D_t/P_{t-1} + GY_t - F_t \tag{8}$$

and based on dividend growth is

$$RXD_t = D_t/P_{t-1} + GD_t - F_t \tag{9}$$

where D_t/P_{t-1} is the real dividend yield; GY_t and GD_t are the estimates of real capital gains using realized earnings and dividend growths, respectively; and F_t is the real risk-free interest rate. We obtain quarterly S&P500 earnings and dividends from Compustat and estimate the real risk premiums quarterly from 2001 to 2008.

Quarterly estimates for the real risk premium are reported in Table 7. Panel A presents the estimates based on earnings growth whereas Panel B reports the estimates based on dividend growth. During bad times, both realized earnings and dividend drop substantially, so the estimated capital gain yield is substantially negative, which in turns causes the risk premium to be negative. This effect is more pronounced for the estimates based on realized earnings than the ones based on dividends. This is true because realized earnings can be negative, but realized dividends are bounded below by zero. For example, at the peak of the sub-prime mortgage crisis in the fourth quarter of 2008, the real risk premium estimate was -274%. The annual average real risk premiums were negative for 2001, 2007 and 2008 when the estimates are based on earnings growth. The annual average real risk premiums were negative in year 2001 and 2008 when estimation is based on dividend growth. In order to obtain a positive average risk premium using the Fama and French (2002) approach, a much longer window will be needed, but then it will have the same drawback as historical risk premium.

To compare the Fama and French (2002) quarterly risk premium measure with the forward-looking risk premium, we re-estimate the quarterly physical forward-looking moments (variance, skewness and kurtosis) at each quarter end. τ is set to be the number of calender days in the forward-looking quarter, but the actual number of trading days in the quarter are applied to estimate forward-looking moments. Then, we calculate quarterly forward-looking risk premium using the most recent estimate of γ available at the quarter end. The Spearman correlation coefficients among the forward-looking risk premium at quarter $t-1$ for quarter t (i.e., $FLRP_{t-1}(\tau)$), the quarterly risk premium based on earnings growth (RXY_t) and that based on dividend growth (RXD_t) are presented in Panel C of Table 7. None of them is significantly correlated with the other, which implies that the risk premium based on the Fama and French (2002) method and the forward-looking risk premium are basically two distinctly different risk premium measures. The Fama and French (2002) risk premium is in essence an ex-post fundamental measure of risk premium, whereas the forward-looking risk premium is an ex-ante measure of risk compensation demanded by investors.

5 Asset Pricing Implications of Forward-Looking Risk Premium

5.1 Change in Forward-Looking Risk Premium and Excess Holding Period Return

A common approach to asset valuation is to set price equal to the present value of its expected future cash flows discounted by the cost of capital (the risk-free interest rate plus a risk premium). An increase in price is therefore related either to an increase of the expected future cash flows or a decrease in the risk premium (assuming the same risk-free rate). French, Schwert and Stambaugh (1987) tested this idea indirectly by assuming that the change in risk premium is positively related to the unexpected change in stock market volatility. Without controlling for the cash flow effect, they found that an unexpected change in market volatility negatively affects the stock's holding period return.

In this section, we test the holding period return implication directly with respect to the change in the forward looking risk premium while controlling for the change in the expected earnings. Our empirical model for this analysis is:

$$R_{mt} - R_{ft} = \alpha + \beta_1 \Delta FLRP_t(\tau) + \beta_2 \Delta EPS_t^e + \epsilon_t \tag{10}$$

Where R_{mt} is the quarterly holding period return (from quarter $t-1$ to t) for the S&P500 index; R_{ft} is the 3-month Treasury bill return from quarter $t-1$ to t; $\Delta FLRP_t(\tau)$ is defined as the change in the forward-looking risk premium from quarter $t-1$ to t (i.e., $FLRP_t(\tau) - FLRP_{t-1}(\tau)$). Similar to Section 4.4.2, quarterly forward-looking risk premiums are constructed for each quarter end. ΔEPS_t^e is the expected change in earnings per share for the S&P500 index. Two proxies are used for EPS_t^e: the actual EPS data from Compustat and the analysts' forecast from I/B/E/S.

Our prediction on the regression coefficients is: $\beta_1 < 0$ and $\beta_2 > 0$, corresponding to an increase in expected risk premium decreases current stock price (holding period return), and an increase in expected future earnings per share increases current stock price. The results reported in Table 8 are consistent with the predictions. Model (1) in Table 8 uses realized EPS as a proxy for expected EPS where Model (2) uses the mean of analysts' EPS forecasts for next quarter as expected EPS. Both tests give us consistent results. The coefficients for $\Delta FLRP_t$ are significantly negative (-0.031 and -0.059) and the ones for ΔEPS_t^e are significantly positive (0.029 and 0.032). The constant term is insignificant in either case. The regression results confirm the theoretical prediction that an increase in discount rate (forward-looking risk premium) negatively affects current stock price (holding period return) while controlling for expected change in future cash flows (change in EPS). A similar conclusion holds when the median of analysts' EPS forecasts instead of the mean is used.

5.2 Liquidity and the Forward-Looking Risk Premium

Illiquidity is commonly perceived as a risk. Equity investors will require a larger risk premium if they may face a large discount on future asset value or have to pay a higher transaction cost to liquidate their positions. Many studies have examined the relationship between stock return and liquidity cross-sectionally, and recognized the existence of liquidity risk (Pastor and Stambaugh, 2003; Acharya and Pedersen, 2005). Amihud (2002) analyzed annual data from 1964 to 1996, and reported a positive relationship between expected market illiquidity and excess returns. His finding serves as direct evidence that illiquidity premium is reflected in excess return. He also found a negative relationship between unexpected illiquidity and contemporaneous excess return. This negative relationship suggests that stock price falls when illiquidity unexpectedly rises, and can be used as indirect evidence that illiquidity affects expected stock returns, assuming firm's profitability is unaffected by market illiquidity.

In this section, we first show that the positive relationship between expected market illiquidity and excess return disappears from the monthly data between 2001 and 2008. However, illiquidity risk premium is still contained in forward-looking risk premium.

Similar to Amihud (2002), monthly market illiquidity denoted by $MILLIQ_m$ is estimated as the average of $|R_{idm}|/VOLD_{idm}$ across all NYSE stocks and over all days in the month, where $|R_{idm}|$ is the absolute return of stock i on day d of month m, and $VOLD_{idm}$ is the corresponding daily trading volume. The monthly unexpected liquidity denoted by $MILLIQ_m^U$ is the residual from applying the AR(1) model to $MILLIQ_m$. Using monthly data from January 2001 to December 2008, we replicate the Amihud (2002) regression:

$$R_{Mt} - R_{ft} = g_0 + g_1 \ln(MILLIQ_{t-1}) + g_2 \ln(MILLIQ_t^U) + g_3 JANDUM_t + w_t \qquad (11)$$

where $R_{Mt} - R_{ft}$ is the excess stock return for month t and $JANDUM_t$ is the dummy variable for January. The regression results are reported in Table 9. Amihud and Hurvich (2004) pointed out that the predictive regression such as equation (11) produces biased estimates, but the bias can be corrected by adjusting the coefficient of the AR(1) model for illiquidity and also adjusting the standard error for g_1. We apply their adjustment and report the t-values computed from the adjusted standard errors.[7] Consistent with Amihud (2002), the monthly unexpected illiquidity negatively affects excess stock returns. However, the lagged illiquidity does not positively affect excess returns. As a robustness check, we re-run the regression using the full monthly sample from 1964 to 2008, the coefficient of $\ln(MILLIQ_{t-1})$ is 0.003 with a t-value equal to 1.65. The results suggest that the lagged illiquidity (as a measure of expected illiquidity) marginally exhibits a positive relationship with excess return, but shows no significant relationship for the more recent period.

The lack of a significant relationship between expected illiquidity and excess return may be indicative of the poor quality of excess return as a proxy for risk premium. Therefore, we use forward-looking risk premium to test whether risk premium is related to expected illiquidity. To be compatible with the monthly horizon used in the liquidity measure, we re-calculate the monthly forward-looking risk premium at each month end using the most

[7] Similar results are obtained if we apply the adjustment procedure as in Amihud (2002).

recent γ available at the time.[8] We run the regression:

$$FLRP_{t-1}(\tau) = \beta_0 + \beta_1 \ln(MILLIQ_{t-1}) + \beta_2 JANDUM_t + \epsilon_t \tag{12}$$

where $FLRP_{t-1}(\tau)$ is the forward-looking risk premium for the next month (month t) at the end of month $t-1$. The Newey-West adjusted standard errors are used to calculate the t-values. The regression results reported in Table 9 show that from 2001 to 2008, the lagged illiquidity (as a measure of expected illiquidity) does not positively affect realized excess return, but it does positively influence forward-looking risk premium.

