Using VIX Futures as Option Market-Makers' Short Hedge

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Abstract

The introduction of VIX futures has been a major financial innovation that will facilitate to a great extent the hedging of volatility risk. Using spot VIX, VIX futures, S&P 500 futures, S&P 500 options and S&P 500 futures options, this study examines alternate models within a *delta-vega* neutral strategy. VIX futures are found to outperform vanilla options in hedging a short position on S&P 500 *futures call options*. In particular, while incorporating stochastic volatility on average outperforms in out-of-sample hedging, adding price jumps further enhances the hedging performance for short-term options during the post-crash-relaxation period.

Keywords: VIX futures; S&P 500 futures options; Forward-start strangle; Stochastic volatility; Price jumps

Classification code: G12, G13, G14

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1. Introduction

The U.S. stock market crashed on October 19, 1987, when the Dow Jones Industrials Average lost 22.6% of its market value and the S&P 500 dropped 20.4% in one day. The 1987 crash brought volatility products to the attention of academics and practitioners. As the booming-crash cycle of financial markets becomes often and makes markets highly uncertain, effectively hedging volatility risk has become urgent for market participants. Volatility and variance swaps have been popular in the OTC equity derivatives market for about a decade. The Chicago Board Options Exchange (CBOE) successively launched the Volatility Index (VIX) futures on March 26, 2004, the three-month S&P 500 variance futures on May 18, 2004, the twelve-month S&P 500 variance futures on March 23, 2006, and the VIX option on February 24, 2006. These were the first of an entire family of volatility products to be traded on exchanges. While implied volatility can also be traded with straddles or by unwinding delta-hedged option positions, VIX futures and options offer a cleaner and less costly exposure which does not need to be adjusted when the market moves. Another attractive feature is that VIX is relatively simple to track and that it can be forecasted from several readily observable variables: the current deviation of VIX from its mean, past realized volatility, the performance of the S&P 500, and even the month of the year.[1] Moran and Dash (2007) and Szado (2009) discuss the benefits of a long exposure to VIX futures and VIX call options. Grant et al. (2007) suggest that VIX calls have the potential to provide particularly effective diversification of equity risk, exhibiting far higher payouts per dollar than S&P 500 puts.

VIX futures and options are important for practices since VIX is implied volatility of the S&P 500 Index (SPX), the most widely followed index of large-cap U.S. stocks and considered as an indicator for the U.S. economy. Many mutual funds, index funds[2] and exchange-traded funds (ETF) attempt to replicate the performance of the SPX by holding the same stocks in the same proportions as the index. In recent years, ETFs and index funds have become the most popular investment products worldwide and the need for hedging price risk and volatility risk of the index-related investment vehicles has become urgent, particularly during periods of extreme market movements.[3] The user base for using volatility instruments as extreme downside

[1] The VIX uses all out-of-the-money calls and puts written on SPX with valid quotes. At-the-money call and put options on SPX are also included with their prices averaged. It attempts to gauge the expected risk-neutral realized return volatility over next 30 days. The VIX relies on the concept of static replication, and thus it is not subjected to a specific option pricing model. In theory, the VIX can be used as the fair value for the 30-day volatility *forward*. The VIX calculation isolates expected volatility from other factors that could affect option prices such as dividends, interest rates, changes in the underlying price and time to expiration.

[2] Popular index funds such as SPDRs, iShares, and the Vanguard 500 are an efficient proxy for the underlying index of the S&P 500.

[3] Recently, the U.S. markets have experienced highly up and down. For example, the SPX reached an all-time high of 1,565.15 on October 9, 2007 during the housing bubble and then lost approximately 57% of its value in one and one-half years between late 2008 and early 2009 surrounding the global financial crisis, reaching a nearly 13-year closing low at 676.53 on March 9, 2009. On September 15, 2008, the failure of the large financial institution Lehman Brothers, rapidly devolved into a global crisis

hedges and/or spread arbitrages continues to expand from sophisticated trading firms and hedge funds to insurance companies, risk managers and fundamental investors. As shown in Figure 1, the explosive growth of the trading volume and open interests of futures and options on VIX in recent years clearly reflects a demand for a tradable vehicle which can be used to hedge or to implement a view on volatility.