The prior literature suggests that illiquidity and volatility are positively related (Grossman and Miller, 1988; Deuskar, 2006; Kang and Yeo, 2008). The positive correlation between illiquidity and forward-looking volatility is confirmed in Table 9. To check whether the positive relationship between forward-looking risk premium and illiquidity is merely an manifestation of the illiquidity-volatility relationship, we take the component, $(\gamma-1)\sigma^2_{Pt}(\tau)$, out of the forward-looking risk premium estimate and re-run the regression in equation (12). The results in Table 9 show that illiquidity still positively affects the forward-looking risk premium after taking out the variance component. Because forward-looking risk premium comprises forward-looking variance, skewness and kurtosis, our results suggest that illiquidity risk premium partly reflects forward-looking skewness and kurtosis. We thus examine the relationship between illiquidity and forward-looking skewness (or kurtosis) and report the results in the last two columns of Table 9. The results show that illiquidity negatively (positively) affects skewness (kurtosis). The relationship between illiquidity and skewness (or kurtosis) are consistent with the intuition that when liquidity dries up, investors face higher uncertainty and may particularly worry about a large drop in stock price (negative skewness) or extreme moves in price (fat tails). In summary, illiquidity risk premium is reflected in forward-looking risk premium, and the relationship is not merely a manifestation of the illiquidity-volatility relationship.

6 Conclusion

We propose a practical model for estimating forward-looking risk premium. First, a forward-looking risk premium formula is developed. Then, the components of this formula – physical forward-looking volatility, skewness and kurtosis, and investors' risk aversion – are estimated. The GARCH model is used to deduce forward-looking physical volatility, skewness and kurtosis needed for the implementation. Investors' risk aversion is estimated by a volatility spread formula that links the gap between risk-neutral volatility and forward-looking physical volatility to forward-looking skewness and kurtosis.

[8] Using the original forward-looking risk premium for 28 calendar days (in section 4.3) does not change our conclusion.

Our empirical analysis uses the S&P500 index return and option data. The estimates for investors' risk aversion are sensible with values in the range from 1.8 to 7.1. The estimated forward-looking risk premium are consistently positive. In sharp contrast, other commonly used risk premium measures, such as historical average excess return, estimates using the CAPM and the Fama-French three-factor model, and the Fama-French (2002) fundamental estimate, are often empirically negative. Obviously, negative risk premiums are theoretically questionable, intuitively unappealing and practically unusable. Furthermore, forward-looking risk premium is higher during the crisis period and lower during the boom time, exhibiting a desirable feature that is in keeping with economic intuition.

Two asset pricing implications related to forward-looking risk premium are also examined in this paper. The change in forward-looking risk premium negatively affects current stock price, and expected illiquidity positively affects forward-looking risk premium. Both are consistent with financial theory and economic intuition. Given the prominent role played by risk premium in finance, our proposed estimation method for forward-looking risk premium can have wide-ranging implications in financial research and practice.

References

Acharya, Viral V. and Lasse Heje Pedersen, 2005, Asset Pricing with Liquidity Risk, *Journal of Financial Economics* 77, 375-410.

Amihud, Yakov, 2002, Illiquidity and Stock Returns: Cross-Section and Time-Series Effects, *Journal of Financial Markets* 5, 21-56.

Amihud, Yakov and Clifford M. Hurvich, 2004, Predictive Regressions: A Reduced-Bias Estimation Method, *Journal of Financial and Quantitaive Analysis* 39, 813-841.

Arnott, Robert D. and Peter L. Bernstein, 2002, What Risk Premium Is "Normal"?, *Financial Analysts Journal* 58, 64-85.

Bakshi, Gurdip and Dilip B. Madan, 2006, A Theory of Volatility Spread, *Management Science* 52, 1945-1956.

Bhar, Ramaprasad, Carl Chiarella and Wolfgang J. Runggaldier, 2004, Inferring the Forward Looking Equity Risk Premium from Derivative Prices,*Studies in Nonlinear Dynamics & Econometrics*, 8, Article 3.

Bliss, R. Robert and Nikolaos Panigirtzoglou, 2004, Option-Implied Risk Aversion Estimates, *Journal of Finance* 59, 407-446.

Britten-Jones, Mark and Anthony Neuberger, 2000, Option Price, Implied Price Processes, and Stochastic Volatility, *Journal of Finance* 55, 839-866.

Campbell, John Y. and Cochrane, John H. , 2000, Explaining the Poor Performance of Consumption-based Asset Pricing Models, *Journal of Finance* 55, 2863-2878.

Carr, Peter and Dilip B. Madan, 2001, Optimal Positioning in Derivative Securities, *Quantitative Finance* 1, 19-37.

Christensen, Bent Jesper and Nagpurnanand R. Prabhala, 1998, The Relation between Implied and Realized Volatility, *Journal of Financial Economics* 50, 125-150.

Damodaran, Aswath, 2008, Equity Risk Premiums (ERP): Determinants, Estimation and Implications, NYU Stern School of Business working paper.

Deuskar, Prachi, 2006, Extrapolative Expectations: Implications for Volatility and Liquidity, NYU Stern School of Business working Paper.

Duan, Jin-Chuan, 1997, Augmented GARCH(p,q) Process and its Diffusion Limit, *Journal of Econometrics* 79, 97-127.

Elton, Edwin J., 1999, Expected Return, Realized Return, and Asset Pricing Tests, *Journal of Finance* 54, 1199-1220.

Engle, Robert and Ng, Victor, 1993, Measuring and Testing of the Impact of News on Volatility, *Journal of Finance* 48, 1749-1778.

Fama, Eugene F. and Kenneth R. French, 1996, Multifactor Explanations of Asset Pricing Anomalies, *Journal of Finance* 51, 55-84.

Fama, Eugene F. and Kenneth R. French, 2002, The Equity Premium, *Journal of Finance* 57, 637-659.

Fama, Eugene F. and James D. Macbeth, 1973, Risk, Return and the Equilibrium: Empirical Tests, *Journal of Political Economy* 82, 607-636.

Fernandez, Pablo, 2009a, Market Risk Premium Used in 2008 by Professors: A Survey with 1,400 Answers, IESE Business School working paper.

Fernandez, Pablo, 2009b, The Equity Premium in 150 Textbooks, IESE Business School working paper.

French, Kenneth R., G.William Schwert and Robert F. Stambaugh, 1987, Expected Stock Returns and Volatility, *Journal of Financial Economics* 19, 3-29.

Graham, John R., and Campbell R. Harvey, 2007, The Equity Risk Premium in January 2007: Evidence from the Global CFO Outlook Survey, *The ICFAI Journal of Financial Risk Management* 4, 46-61.

Grossman, Sanford J., and Merton H. Miller, 1988, Liquidity and Market Structure, *Journal of Finance* 43, 617-637.

Grossman, Sanford J., and Robert J. Shiller (1981), The determinants of the variability of stock market prices, *American Economic Review* 71, 222-227.

Hansen, Lars Peter, and Kenneth J. Singleton, 1983, Stochastic consumption, risk aversion, and the temporal behavior of asset returns, *Journal of Political Economy* 91, 249-268.

Harvey, Campbell R. and Akhtar Siddique, 2000, Conditional Skewness in Asset Pricing Tests, *Journal of Finance* 55, 1263-1295.

Jackwerth, Jens Carsten , and Mark Rubinstein, 1996, Recovering probability distributions from option prices, *Journal of Finance* 51, 1611-1631.

Jiang, George J. and Yisong S. Tian, 2005, The Model-Free Implied Volatility and its Information Content, *Review of Financial Studies* 18, 1305-1342.

Kang, Wenjin and Wee Yong Yeo, 2008, Liquidity Beyond the Best Quote: A Study of the NYSE Limit Order Book, National University of Singapore working paper.

Kraus, Alan and Robert H. Litzenberger, 1976, Skewness Preference and the Valuation of Risk Assets, *Journal of Finance* 31, 1085-1100.

Merton, Robert C., 1980, On Estimating the Expected Return on the Market: An Exploratory Investigation, *Journal of Financial Economics* 8, 323-361.

Pastor, Lubos and Robert F. Stambaugh, 2003, Liquidity Risk and Expected Stock Retums, *Journal of Political Economy* 111, 642-685.

Pitt, Michael K., 2002, Smooth Particle Filters for Likelihood Evaluation and Maximisation, University of Warwick working paper

Santa-Clara, Pedro and Shu Yan, 2010, Crashes, Volatility and the Equity Premium: Lessons From S&P 500 Options, *The Review of Economics and Statistics* 92, 435-451.

Welch, Ivo, 2000, Views of Financial Economists on the Equity Premium and on Professional Controversies, *Journal of Business* 73, 501-537.

Appendix A. Derivation of Equation (1)

First expand

$$\begin{aligned} e^{R_t(\tau)-\mu_{Qt}(\tau)} &= 1 + R_t(\tau) - \mu_{Qt}(\tau) + \frac{(R_t(\tau)-\mu_{Qt}(\tau))^2}{2} + \frac{(R_t(\tau)-\mu_{Qt}(\tau))^3}{6} \\ &\quad + \frac{(R_t(\tau)-\mu_{Qt}(\tau))^4}{24} + O\left[(R_t(\tau)-\mu_{Qt}(\tau))^5\right]. \end{aligned}$$

Taking expectation with respect to measure Q and recognizing $E_t^Q\left(e^{R_t(\tau)}\right) = e^{r_t(\tau)-\delta_t(\tau)}$ in turn give rise to

$$e^{r_t(\tau)-\delta_t(\tau)-\mu_{Qt}(\tau)} = 1 + \frac{\sigma^2_{Qt}(\tau)}{2} + \frac{\theta_{Qt}(\tau)\sigma^3_{Qt}(\tau)}{6} + \frac{\kappa_{Qt}(\tau)\sigma^4_{Qt}(\tau)}{24} + o\left[\sigma^4_{Qt}(\tau)\right].$$

Therefore, the equilibrium risk-free interest rate can be written as

$$\begin{aligned} r_t(\tau) &= \delta_t(\tau) + \mu_{Qt}(\tau) + \ln\left(1 + \frac{1}{2}\sigma^2_{Qt}(\tau) + \frac{1}{6}\theta_{Qt}(\tau)\sigma^3_{Qt}(\tau) + \frac{1}{24}\kappa_{Qt}(\tau)\sigma^4_{Qt}(\tau) + o\left[\sigma^4_{Qt}(\tau)\right]\right) \\ &= \delta_t(\tau) + \mu_{Qt}(\tau) + \frac{1}{2}\sigma^2_{Qt}(\tau) + \frac{1}{6}\sigma^3_{Qt}(\tau)\theta_{Qt}(\tau) + \frac{1}{24}\sigma^4_{Qt}(\tau)[\kappa_{Qt}(\tau)-3] + o\left[\sigma^4_{Qt}(\tau)\right]. \end{aligned}$$

The second equality comes from a second-order Taylor expansion of the logarithmic function around 1. The term $o\left[\sigma^4_{Qt}(\tau)\right]$ can be ignored because the typical estimate suggests that $\sigma_{Qt}(\tau)$ on an annualized basis is less than 1. Applying to the monthly or quarterly cumulative return, it would be even smaller.

Appendix B. Derivation of Equation (3)

Our derivations for the following equations are essentially same as Bakshi and Madan (2006) except for two subtle points. First, our approximation ignores terms with an order higher than $\sigma^4_{Qt}(\tau)$ whereas their approach drops terms with an order of γ^3 or higher. Since the estimated risk-aversion coefficient is typically large (Bakshi and Madan (2006)'s own estimate for γ is around 17), it is questionable to ignore terms in the order of γ^3 or higher. However, the volatility for an equity index such as S&P 500 is typically below 20% per annum, which makes its 5-th or higher power indeed negligible. Second, Bakshi and Madan (2006) assumed that the physical first moment equals zero, which turns out to be not needed.

Instead of dealing with the moment generating function of $R_t(\tau)$ directly, it is analytically more convenient to compute that for $R^*_t(\tau) = R_t(\tau) - \mu_{Pt}(\tau)$. We have

$$\mathcal{C}_t(\lambda) \equiv E_t^P\left(e^{\lambda R^*_t(\tau)}\right) = 1 + \frac{\lambda^2}{2}\sigma^2_{Pt}(\tau) + \frac{\lambda^3}{6}\theta_{Pt}(\tau)\sigma^3_{Pt}(\tau) + \frac{\lambda^4}{24}\kappa_{Pt}(\tau)\sigma^4_{Pt}(\tau) + o\left[\lambda^4\sigma^4_{Pt}(\tau)\right].$$

This in turn allows one to express the moment generating function of $R_t^*(\tau)$ under measure Q using $\mathcal{C}_t(\cdot)$ as follows:

$$E_t^Q\left(e^{\lambda R_t^*(\tau)}\right) = \frac{E_t^P\left(e^{\lambda R_t^*(\tau)}e^{-\gamma R_t(\tau)}\right)}{E_t^P\left(e^{-\gamma R_t(\tau)}\right)} = \frac{E_t^P\left(e^{(\lambda-\gamma)R_t^*(\tau)}e^{-\gamma\mu_{Pt}}\right)}{E_t^P\left(e^{-\gamma R_t^*(\tau)}e^{-\gamma\mu_{Pt}}\right)} = \frac{\mathcal{C}_t(\lambda-\gamma)}{\mathcal{C}_t(-\gamma)}.$$

Thus,

$$\begin{aligned}
&E_t^Q\left(R_t^*(\tau)\right) = \frac{\mathcal{C}_t'(\lambda-\gamma)|_{\lambda=0}}{\mathcal{C}_t(-\gamma)} \\
= \;& \left(1+\frac{\gamma^2}{2}\sigma_{Pt}^2(\tau)+O\left[\sigma_{Pt}^3(\tau)\right]\right)^{-1}\left(-\gamma\sigma_{Pt}^2(\tau)+\frac{\gamma^2}{2}\theta_{Pt}(\tau)\sigma_{Pt}^3(\tau)-\frac{\gamma^3}{6}\kappa_{Pt}(\tau)\sigma_{Pt}^4(\tau)+o\left[\sigma_{Pt}^4(\tau)\right]\right) \\
= \;& -\gamma\sigma_{Pt}^2(\tau)+\frac{\gamma^2}{2}\sigma_{Pt}^3(\tau)\theta_{Pt}(\tau)-\frac{\gamma^3}{6}\sigma_{Pt}^4(\tau)[\kappa_{Pt}(\tau)-3]+o\left[\sigma_{Pt}^4(\tau)\right]
\end{aligned}$$

$$\begin{aligned}
&E_t^Q\left[(R_t^*(\tau))^2\right] = \frac{\mathcal{C}_t''(\lambda-\gamma)|_{\lambda=0}}{\mathcal{C}_t(-\gamma)} \\
= \;& \left(1+\frac{\gamma^2}{2}\sigma_{Pt}^2(\tau)+O\left[\sigma_{Pt}^3(\tau)\right]\right)^{-1}\left(\sigma_{Pt}^2(\tau)-\gamma\theta_{Pt}(\tau)\sigma_{Pt}^3(\tau)+\frac{\gamma^2}{2}\kappa_{Pt}(\tau)\sigma_{Pt}^4(\tau)+o\left[\sigma_{Pt}^4(\tau)\right]\right) \\
= \;& \sigma_{Pt}^2(\tau)-\gamma\sigma_{Pt}^3(\tau)\theta_{Pt}(\tau)+\frac{\gamma^2}{2}\sigma_{Pt}^4(\tau)[\kappa_{Pt}(\tau)-1]+o\left[\sigma_{Pt}^4(\tau)\right]
\end{aligned}$$

$$\begin{aligned}
&E_t^Q\left[(R_t^*(\tau))^3\right] = \frac{\mathcal{C}_t'''(\lambda-\gamma)|_{\lambda=0}}{\mathcal{C}_t(-\gamma)} \\
= \;& \left(1+\frac{\gamma^2}{2}\sigma_{Pt}^2(\tau)+O\left[\sigma_{Pt}^3(\tau)\right]\right)^{-1}\left(\theta_{Pt}(\tau)\sigma_{Pt}^3(\tau)-\gamma\kappa_{Pt}(\tau)\sigma_{Pt}^4(\tau)+o\left[\sigma_{Pt}^4(\tau)\right]\right) \\
= \;& \theta_{Pt}(\tau)\sigma_{Pt}^3(\tau)-\gamma\kappa_{Pt}(\tau)\sigma_{Pt}^4(\tau)+o\left[\sigma_{Pt}^4(\tau)\right]
\end{aligned}$$

$$\begin{aligned}
&E_t^Q\left[(R_t^*(\tau))^4\right] = \frac{\mathcal{C}_t''''(\lambda-\gamma)|_{\lambda=0}}{\mathcal{C}_t(-\gamma)} \\
= \;& \left(1+\frac{\gamma^2}{2}\sigma_{Pt}^2(\tau)+O\left[\sigma_{Pt}^3(\tau)\right]\right)^{-1}\left(\kappa_{Pt}(\tau)\sigma_{Pt}^4(\tau)+o\left[\sigma_{Pt}^4(\tau)\right]\right) \\
= \;& \kappa_{Pt}(\tau)\sigma_{Pt}^4(\tau)+o\left[\sigma_{Pt}^4(\tau)\right],
\end{aligned}$$

The above results immediately give rise to the risk neutral expected return, variance, skewness and kurtosis as follows:

$$
\begin{aligned}
\mu_{Qt}(\tau) &= E_t^Q\left(R_t^*(\tau)\right) + \mu_{Pt}(\tau) \\
&= \mu_{Pt}(\tau) - \gamma\sigma_{Pt}^2(\tau) + \frac{\gamma^2}{2}\sigma_{Pt}^3(\tau)\theta_{Pt}(\tau) - \frac{\gamma^3}{6}\sigma_{Pt}^4(\tau)[\kappa_{Pt}(\tau)-3] \\
&\quad + o\left[\sigma_{Pt}^4(\tau)\right]. \qquad (13) \\
\sigma_{Qt}^2(\tau) &= E_t^Q\left[(R_t^*(\tau))^2\right] - [\mu_{Qt}(\tau) - \mu_{Pt}(\tau)]^2 \\
&= \sigma_{Pt}^2(\tau) - \gamma\sigma_{Pt}^3(\tau)\theta_{Pt}(\tau) + \frac{\gamma^2}{2}\sigma_{Pt}^4[\kappa_{Pt}(\tau)-3] + o\left[\sigma_{Pt}^4(\tau)\right]. \qquad (14) \\
\theta_{Qt}(\tau)\sigma_{Qt}^3(\tau) &= E_t^Q\left[(R_t^*(\tau))^3\right] - 3\sigma_{Qt}^2(\tau)[\mu_{Qt}(\tau)-\mu_{Pt}(\tau)] - [\mu_{Qt}(\tau)-\mu_{Pt}(\tau)]^3 \\
&= \sigma_{Pt}^3(\tau)\theta_{Pt}(\tau) - \gamma\sigma_{Pt}^4[\kappa_{Pt}(\tau)-3] + o\left[\sigma_{Pt}^4(\tau)\right]. \qquad (15) \\
\kappa_{Qt}(\tau)\sigma_{Qt}^4(\tau) &= E_t^Q\left[(R_t^*(\tau))^4\right] - 4\theta_{Qt}\sigma_{Qt}^3(\tau)[\mu_{Qt}(\tau)-\mu_{Pt}(\tau)] \\
&\quad - 6\sigma_{Qt}^2(\tau)[\mu_{Qt}(\tau)-\mu_{Pt}(\tau)]^2 - [\mu_{Qt}(\tau)-\mu_{Pt}(\tau)]^4 \\
&= \kappa_{Pt}(\tau)\sigma_{Pt}^4(\tau) + o\left[\sigma_{Pt}^4(\tau)\right]. \qquad (16)
\end{aligned}
$$

Note that equation (14) can be rewritten as a volatility spread as in equation (2).