[Figure 1 about here]

There is also a hedge need for financial intermediaries such as option market-makers and hedge funds who provide liquidity to end-users by taking the other side of the end-user net demand. In reality, however, even market-makers cannot hedge options perfectly because of the impossibility of trading continuously, stochastic volatility, jumps in the underlying and transaction costs (Gârleanu, Pedersen and Poteshman, 2009). In light of these facts, this study considers how options are hedged using VIX futures by competitive risk-averse market-makers who face stochastic volatility and jumps. The risks of an option writer can be partitioned into price risk and volatility risk. Carr and Madan (1998) suggest options on a straddle and Brenner et al. (2006) construct a straddle to hedge volatility risk. Those straddle positions look for a

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resulting in a number of bank failures and sharp reductions in the value of equities and commodities and quickly spread into a global economic shock and recession. Stock market crashes often follow speculative market bubbles of a prolonged period of rising stock prices and excessive economic optimism and then are driven by panic as much as by underlying economic factors.

large price move in the underlying and thus both *delta* and *vega* risk must be simultaneously hedged. A slightly dissenting focus is Rebonato (1999), who constructs a *forward-start* strangle, consisting of two wide strangles with different maturities, so that the changes of underlying stock price will not affect the payoff of the portfolio. This *forward-start* strangle hedges *forward* volatility risk without exposure to *delta* and *gamma* risk. Other than using vanilla options, Neuberger (1994) adopts the log contract to hedge volatility. Finally, in their simulation study, Psychoyios and Skiadopoulos (2006) hedge the instantaneous volatility using either a volatility call option or a traditional option. They conclude that a vanilla option is a more efficient instrument to hedge the volatility risk arisen from a short position on a European call option than a volatility option.

The introduction of VIX futures and options has been a major financial innovation that facilitates to a great extent the hedging of volatility risk. Since VIX futures and options settle to the implied volatility of the S&P 500, they are effective to cross-hedge the *vega* risk of stock options and stock indexes correlated to the S&P 500, whether these are exchange-traded or embedded in other assets. The hedging effectiveness of the new VIX derivatives is an important issue that has not yet been concluded in the literature. This paper addresses this issue by building a *delta-vega* neutral strategy from market-makers' perspective and demonstrates to what extent the risk factors such as stochastic volatility and price jumps in the S&P 500 price dynamics influence hedging effectiveness of using VIX futures as upside hedges of a short position on a futures call option. A short position on the SPX futures call option is chosen as our target asset because its *vega* risk consists of (i) *spot* volatility randomness over the period from current time and the option maturity, and (ii) *forward* volatility changes between the option maturity and the underlying futures expiry. The current price of VIX futures reflects the market's expectation of the VIX level at expiration, and thus VIX futures, at least in theory, should provide an effective hedge on *vega* risk of SPX futures options. In particular, price fluctuations in VIX futures, SPX futures, SPX options and SPX futures options arise endogenously through S&P 500 stocks' response to the macroeconomic conditions. Consistent modeling of SPX, SPX derivatives, VIX and VIX derivatives helps address the "hedging effectiveness of VIX futures" in a unified framework. Our model generates interesting dynamics for S&P 500, including stochastic volatility and price jumps. It also provides a novel procedure to estimate state-dependent hedge ratios. For the purpose of comparison, a *forward-start* strangle portfolio, rather than a *forward-start* straddle, is constructed to hedge the *forward* volatility risk arisen from a short position on a futures call option.[4]

Recent empirical work on index options identifies factors such as stochastic volatility, jumps in prices and jumps in volatility. The results in the literature regarding these issues are mixed. For example, tests using option data disagree over the importance of price jumps: Bakshi et al. (1997) find substantial benefits from including jumps in prices, whereas Bates (2000) and others find that such benefits are economically small, if not negligible. Pan (2002) finds that pricing errors decrease when price jumps are added for certain strike-maturity combinations, but increase for others. Eraker (2004) finds that adding jumps in returns and volatility decreases errors by only 1%. Bates (2000) finds a 10% decrease, but it falls to around 2% when time-series consistency is imposed. Broadie et al. (2007) find strong evidence for jumps in prices and modest evidence for jumps in volatility for the cross section of SPX futures option prices from 1987 to 2003. Furthermore, while studies using the time series of returns unanimously support jumps in prices, they disagree with respect to the importance of jumps in volatility. To learn about rare jumps and stochastic