Substitute equations (13)-(16) to the risk-free interest rate equation (1), we can express the equilibrium risk-free interest rate in terms of physical moments:

$$
\begin{aligned}
r_t(\tau) \approx\ & \delta_t(\tau) + \mu_{Pt}(\tau) - \left(\gamma - \frac{1}{2}\right)\sigma_{Pt}^2(\tau) + \frac{3\gamma^2 - 3\gamma + 1}{6}\sigma_{Pt}^3(\tau)\theta_{Pt}(\tau) \\
& - \frac{4\gamma^3 - 6\gamma^2 + 4\gamma - 1}{24}\sigma_{Pt}^4(\tau)[\kappa_{Pt}(\tau) - 3].
\end{aligned}
$$

Consequently, the equity risk premium can be expressed as

$$
\begin{aligned}
\mu_{Pt}(\tau) + \delta_t(\tau) - r_t(\tau) \approx\ & \left(\gamma - \frac{1}{2}\right)\sigma_{Pt}^2(\tau) - \frac{3\gamma^2 - 3\gamma + 1}{6}\sigma_{Pt}^3(\tau)\theta_{Pt}(\tau) \\
& + \frac{4\gamma^3 - 6\gamma^2 + 4\gamma - 1}{24}\sigma_{Pt}^4(\tau)[\kappa_{Pt}(\tau) - 3].
\end{aligned}
$$

Remark: Ignoring terms with an order of γ^3 or higher as in Bakshi and Madan (2006) does not affect the volatility spread equation, but it alters the equation for the risk-neutral first moment. Were γ small, the term $\frac{\gamma^3}{6}\sigma_{Pt}^4(\tau)[\kappa_{Pt}(\tau)-3]$ in (13) could be grouped into $O(\gamma^3)$ and ignored. Interestingly, all $O(\gamma^3)$ except for $\frac{\gamma^3}{6}\sigma_{Pt}^4(\tau)[\kappa_{Pt}(\tau)-3]$ are in an order higher than $\sigma_{Pt}^4(\tau)$, and are thus in agreement with our approach.

Appendix C. Risk-Neutral Variance as an Option Portfolio

A continuous twice-differentiable function payoff function $f(S)$ can be represented in a integral form as follows (see Carr and Madan (2001) and Bakshi and Madan (2006)):

$$f(S) = f(\bar{S}) + (S - \bar{S})f_S(\bar{S}) + \int_{\bar{S}}^{\infty} f_{SS}(k)(S-k)^+ dk + \int_0^{\bar{S}} f_{SS}(k)(k-S)^+ dk$$

where $f_S(\cdot)$ and $f_{SS}(\cdot)$ are the first and second derivatives, respectively. Expand $f(S_{t+\tau})$ around K_t, a point close to the forward price $F_t(\tau) = S_t e^{(r_t(\tau)-\delta_t(\tau))\tau}$.

Apply the risk-neutral measure at time t to yield

$$\begin{aligned}
E_t^Q\left[f(S_{t+\tau})\right] &= f(K_t) + (E_t^Q\left[S_{t+\tau}\right] - K_t)f_S(K_t) + \int_{K_t}^{\infty} f_{SS}(k)E_t^Q\left[(S_{t+\tau}-k)^+\right]dk \\
&\quad + \int_0^{K_t} f_{SS}(k)E_t^Q\left[(k-S_{t+\tau})^+\right]dk \\
&= f(K_t) + \left[S_t(\tau)e^{(r_t(\tau)-\delta_t(\tau))\tau} - K_t\right]f_S(K_t) + e^{r_t(\tau)\tau}\int_{K_t}^{\infty} f_{SS}(k)C(k;S_t,\tau)dk \\
&\quad + e^{r_t(\tau)\tau}\int_0^{K_t} f_{SS}(k)P(k;S_t,\tau)dk
\end{aligned}$$

where $C(k;S_t,\tau)$ and $P(k;S_t,\tau)$ are the time-t European call and put option prices with strike price k and maturity τ.

Define $w_1(k) \equiv \frac{1}{k^2}$, $w_2(k;S_t) \equiv \frac{2\left[1-\ln\left(\frac{k}{S_t}\right)\right]}{k^2}$. The following expressions can be easily derived:

$$\begin{aligned}
E_t^Q\left[R_t(\tau)\right] &= \ln\left(\frac{K_t}{S_t}\right) + \frac{F_t(\tau)-K_t}{K_t} - e^{r_t(\tau)\tau}\int_{K_t}^{\infty} w_1(k)C(k;S_t,\tau)dk \\
&\quad - e^{r_t(\tau)\tau}\int_0^{K_t} w_1(k)P(k;S_t,\tau)dk \\
E_t^Q\left[R_t^2(\tau)\right] &= \left[\ln\left(\frac{K_t}{S_t}\right)\right]^2 + 2\left(\frac{F_t(\tau)-K_t}{K_t}\right)\ln\left(\frac{K_t}{S_t}\right) \\
&\quad + e^{r_t(\tau)\tau}\int_{K_t}^{\infty} w_2(k;S_t)C(k;S_t,\tau)dk + e^{r_t(\tau)\tau}\int_0^{K_t} w_2(k;S_t)P(k;S_t,\tau)dk
\end{aligned}$$

They then give rise to the risk-neutral variance as a portfolio of options by noting that $\sigma_{Qt}^2(\tau) = E_t^Q\left[R_t^2(\tau)\right] - \left(E_t^Q\left[R_t(\tau)\right]\right)^2$.

Appendix D. Cumulative Return Moments under NGARCH(1,1)

Under the NGARCH(1,1) model in equations (5)-(6), the conditional variance of the cumulative return over τ-days can be derived as follows:

$$
\begin{aligned}
\sigma_{Pt}^2(\tau) &= E_t^P\left[\ln\left(\frac{S_{t+\tau}}{S_t}\right) - E_t^P\left(\ln\left(\frac{S_{t+\tau}}{S_t}\right)\right)\right]^2 \\
&= E_t^P\left[\sum_{i=1}^{\tau}\ln\left(\frac{S_{t+i}}{S_{t+i-1}}\right) - E_t^P\left(\sum_{i=1}^{\tau}\ln\left(\frac{S_{t+i}}{S_{t+i-1}}\right)\right)\right]^2 \\
&= E_t^P\left[\mu\tau + \sum_{i=1}^{\tau}\sigma_{t+i}\varepsilon_{t+i} - E_t^P\left(\mu\tau + \sum_{i=1}^{\tau}\sigma_{t+i}\varepsilon_{t+i}\right)\right]^2 \\
&= E_t^P\left[\sum_{i=1}^{\tau}\sigma_{t+i}\varepsilon_{t+i}\right]^2 \\
&= \sum_{i=1}^{\tau}E_t^P\left(\sigma_{t+i}^2\right)
\end{aligned}
$$

It is clear that $\sigma_{Pt}^2(1) = \sigma_{t+1}^2$. When $\tau \geq 2$, then,

$$
\begin{aligned}
\sigma_{Pt}^2(\tau) &= \sigma_{t+1}^2 + \sum_{i=1}^{\tau-1}E_t^P\left[\beta_0 + \beta_1\sigma_{t+i}^2 + \beta_2\sigma_{t+i}^2(\varepsilon_{t+i}-\eta)^2\right] \\
&= \sigma_{t+1}^2 + \sum_{i=1}^{\tau-1}\left\{\beta_0 + \left[\beta_1 + \beta_2(1+\eta^2)\right]E_t^P\left(\sigma_{t+i}^2\right)\right\} \qquad (17)
\end{aligned}
$$

Let $\lambda = \beta_1 + \beta_2(1+\eta^2)$. Recursively apply conditional expectation to equation (6) to yield

$$
E_t^P\left(\sigma_{t+i}^2\right) = \frac{\beta_0}{1-\lambda} + \lambda^{i-1}\left(\sigma_{t+1}^2 - \frac{\beta_0}{1-\lambda}\right).
$$

Plugging the above result into equation (17) gives rise to

$$
\begin{aligned}
\sigma_{Pt}^2(\tau) &= \sigma_{t+1}^2 + \sum_{i=1}^{\tau-1} \left[\beta_0 + \lambda E_t^P\left(\sigma_{t+i}^2\right)\right] \\
&= \sigma_{t+1}^2 + \sum_{i=1}^{\tau-1} \left[\frac{\beta_0}{1-\lambda} + \lambda^i \left(\sigma_{t+1}^2 - \frac{\beta_0}{1-\lambda}\right)\right] \\
&= \left(\sum_{i=0}^{\tau-1} \lambda^i\right) \sigma_{t+1}^2 + \left(\sum_{i=1}^{\tau-1} (1-\lambda^i)\right) \frac{\beta_0}{1-\lambda} \\
&= \frac{1-\lambda^\tau}{1-\lambda} \sigma_{t+1}^2 + \frac{(\tau-1)\beta_0}{1-\lambda} - \frac{\lambda(1-\lambda^{\tau-1})\beta_0}{(1-\lambda)^2}
\end{aligned}
$$

Note that the above formula also applies in the case of $\tau = 1$.