[4] On the one hand, since the present value profile of straddle as a function of spot around the at-the-money level is less flat than strangle, the *delta* and *gamma* of a straddle are less close to zero than a strangle. On the other hand, VIX futures offer a way for investors to buy and sell option volatility without having to explicitly deal with factors that have an impact on the value of the SPX option position such as *delta* and *gamma*. This study thus uses a *forward-start* strangle, instead of a *forward-start* straddle, as a benchmark to hedge the *forward* volatility risk arisen from a short position on a futures call option.

volatility, and investors' attitudes toward the risks these factors embody, Figure 2 displays a time-series plot of the VIX against the SPX over January 1990 to June 2009. In empirical index option hedging, Bakshi et al. (1997) find that once the stochastic volatility is modeled, the hedging performance may be improved by incorporating neither price jumps, nor stochastic interest rates into the SPX option pricing framework. Bakshi and Kapadia (2003) use Heston's (1993) stochastic-volatility option pricing model to construct a *delta*-hedged strategy for a long position on SPX call options. They find that the volatility risk is priced and the price jump affects the hedging efficiency. Vishnevskaya (2004) follows the structure of Bakshi and Kapadia (2003) and constructs a *delta-vega*-hedged portfolio consisting of the underlying stock, another option and the money-market fund, for a long position on the SPX call option. His result suggests the existence of some other sources of risk.

Guided by previous studies, this study considers both stochastic volatility and price jumps, denoted the "SVJ" model.[5] The setup contains the competing stochastic-volatility (SV) futures option formula as special cases. Since SPX futures option contracts are American-style, it is important to take into account the extra value

[5] Our results could be generalized by introducing a time-varying jump intensity, jumps in the volatility, or a more complicated correlation structure for the state variables. While such generalization would add realism, this study wants to test the effect of the VIX futures in the presence of the most basic sources of incomplete market risk considered here.

accruing from the ability to exercise the options prior to maturity. One can follow such a nonparametric approach as in Aït-Sahalia and Lo (1998) and Broadie et al. (2000) to price American options. Closed-form option pricing formulas, however, make it possible to derive hedge ratios analytically. Therefore, for options with early exercise potential, this paper computes a quadratic approximation for evaluating American futures options. The approximation is based on the one developed by MacMillan (1987), examined by Barone-Adesi and Whaley (1987) for the constant-volatility process, extended by Bates (1991) for the jump-diffusion process, and modified by Bates (1996) for the SV and SVJ processes. For the SV and SVJ processes, this approximation is consistent with Bates (1996) for evaluating American currency futures options.

In sum, this study examines using VIX futures as *vega* hedges required for a short position on the SPX futures call option, and compares its effectiveness against traditional "synthetic long volatility" hedging instruments such as *forward-start* strangle. The study also considers how options are priced by competitive risk-averse market-makers who cannot hedge perfectly due to stochastic volatility and price jumps in the underlying SPX. Our findings reveal that VIX futures generally outperform *forward-start* strangle over the out-of-sample hedging period, August 2006–June 2009.

These diagnostics document the extraordinary significance in our hedging exercise that occurred following the stock market crash on September 15, 2008. Based on empirical analyses, this paper concludes that using VIX futures as *vega* hedges of a short position on a SPX futures call option on average provides superior hedging effectiveness than a *forward-start* strange portfolio. In addition, adding price jumps into the SPX price process further improve hedging performance for short-term cases during the post-crash-relaxation period.

The rest of this paper proceeds as follows. Next section illustrates hedging strategies. Pricing models for calculating *delta* and *vega* hedge ratios are presented in Section 3. Section 4 summarizes data and model parameter estimation procedure. Section 5 presents summary statistics of parameter estimates and in-sample pricing errors. Section 6 analyzes out-of-sample hedging results. Section 7 finally concludes.

2. Hedging Strategies

A time-t short position on the T_1-matured call option written on T_2-matured SPX futures is used as the target portfolio, i.e. $TAR_t = -C_t^A(F)$ for $t < T_1 < T_2$. This study then constructs two hedging schemes to hedge the target position.

Hedging Scheme 1 (HS1): *The instrument portfolio consists of* $N_{1,t}$ *shares of underlying SPX futures, and* $N_{2,t}$ *shares of forward-start strangle portfolios. One unit of forward-start strangle portfolio consists of a short position on a* T_1*-matured strangle and a long position on a* T_2*-matured strangle, given by*

$$INST_t = -c_t^E(S,T_1,K_2) - p_t^E(S,T_1,K_1) + c_t^E(S,T_2,K_2) + p_t^E(S,T_2,K_1) \tag{1}$$

where $c_t^E(S,T_1,K_2)$ *and* $c_t^E(S,T_2,K_2)$ *are* K_2 *-strike SPX call options with maturities* T_1 *and* T_2*, respectively.* $p_t^E(S,T_1,K_1)$ *and* $p_t^E(S,T_2,K_1)$ *are* K_1*-strike SPX put options with maturities* T_1 *and* T_2*, respectively.*

Hedging Scheme 2 (HS2): *The instrument portfolio consists of* $N_{1,t}$ *shares of underlying SPX futures, and* $N_{2,t}$ *shares of the VIX futures, i.e.,*

$$INST_t = \mathrm{F}_t^{\mathrm{VIX}}(T_1) \tag{2}$$

where $\mathrm{F}_t^{\mathrm{VIX}}(T_1)$ *is the time-t price of the VIX futures with expiry* T_1.

The gain or loss of this hedged portfolio is expressed by

$$\pi_t = N_{1,t}F_t(T_2) + N_{2,t}INST_t - C_t^A(F) \tag{3}$$

Further add the constraints of *delta*-neutral and *vega*-neutral by

$$\frac{\partial \pi_t}{\partial F_t} = N_{1,t} + N_{2,t}\frac{\partial INST_t}{\partial F_t} - \frac{\partial C_t^A(F)}{\partial F_t} = 0 \tag{4}$$

$$\frac{\partial \pi_t}{\partial \nu_t} = N_{2,t}\frac{\partial INST_t}{\partial \nu_t} - \frac{\partial C_t^A(F)}{\partial \nu_t} = 0 \tag{5}$$

The shares of instrument assets are computed as

$$N_{1,t} = \frac{\partial C_t^A(F)}{\partial F_t} - N_{2,t}\frac{\partial INST_t}{\partial F_t} \tag{6}$$

$$N_{2,t} = \left(\frac{\partial C_t^A(F)}{\partial \nu_t}\right) / \left(\frac{\partial INST_t}{\partial \nu_t}\right) \tag{7}$$

The illustrations of $\partial C_t^A(F)/\partial F_t$, $\partial INST_t/\partial F_t$, $\partial C_t^A(F)/\partial \nu_t$ and $\partial INST_t/\partial \nu_t$ for alternate models are provided in the following section.

Next, this study couples these two hedging schemes with the SV and the SVJ option models to construct four hedging strategies: HS1-SV, HS1-SVJ, HS2-SV and HS2-SVJ. Assuming that there are no arbitrage opportunities, the hedged portfolio π_t should earn the risk-free interest rate *r*. In other words, the change in the value of this hedged portfolio over Δt is expressed as

$$\Delta\pi_{t+\Delta t} = \pi_{t+\Delta t} - \pi_t = \pi_t(e^{r\Delta t} - 1) \tag{8}$$

where $\Delta\pi_{t+\Delta t} = N_{1,t}[F_{t+\Delta t}(T_2) - F_t(T_2)] + N_{2,t}[INST_{t+\Delta t} - INST_t] - [C_{t+\Delta t}^A(F) - C_t^A(F)]$.

The hedging error as a function of rebalancing frequency Δt is defined as the additional profit (loss) over the risk-free return and it can be written as

$$\begin{aligned} HE_t(t+\Delta t) &= \Delta\pi_{t+\Delta t} - \pi_t(e^{r\Delta t} - 1) \\ &= N_{1,t}[F_{t+\Delta t}(T_2) - F_t(T_2)] + N_{2,t}[INST_{t+\Delta t} - INST_t] - [C_{t+\Delta t}^A(F) - C_t^A(F)] \\ &\quad - (e^{r\Delta t} - 1)[N_{1,t}F_t(T_2) + N_{2,t}INST_t - C_t^A(F)] \end{aligned} \tag{9}$$

Finally, compute the average absolute hedging error:

$$HE(\Delta t) = (1/M)\sum_{l=1}^{M} |HE_{t+(l-1)\Delta t}(t + l\Delta t)e^{r\Delta t(M-l)}| \tag{10}$$

, and the average dollar-value hedging error:

$$\overline{HE}(\Delta t) = (1/M)\sum_{l=1}^{M} HE_{t+(l-1)\Delta t}(t + l\Delta t)e^{r\Delta t(M-l)} \tag{11}$$

where $M = (T_1 - t)/\Delta t$ and T_1 is the expiry of the target SPX futures call option. Measurements of hedging performances are based on the framework proposed in previous papers, for instance, Bakshi et al. (1997) and Psychoyios and Skiadopoulos (2006).