Table 1: Summary statistics of return moments

This table shows the summary statistics for the 28-calender day risk-neutral volatility (σ_Q) and physical forward-looking volatility (σ_P), skewness (θ_P) and kurtosis (κ_P). The sample period is from January 1996 to October 2009. All volatilities are expressed in annualized percentage terms. Risk-neutral volatility (σ_Q) is calculated using S&P500 index option prices. Physical forward-looking volatility (σ_P) is calculated analytically using the NGARCH(1,1) model estimated to a 5-year moving window of the S&P500 index returns. Physical forward-looking skewness (θ_P) and kurtosis (κ_P) are computed by smoothed bootstrap simulations. τ equals 28 calendar days.

	$\sigma_Q(\tau)$	$\sigma_P(\tau)$	$\theta_P(\tau)$	$\kappa_P(\tau)$
# of months	166	166	166	166
Mean	22.00	18.10	-1.05	6.44
Standard Deviation	10.17	10.12	0.39	2.46
Minimum	9.95	7.48	-1.92	3.58
Maximum	84.29	82.46	-0.25	15.89

Table 2: Investors' risk aversion estimates

This table reports the GMM estimation results based on the following orthogonality condition:

$$E\left\{\frac{\sigma_{Qt}^2(\tau)-\sigma_{Pt}^2(\tau)}{\sigma_{Pt}^2(\tau)}+\gamma\theta_{Pt}(\tau)\sigma_{Pt}(\tau)-\frac{\gamma^2}{2}\sigma_{Pt}^2(\tau)[\kappa_{Pt}(\tau)-3]\middle| I_t\right\}=0$$

For every month, we use five-year data preceding the month to estimate the risk aversion (γ). The instruments are constant and the risk-neutral variances being lagged one, two and three periods. The *t*-statistics for γ, denoted by *t*(γ), the J statistics for model over-identification (J-stat) and J-stat's p-value (Model-p) are provided.

End Month	γ	t(γ)	J-stat	Model-p
Jan-01	4.94	3.17	6.46	0.09
Feb-01	4.86	3.09	5.97	0.11
Mar-01	5.15	3.45	4.62	0.20
Apr-01	5.09	3.61	4.37	0.22
May-01	5.12	3.58	4.20	0.24
Jun-01	5.39	3.89	3.55	0.31
Jul-01	5.45	4.22	3.17	0.37
Aug-01	5.47	4.35	3.03	0.39
Sep-01	5.86	4.69	1.89	0.60
Oct-01	6.95	5.97	1.25	0.74
Nov-01	7.10	6.36	1.33	0.72
Dec-01	6.96	6.13	1.38	0.71
Jan-02	6.58	6.03	1.66	0.65
Feb-02	6.44	6.07	1.45	0.69
Mar-02	6.31	6.09	1.39	0.71
Apr-02	6.23	5.95	1.86	0.60
May-02	6.18	6.12	1.81	0.61
Jun-02	5.91	5.70	1.67	0.64
Jul-02	5.21	4.90	1.40	0.70
Aug-02	5.21	5.28	1.48	0.69
Sep-02	5.39	5.28	1.15	0.77
Oct-02	5.01	7.04	1.60	0.66
Nov-02	5.76	6.85	1.26	0.74
Dec-02	4.54	6.21	1.56	0.67
Jan-03	4.34	6.64	2.35	0.50
Feb-03	4.35	6.93	2.09	0.55
Mar-03	4.34	6.93	2.34	0.51
Apr-03	4.27	7.04	2.57	0.46
May-03	4.16	6.92	2.95	0.40
Jun-03	4.01	6.68	3.37	0.34
Jul-03	4.01	6.15	3.65	0.30
Aug-03	3.87	6.56	3.59	0.31
Sep-03	3.76	6.01	4.19	0.24
Oct-03	3.93	5.92	3.14	0.37
Nov-03	3.65	6.01	2.19	0.53
Dec-03	3.53	5.61	2.30	0.51
Jan-04	3.22	5.65	2.79	0.42
Feb-04	3.40	5.20	2.22	0.53
Mar-04	3.17	5.48	2.05	0.56
Apr-04	2.96	5.23	2.33	0.51
May-04	3.01	5.20	2.04	0.57
Jun-04	2.80	5.25	2.33	0.51
Jul-04	2.73	5.12	2.23	0.53
Aug-04	2.71	5.09	2.23	0.53
Sep-04	2.43	5.06	2.71	0.44
Oct-04	2.29	4.72	3.58	0.31
Nov-04	2.61	4.98	2.40	0.49
Dec-04	2.50	5.11	1.81	0.61
Jan-05	1.84	3.58	2.94	0.40
Feb-05	1.94	4.22	3.25	0.35
Mar-05	1.94	4.02	3.17	0.37
Apr-05	1.85	3.44	3.47	0.32
May-05	2.05	4.21	3.31	0.35

End Month	γ	t(γ)	J-stat	Model-p
Jun-05	2.24	4.78	3.09	0.38
Jul-05	2.27	4.90	3.05	0.38
Aug-05	2.42	5.30	2.87	0.41
Sep-05	2.41	5.16	2.93	0.40
Oct-05	2.51	5.40	2.92	0.40
Nov-05	2.66	5.65	2.87	0.41
Dec-05	2.85	6.14	2.58	0.46
Jan-06	3.09	6.00	2.37	0.50
Feb-06	3.14	5.70	2.56	0.47
Mar-06	3.31	6.06	2.49	0.48
Apr-06	3.80	6.17	1.74	0.63
May-06	3.90	6.29	1.55	0.67
Jun-06	3.84	6.16	1.73	0.63
Jul-06	4.18	5.91	1.44	0.70
Aug-06	3.84	6.28	1.91	0.59
Sep-06	4.03	5.80	1.82	0.61
Oct-06	3.99	5.25	1.64	0.65
Nov-06	3.82	5.27	0.77	0.86
Dec-06	3.96	5.21	1.67	0.64
Jan-07	4.28	6.90	1.36	0.71
Feb-07	4.49	8.13	1.69	0.64
Mar-07	4.60	8.70	1.70	0.64
Apr-07	4.71	9.35	1.66	0.65
May-07	4.95	10.00	1.67	0.64
Jun-07	4.81	9.74	2.36	0.50
Jul-07	6.19	12.91	0.58	0.90
Aug-07	5.46	10.46	2.42	0.49
Sep-07	6.38	18.71	2.13	0.55
Oct-07	6.38	7.90	2.44	0.49
Nov-07	5.60	9.02	2.53	0.47
Dec-07	5.67	7.75	2.99	0.39
Jan-08	6.19	9.24	2.81	0.42
Feb-08	5.61	5.05	3.00	0.39
Mar-08	6.20	5.78	3.73	0.29
Apr-08	4.16	3.57	2.88	0.41
May-08	4.58	4.82	2.31	0.51
Jun-08	4.66	5.67	2.04	0.56
Jul-08	4.81	6.40	1.82	0.61
Aug-08	5.18	7.17	2.23	0.53
Sep-08	4.77	6.31	1.96	0.58
Oct-08	3.96	5.40	3.14	0.37
Nov-08	3.49	5.69	3.49	0.32
Dec-08	3.47	3.24	3.27	0.35
Jan-09	3.33	3.02	3.15	0.37
Feb-09	2.98	2.62	3.57	0.31
Mar-09	2.99	2.84	3.71	0.29
Apr-09	3.35	3.19	3.24	0.36
May-09	3.61	3.42	3.09	0.38
Jun-09	3.85	3.70	2.80	0.42
Jul-09	4.18	3.74	2.59	0.46
Aug-09	4.67	3.64	2.44	0.49
Sep-09	5.07	3.41	2.43	0.49
Oct-09	5.12	3.40	2.42	0.49
Average	4.25	5.77	2.52	0.49

Table 3: S&P500 forward-looking risk premium estimate

This table reports monthly forward-looking risk premium estimates based on the S&P500 index values and options from January 2001 to October 2009. The numbers reported are annualized but not in percentage. Note that one month is 28 days from the observation date in the month to the subsequent option maturity date (every third Friday of the month). For example, Jan 2001 is from January 19, 2001 to February 16, 2001. The numbers in bold are the particularly large risk premiums.