3. Empirical Pricing Models

Hedging strategies are constructed using SPX futures, SPX options, SPX futures options, VIX futures and VIX. Therefore, their fair value and related greeks are required for further empirical analyses. The most general process considered in this paper is the jump-diffusion and stochastic volatility (SVJ) process of Bates (1996) and Bakshi et al. (1997). This general process contains stochastic volatility (SV) of Heston (1993) as a special case.

3.1 The SVJ Process for SPX Prices

Contingent claims are priced as if investors were risk-neutral and under the SVJ model the SPX price follows the jump-diffusion with stochastic volatility

$$dS_t = (b - \lambda_J \mu_J) S_t + \sqrt{v_t} S_t \, d\omega_{S,t} + J_t S_t dN_t \qquad (12)$$

where b is the cost of carry coefficient (0 for futures options and $r - \delta$ for stock options with a cash dividend yield δ). J_t is the percentage jump size with mean μ_J. The jumps in the asset log-price are assumed to be normally distributed, i.e., $\ln(1 + J_t) \sim N(\theta_J, \sigma_J^2)$. Satisfying the no-arbitrage condition, $\mu_J = \exp(\theta_J + \sigma_J^2 / 2) - 1$. N_t is the jump frequency following a Poisson process with mean λ_J. The instantaneous variance v_t of the index follows a risk-neutral mean-reverting square root process

$$dv_t = (\theta_v - \kappa_v v_t) dt + \sigma_v \sqrt{v_t} \, d\omega_{v,t} \qquad (13)$$

where κ_v is the speed of mean-reverting adjustment of v_t; θ_v / κ_v is the long-run mean of v_t; σ_v is the variation coefficient of v_t; and $\omega_{S,t}$ and $\omega_{v,t}$ are two correlated Brownian motions with the correlation coefficient $\rho \, dt = corr(d\omega_{S,t}, d\omega_{v,t})$.

3.2 Fair Value to SPX Options

SPX options are European-style. Bakshi et al. (1997) provide the time-*t* value of SPX call and put options with strike *K* and maturity T for the SVJ model:

$$c_t^E(S,T,K) = S_t e^{-\delta(T-t)}\Pi_1 - Ke^{-r(T-t)}\Pi_2 \quad (14)$$

$$p_t^E(S,T,K) = S_t e^{-\delta(T-t)}(\Pi_1 - 1) - Ke^{-r(T-t)}(\Pi_2 - 1) \quad (15)$$

where Π_1 and Π_2 are risk-neutral probabilities that are recovered from inverting the characteristic functions f_1 and f_2, respectively,

$$\Pi_j(T-t, K; S_t, r, \nu_t) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\varphi \ln K} f_j(t, T-t; S_t, r, \nu_t; \varphi)}{i\varphi}\right] d\varphi \quad (16)$$

for j = 1, 2. The characteristic functions f_1 and f_2 for the SVJ model are given in equations (A12) and (A13) of Bakshi et al. (1997). *Delta* and *vega* of the European SPX options are given in equation (13) of Bakshi et al. (1997). Finally, *delta* and *vega* of the *forward-start* strangle portfolio can be calculated straightforward.

3.3 Fair Value to SPX Futures Options

Since SPX futures options are American-style, it is important, in principle, to take into account the extra value accruing from the ability to exercise the options prior to maturity. Referred to Bates (1996), the futures call option is

$$C_t^A(F, T_1, K) \equiv \begin{cases} C_t^E(F, T_1, K) + KA_2\left(\dfrac{F_t(T_2)/K}{y_c^*}\right)^{q_2} & \text{if} \quad F_t(T_2)/K < y_c^* \\ F_t(T_2) - K & \text{if} \quad F_t(T_2)/K \geq y_c^* \end{cases} \quad (17)$$