	Average	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
2001	0.423	0.205	0.263	0.669	0.213	0.136	0.259	0.236	0.150	**2.088**	0.366	0.236	0.252
2002	0.452	0.331	0.312	0.260	0.281	0.310	0.569	**1.672**	0.288	0.691	0.308	0.187	0.212
2003	0.143	0.352	0.220	0.122	0.150	0.131	0.122	0.138	0.111	0.092	0.121	0.097	0.056
2004	0.052	0.047	0.070	0.093	0.061	0.069	0.045	0.068	0.049	0.029	0.046	0.030	0.022
2005	0.025	0.029	0.018	0.026	0.036	0.023	0.016	0.018	0.025	0.028	0.042	0.021	0.017
2006	0.038	0.038	0.026	0.024	0.031	0.055	0.070	0.086	0.039	0.032	0.022	0.018	0.020
2007	0.109	0.023	0.024	0.062	0.039	0.036	0.081	0.100	0.167	0.157	0.177	0.270	0.168
2008	1.011	0.481	0.186	0.426	0.103	0.107	0.187	0.208	0.134	0.430	**5.393**	**3.545**	0.934
2009	0.244	0.627	0.649	0.334	0.154	0.209	0.132	0.078	0.081	0.057	0.122		

Table 4: S&P500 historical risk premium estimates

This table reports historical risk premium estimates using the S&P500 index returns from January 2001 to October 2009. Panel A reports the historical risk premium estimated by averaging excess returns over 3 years, and Panel B reports historical risk premium estimated by averaging excess returns over 5 years. Note that one month is 28 days from the observation date in the month to the subsequent option maturity date (every third Friday of the month). For example, Jan 2001 is from January 19, 2001 to February 16, 2001. The numbers reported are annualized but not in percentage.

Panel A: 3-year historical risk premium

	Average	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
2001	-0.018	0.048	0.037	-0.020	-0.033	-0.005	-0.021	-0.019	0.004	-0.014	-0.060	-0.065	-0.073
2002	-0.136	-0.072	-0.084	-0.088	-0.101	-0.105	-0.134	-0.153	-0.169	-0.160	-0.195	-0.187	-0.188
2003	-0.153	-0.179	-0.195	-0.221	-0.191	-0.172	-0.158	-0.152	-0.158	-0.128	-0.114	-0.094	-0.079
2004	-0.015	-0.067	-0.030	-0.007	-0.036	-0.046	-0.026	-0.021	-0.007	0.029	0.011	0.008	0.018
2005	0.074	0.029	0.022	0.025	0.031	0.040	0.074	0.113	0.104	0.134	0.101	0.096	0.112
2006	0.093	0.135	0.142	0.128	0.110	0.096	0.072	0.074	0.069	0.070	0.072	0.077	0.064
2007	0.068	0.059	0.062	0.056	0.069	0.080	0.082	0.093	0.067	0.071	0.079	0.050	0.043
2008	-0.024	0.038	0.013	0.015	0.026	0.027	0.016	-0.016	-0.011	-0.013	-0.088	-0.140	-0.151
2009	-0.136	-0.148	-0.169	-0.211	-0.172	-0.130	-0.116	-0.122	-0.101	-0.097	-0.094		

Panel B: 5-year historical risk premium

	Average	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
2001	0.071	0.104	0.104	0.082	0.073	0.082	0.086	0.082	0.072	0.052	0.034	0.037	0.039
2002	-0.015	0.034	0.023	0.040	0.026	0.003	-0.013	-0.037	-0.043	-0.052	-0.061	-0.051	-0.045
2003	-0.058	-0.056	-0.079	-0.089	-0.080	-0.065	-0.063	-0.053	-0.035	-0.027	-0.042	-0.048	-0.055
2004	-0.054	-0.045	-0.045	-0.051	-0.056	-0.054	-0.060	-0.053	-0.061	-0.049	-0.057	-0.059	-0.055
2005	-0.044	-0.049	-0.044	-0.057	-0.054	-0.054	-0.052	-0.049	-0.050	-0.037	-0.039	-0.029	-0.017
2006	0.014	-0.018	0.001	0.014	0.002	0.000	0.001	0.005	0.021	0.039	0.036	0.033	0.035
2007	0.074	0.043	0.041	0.037	0.051	0.064	0.087	0.102	0.089	0.106	0.099	0.087	0.084
2008	0.030	0.092	0.083	0.070	0.062	0.058	0.050	0.036	0.030	0.023	-0.025	-0.052	-0.066
2009	-0.058	-0.069	-0.079	-0.101	-0.076	-0.061	-0.054	-0.048	-0.036	-0.030	-0.028		

Table 5: S&P500 Fama-Mecbeth risk premium estimates

This table reports the S&P500 risk premiums calculated from the CAPM (Panel A) and the Fama-French three-factor model (Panel B) by applying the Fama-Macbeth estimation method. Every risk premium is estimated using monthly returns in the five-year period immediately before the month in question. The numbers reported are annualized but not in percent.

Panel A: CAPM

	Average	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
2001	0.013	0.004	0.041	0.023	0.012	0.018	0.016	0.017	0.011	0.001	-0.010	0.008	0.016
2002	-0.001	0.019	0.017	0.010	0.017	0.009	-0.002	-0.010	-0.016	-0.017	-0.028	-0.014	0.010
2003	0.022	0.004	0.005	0.000	0.002	0.008	0.027	0.029	0.037	0.047	0.039	0.038	0.034
2004	0.015	0.030	0.031	0.033	0.028	0.021	0.022	0.016	0.009	0.006	0.002	0.000	-0.011
2005	-0.032	-0.022	-0.033	-0.050	-0.049	-0.042	-0.028	-0.042	-0.032	-0.037	-0.027	-0.022	-0.001
2006	0.002	0.008	-0.024	-0.006	0.009	0.000	-0.003	-0.003	0.002	0.012	0.022	0.009	0.001
2007	0.022	-0.002	0.000	0.010	0.007	0.018	0.025	0.034	0.038	0.036	0.045	0.036	0.012
2008	0.003	0.022	0.020	0.022	0.020	0.016	0.003	0.003	0.000	-0.001	-0.011	-0.024	-0.03

Panel B: Fama-French 3-factor model

	Average	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
2001	-0.005	-0.006	0.015	0.004	-0.003	0.002	-0.002	-0.003	-0.007	-0.015	-0.027	-0.012	-0.004
2002	-0.016	-0.001	-0.002	-0.008	-0.002	-0.009	-0.017	-0.023	-0.031	-0.028	-0.040	-0.028	-0.008
2003	-0.007	-0.017	-0.016	-0.021	-0.022	-0.015	-0.001	-0.001	0.002	0.009	0.001	-0.002	-0.001
2004	-0.009	-0.005	-0.001	0.003	-0.003	-0.008	-0.004	-0.007	-0.011	-0.014	-0.016	-0.018	-0.023
2005	-0.023	-0.028	-0.030	-0.028	-0.033	-0.033	-0.025	-0.027	-0.022	-0.022	-0.014	-0.014	-0.002
2006	0.003	0.000	-0.016	-0.005	0.005	-0.002	0.000	0.000	0.004	0.012	0.022	0.011	0.002
2007	0.017	-0.001	0.000	0.008	0.005	0.013	0.019	0.026	0.030	0.027	0.036	0.029	0.007
2008	0.009	0.017	0.017	0.020	0.020	0.017	0.008	0.009	0.010	0.009	0.0017	-0.01	-0.016

Table 6: Correlation between risk premium estimates

This table presents the correlation coefficients between different measures of the S&P500 index risk premium. FLRP denotes the forward-looking risk premium. Historical 5yr and Historical 3 yr are the historical risk premiums measured over five and three years, respectively. The risk premiums using the CAPM and the Fama-French three-factor model are denoted as CAPM and FF3, respectively. ***, **, and * means statistical significance at the 1%, 5%, and 10% levels, respectively.

	FLRP	Historical 5yr	Historical 3yr	CAPM	FF3
FLRP	1				
Historical 5yr	-0.036	1			
Historical 3yr	-0.249^{***}	0.506^{***}	1		
CAPM	-0.145	0.430^{***}	-0.091	1	
FF3	-0.042	0.705^{***}	0.344^{***}	0.750^{***}	1

Table 7: S&P500 Fama-French (2002) equity premium

This table reports the S&P500 risk premiums using the Fama and French (2002) approach. The numbers reported are real equity premium for the S&P500 index being estimated with earnings or dividend growth. The results are quarterly from 2001 to 2008. RXY_t and RXD_t are the estimates for the S&P real risk premium based on earnings and dividend growth, respectively. D_t/P_{t-1} is real dividend yield. GY_t and GD_t are real earnings and dividend growth rates, respectively. F_t is the three-month T-bill rate adjusted by inflation. All numbers reported are annualized but not in percentage.