where $C_t^E(F, T_1, K) = e^{-r(T_1 - t)}[F_t(T_2)\Pi_1' - K\Pi_2']$ is the time-t price of a European-style futures call option with strike K and expiry T_1. $F_t(T_2)$ is the time-*t* futures price with maturity T_2. For *j*=1, 2, Π'_j are the relevant tail probabilities for evaluating futures call options, which are the same to Π_j in equation (16) except replacing K with $Ke^{-(r-\delta)(T_2-T_1)}$. $A_2 = (y_c^*/q_2)[1-\partial C_t^E(y_c^*, T_1, 1)/\partial F_t]$. For the SVJ process, q_2 is the positive root to the equation of q given as follows

$$\frac{1}{2}\bar{v}q^2 + \left(-\lambda_J\mu_J - \frac{1}{2}\bar{v}\right)q - \frac{r}{1-e^{-r(T_1-t)}} + \lambda_J[(1+\mu_J)^q e^{q(q-1)\sigma_J^2/2} - 1] = 0 \tag{18}$$

$\bar{v}$ is the expected average variance over the lifetime of the option conditional on no jumps, i.e., $\bar{v} = \frac{\theta_v}{\kappa_v} + \left(v_t - \frac{\theta_v}{\kappa_v}\right)\frac{[1-e^{-\kappa_v(T_1-t)}]}{\kappa_v(T_1-t)}$. The critical futures price/strike price ratio $y_c^* \geq 1$ above which the futures call is exercised immediately is given implicitly by

$$y_c^* - 1 = C_t^E(y_c^*, T_1, 1) + \left(\frac{y_c^*}{q_2}\right)\left[1 - \frac{\partial C_t^E(y_c^*, T_1, 1)}{\partial F_t}\right] \tag{19}$$

The closed form solutions to the parameters q_2 and y_c^* are provided for given model parameters and for given maturity T_1. Since linear homogeneity in underlying asset and strike holds for European options, by Euler theorem the following equations sustain:

$$C_t^E(y_c, T_1, 1) = \frac{1}{K}C_t^E(F, T_1, K) = e^{-r(T_1-t)}(y_c\Pi'_1 - \Pi'_2) \tag{20}$$

$$\frac{\partial C_t^E(y_c, T_1, 1)}{\partial F_t} = \frac{1}{K}\frac{\partial C_t^E(F, T_1, K)}{\partial F_t} = \frac{1}{K}e^{-r(T_1-t)}\Pi'_1 \tag{21}$$

where $y_c = F_t(T_2)/K$. Replacing $C_t^E(y_c, T_1, 1)$ and $\frac{\partial C_t^E(y_c, T_1, 1)}{\partial F_t}$ of y_c^* in equation (19) with equations (20) and (21), the solution to y_c^* is given by

$$y_c^* = \frac{1 - e^{-r(T_1 - t)}\Pi_2'}{1 - \frac{1}{q_2} + \left(\frac{1}{q_2 K} - 1\right)e^{-r(T_1 - t)}\Pi_1'} \tag{22}$$

Thus, the solution to A_2 is computed as

$$A_2 = \frac{[1 - e^{-r(T_1 - t)}\Pi_2']}{\left[q_2 - 1 + \left(\frac{1}{K} - q_2\right)e^{-r(T_1 - t)}\Pi_1'\right]}\left[1 - \frac{1}{K}e^{-r(T_1 - t)}\Pi_1'\right] \tag{23}$$

Finally, the calculation for *delta* and *vega* of the futures call option is straightforward.

3.4 Fair Value to VIX Futures

From Lin (2007), the time-*t* fair price of the VIX futures expiring at T_1 under the SVJ model is given by

$$\mathrm{F}_t^{\mathrm{VIX}}(T_1) \equiv \sqrt{\zeta_2 + \frac{1}{\tau_0}(a_{\tau_0}\alpha_{T_1 - t}\nu_t + a_{\tau_0}\beta_{T_1 - t} + b_{\tau_0})} - \frac{\left(\frac{a_{\tau_0}}{\tau_0}\right)^2 (C_{T_1 - t}\nu_t + D_{T_1 - t})}{8\left[\zeta_2 + \frac{1}{\tau_0}(a_{\tau_0}\alpha_{T_1 - t}\nu_t + a_{\tau_0}\beta_{T_1 - t} + b_{\tau_0})\right]^{3/2}} \tag{24}$$