Panel A: Real risk premium using earnings growth ($RXY_t = D_t/P_{t-1} + GY_t - F_t$)

	Average	Quarter1	Quarter2	Quarter3	Quarter4
2001	-0.665	-0.412	-0.785	-0.944	-0.521
2002	0.122	-0.005	0.326	0.481	-0.316
2003	0.615	0.382	0.550	0.468	1.061
2004	0.197	0.283	0.325	0.123	0.059
2005	0.168	0.117	0.189	0.182	0.186
2006	0.127	0.128	0.074	0.185	0.122
2007	-0.217	0.048	0.055	-0.329	-0.641
2008	-1.021	-0.359	-0.570	-0.420	-2.737

Panel B: Real risk premium using dividend growth ($RXD_t = D_t/P_{t-1} + GD_t - F_t$)

	Average	Quarter1	Quarter2	Quarter3	Quarter4
2001	-0.025	-0.244	0.025	0.288	-0.166
2002	0.078	-0.210	0.389	-0.237	0.368
2003	0.200	-0.302	0.173	0.238	0.692
2004	0.070	-0.377	0.097	0.191	0.369
2005	0.127	0.003	0.005	0.034	0.467
2006	0.103	-0.129	0.042	0.014	0.486
2007	0.080	-0.229	0.071	0.085	0.395
2008	-0.055	-0.290	0.017	-0.037	0.091

Panel C: Correlations

	$FLRP_{t-1}(t)$	RXD_t	RXY_t
$FLRP_{t-1}(t)$	1		
RXD_t	-0.068	1	
RXY_t	-0.248	0.073	1

Table 8: S&P 500 excess return and change in forward-looking risk premium

This table reports the regression coefficients from the time series regression of quarterly S&P500 excess holding period return on change in forward-looking risk premium ($\Delta FLRP_t(\tau)$) and change in expected future earnings (ΔEPS_t^e). The sample is from 2001 to 2008. Rm,t is quarterly return for the S&P500 index. $\Delta FLRP_t(\tau)$ is defined as the quarterly change in $FLRP_t(\tau)$, where $FLRP_t(\tau)$ is the forward-looking risk premium at the t for quarter t+1. ΔEPS_t^e is the change in expected quarterly earnings. Model (1) uses realized earnings in current quarter as a proxy for next quarter expected earnings. Model (2) uses analysts' forecasted next quarter earnings from I/B/E/S. *t*-values are reported in parenthesis. ***,**, and * means statistical significance at the 1%, 5%, 10% levels, respectively.

	R_{mt} - R_{ft}	
	(1)	(2)
Const.	-0.003	0.013
	(-0.24)	(1.09)
$\Delta FLRP_t$	-0.031^{*}	-0.059^{***}
	(-1.85)	(-3.47)
ΔEPS_t^e	0.029^{***}	0.032^{***}
	(4.04)	(2.82)
Adj. R-sq	45%	59%
Nobs.	31	27

Table 9: Market illiquidity and risk premium

This table reports the coefficients for monthly regression from January 2001 to December 2008. $MILLIQ_{t-1}$ is the Amihud (2002) aggregate market illiquidity measure; $MILLIQ_t^u$ is the illiquidity shock in month *t*; $JANDUM_t$ is the dummy variable for January; R_{mt} is the monthly return for S&P 500 index; R_{ft} is the 30-day T-bill rate; $FLRP_{t-1}(\tau)$ is the forward-looking risk premium for next month at month *t*-1; $Variance_{t-1}(\tau)$ is the forward-looking variance for next month at time *t*-1, which is computed from the GARCH model; $FLRP_{t-1}(ex.var)$ is the forward-looking risk premium minus $(\gamma-1)\sigma^2 P_t(\tau)$; $Skewness_{t-1}(\tau)$ and $Kurtosis_{t-1}(\tau)$ are the forward-looking skewness and kurtosis for next month available at month *t*-1. The numbers in brackets are *t*-values; for model (1), they are based on the adjusted standard errors as in Amihud and Hurvich (2004); for models (2) to (6), they are based on the Newey-West adjusted standard errors. ***, **, and * means statistical significance at the 1%, 5%, 10% levels, respectively.

	$R_{mt}-R_{ft}$	$FLRP_{t-1}(\tau)$	$Variance_{t-1}(\tau)$	$FLRP_{t-1}(\tau)$ (ex.var)	$Skewness_{t-1}(\tau)$	$Kurtosis_{t-1}(\tau)$
	(1)	(2)	(3)	(4)	(5)	(6)
Const.	-0.0251	3.2153***	0.0375***	1.0136***	-4.3314***	24.1236***
	(-0.43)	(2.73)	(2.51)	(2.32)	(-11.02)	(9.84)
Ln $MILLIQ_{t-1}$	-0.0017	0.1851***	0.0020***	0.0597***	-0.2045***	1.1304***
	(-0.43)	(2.63)	(2.37)	(2.28)	(-8.21)	(7.54)
Ln $MILLIQ_t^U$	-0.0269***					
	(-4.38)					
$JANDUM_t$	0.0108	-0.0757	-0.0011*	-0.0234	0.1123***	-0.7127***
	(0.64)	(-1.43)	(-1.76)	(-1.20)	(2.91)	(-3.13)
Adj. R-sq	18%	38%	30%	34%	66%	64%
Nobs.	96	96	96	96	96	96

Figure 1: Time series plots of risk-neutral and physical forward-looking volatilities

This figure presents the time-series of the 28-canlender day risk neutral volatility and physical forward-looking volatility for the S&P500 index return. The data period is from January 1996 to October 2009. Volatilities plotted are annualized.

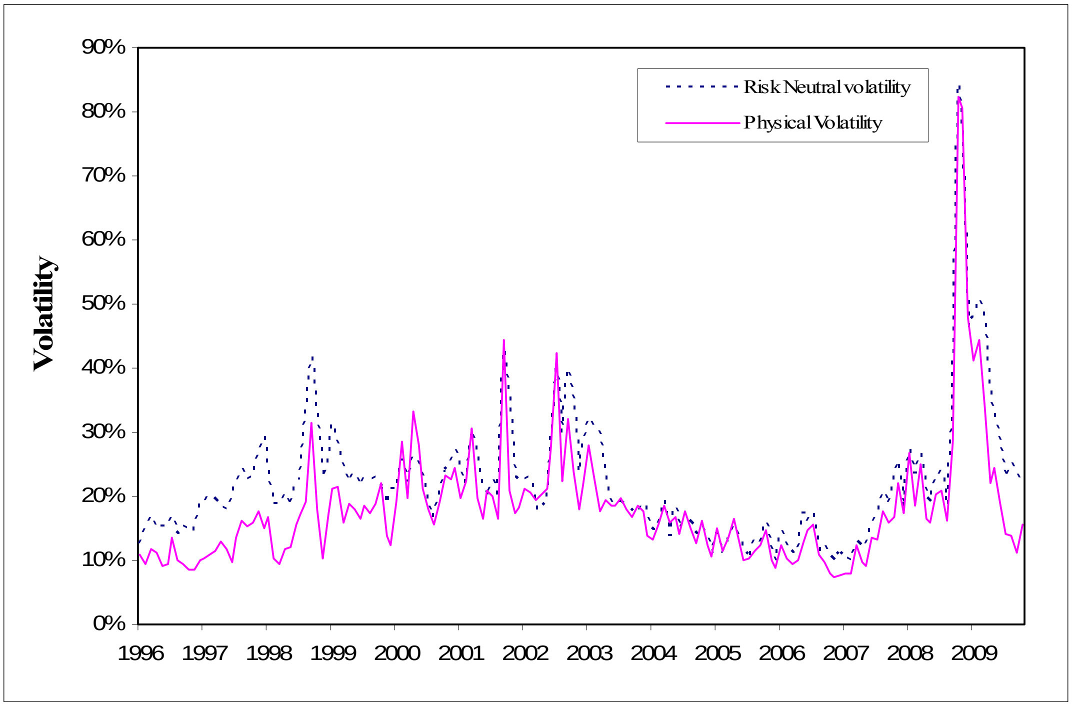
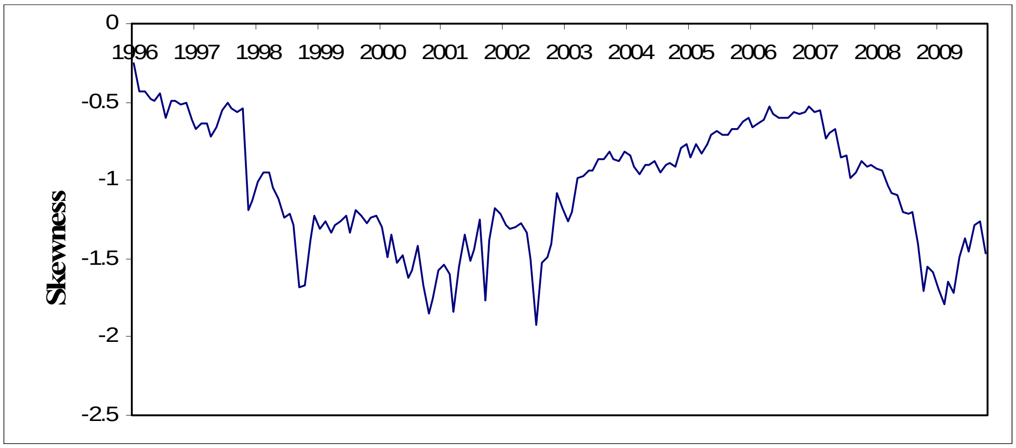


Figure 2: Time series plots of physical forward-looking skewness and kurtosis

This figure presents the time-series plots of the 28-calender day physical forward-looking skewness (Panel A) and kurtosis (Panel B) using the S&P500 index data from January 1996 to October 2009.

Panel A: Physical forward-looking skewness



Panel B: Physical forward-looking kurtosis

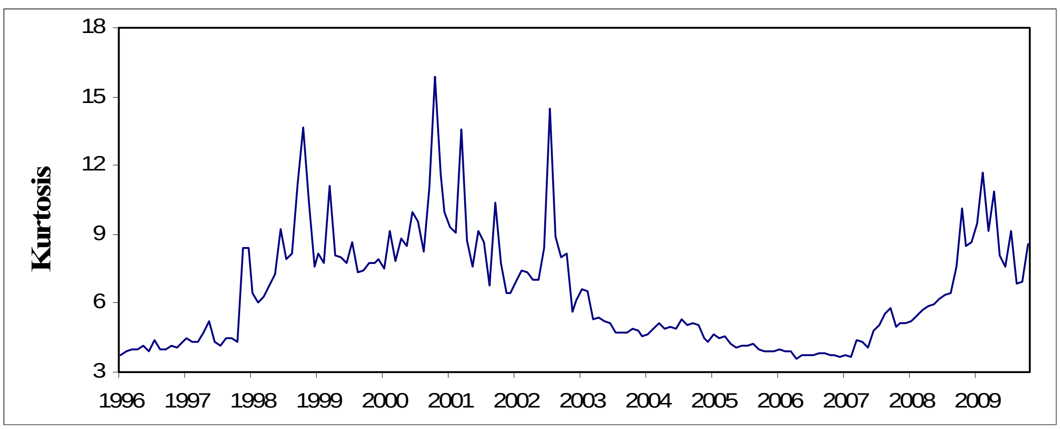


Figure 3: Market risk premiums

This figure presents the times series plot of S&P500 historical and forward-looking risk premiums with NBER recession periods. The sample period is from Jan. 2001 to Oct. 2009.

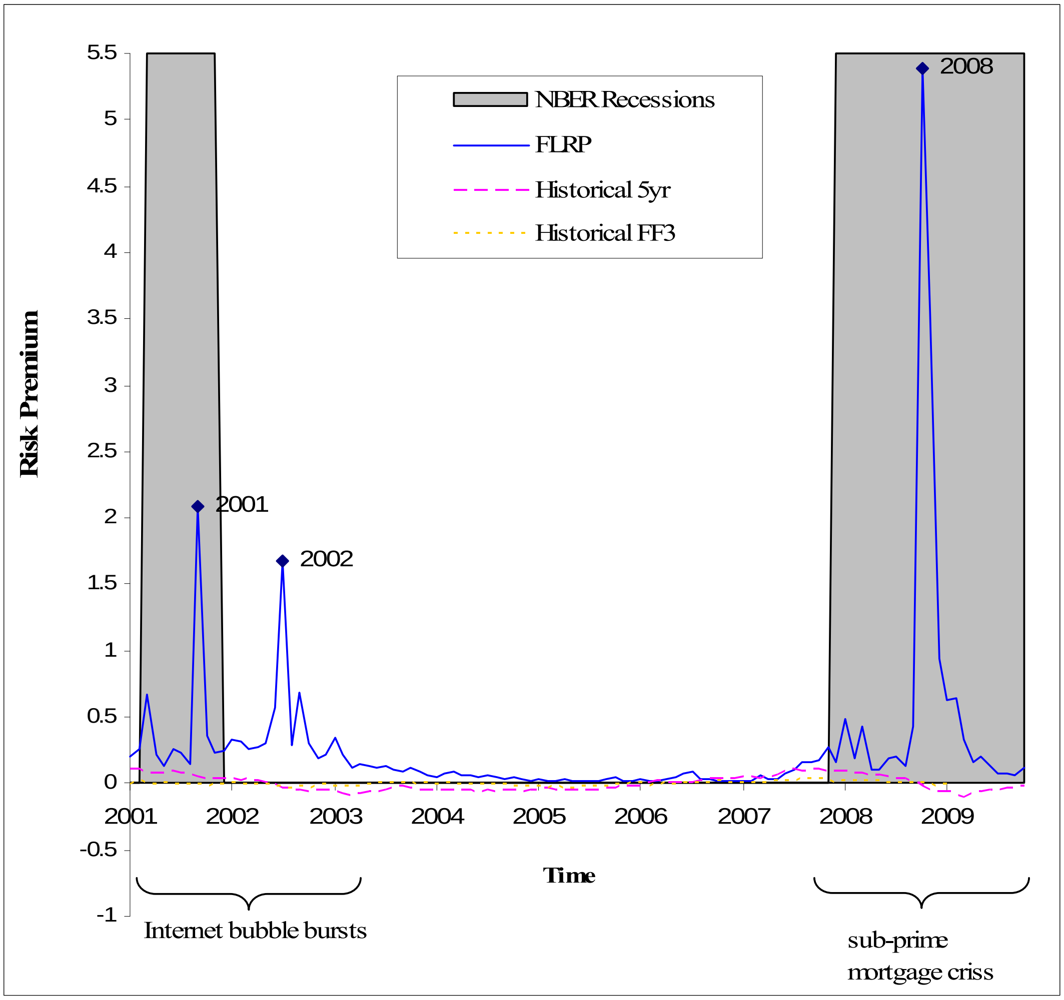
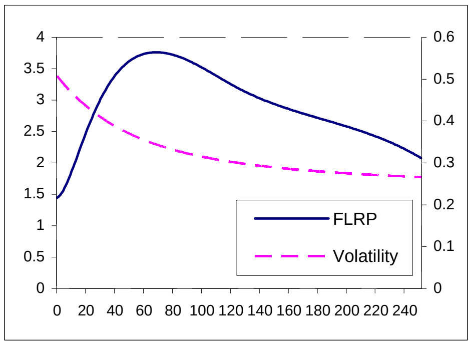


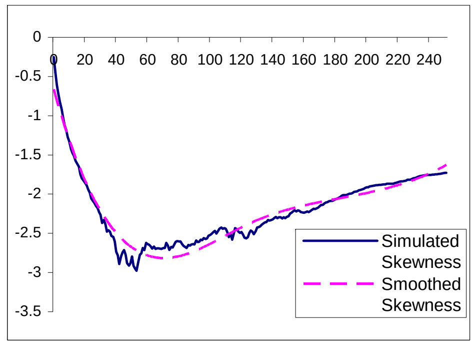
Figure 4: Term structure of forward-looking risk premiums

This figure plots the smoothed forward-looking risk premium up to next 1 year (252 trading days) along with the physical forward-looking volatility, skewness (smoothed and unsmoothed) and kurtosis (smoothed and unsmoothed). The three plots at the top are for September 2001 when the market was volatile. The bottom three plots are for September 2003 when the market was relatively quiet. The left figures (a1 and b1) are the term structures of forward-looking risk premiums and volatilities where the left axis is for the risk premium and the right axis is for the volatility. The middle plots (a2 and b2) are the term structures of physical forward-looking skewness (smoothed and unsmoothed), whereas and the right plots (a3 and b3) are the term structures of physical forward-looking kurtosis (smoothed and unsmoothed).

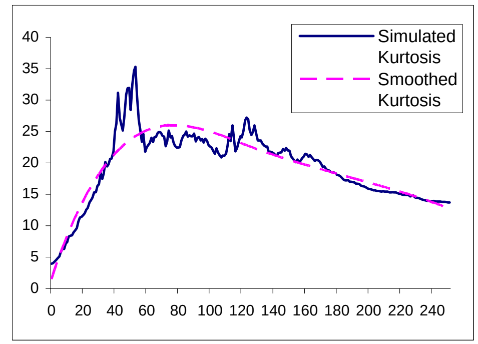
(a) High volatility state (September 2001)



(a1)

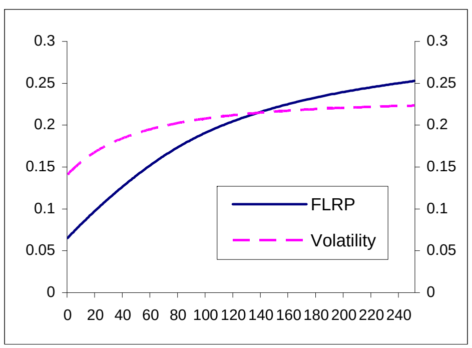


(a2)

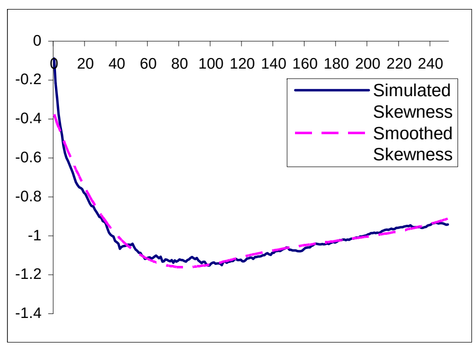


(a3)

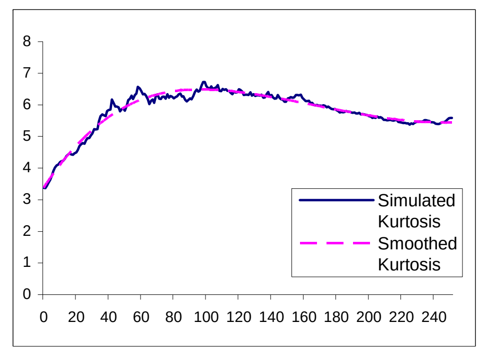
(b) Low volatility state (September 2003)



(b1)



(b2)



(b3)