where $\tau_0 = 30/365$, $\zeta_2 = 2\lambda_J(\mu_J - \theta_J)$, $\alpha_{T_1 - t} = e^{-\kappa_\nu (T_1 - t)}$, $\beta_{T_1 - t} = \frac{\theta_\nu}{\kappa_\nu}(1 - \alpha_{T_1 - t})$, $a_{\tau_0} = \frac{(1 - e^{-\kappa_\nu \tau_0})}{\kappa_\nu}$, $b_{\tau_0} = \frac{\theta_\nu}{\kappa_\nu}(\tau_0 - a_{\tau_0})$, $C_{T_1 - t} = \frac{\sigma_\nu^2}{\kappa_\nu}(\alpha_{T_1 - t} - \alpha_{T_1 - t}^2)$, $D_{T_1 - t} = \frac{\sigma_\nu^2\theta_\nu}{2\kappa_\nu^2}(1 - \alpha_{T_1 - t})^2$, $\nu_t \equiv \frac{\tau_0(\mathrm{VIX}_t^2 - \zeta_2) - b_{\tau_0}}{a_{\tau_0}}$.

Pricing formulas and related greeks for the SV model are obtained as a special case of the general model with price jumps restricted to zero, i.e., $J_t dN_t = 0$ and thus $\lambda_J = \mu_J = \sigma_J = 0$.

4. Data and Estimation Procedure

The sample period spans from July 3, 2006, through June 30, 2009. Spot VIX, spot SPX, daily midpoints of the last bid and last ask quotations for SPX options, and daily settlement prices for VIX futures are obtained from the CBOE. Daily settlement prices for SPX futures and daily settlement SPX futures options are obtained from CME. The averages of U.S. Treasury bill bid and ask discounts, collected from Datastream, with maturity closest to an option's maturity are used as risk-free interest rates. Because SPX option contracts are European-style, index levels are adjusted for dividends by the subtraction of the present value of future cash dividend payments (from S&P Corporation) before each option's expiration date.

The contracts that are selected for empirical analyses are described as follows: First, the selected SPX futures contracts expire in March, June, September and December. Second, the SPX futures call options that expire in February, May, August and November are selected as the target portfolio. Third, the *forward-start* strangle portfolio involves in two strangles. This study uses the SPX options contracts that expire in February, May, August and November to construct a short-term strangle, and that expire in March, June, September, and December for another long-term strangle. Finally, the VIX futures that expire in February, May, August and November are selected as the hedging instrument.

Several exclusion filters are applied to construct the option price data. First, as options with less than six days to expiration may induce liquidity-related biases, they are excluded from the sample. Second, to mitigate the impact of price discreteness on option valuation, option prices lower than 3/8 are excluded. Finally, option prices that violate the upper and lower boundaries are not included in the sample. Based on these criteria, 29.90% and 10.50% of the original selected SPX options and SPX futures call options are eliminated, respectively. A total of 24,761 records of joint SPX futures call options (20,839 records), SPX options (3,200 observations), and VIX futures (722 records) are used for parameter estimation.

Table 1 reports descriptive properties of the SPX futures call options, SPX futures, VIX futures and VIX for each moneyness-maturity category where *moneyness* is defined as the SPX futures price divided by the SPX futures call option's strike price, i.e., $F_t(T_2)/K$. Out of 20,839 SPX futures call option observations, about 50% is out-of-the-money (OTM) and 35% is at-the-money (ATM). By maturity, 6,221 transactions are under 30 days to maturity, 8,613 are between 30 and 60 days to maturity, and 6,005 are more than 60 days to maturity. The average futures call price ranges from 5.57 points for short-term (<30 days) deep out-of-the-money (DOTM) call options to 118.56 points for medium-term (30－60 days) deep in-the-money (DITM) call options. Its underlying SPX futures price ranges from 947.85 points for short-term DOTM calls to 1,347.89 points for medium-term ATM calls.

The average VIX futures prices range from 21.02 corresponding to the medium-term ATM calls to 46.23 for the short-term DOTM calls. The VIX varies from 20.68 (for medium-term ATM call options) to 50.87 (for short-term DOTM call options). The price of a VIX futures contract to VIX is an analogy to what a 30-day forward interest rate is to a 30-day spot interest rate. The price of a VIX futures contract can be lower, equal to or higher than VIX, depending on whether the market expects volatility to be lower, equal to or higher in the 30-day forward period (covered by the VIX futures) than in the 30-day spot period (covered by VIX). Owing to the tendency of VIX to have a negative correlation with the SPX, the VIX futures prices have traded at a discount to the VIX before contract settlement especially during the in-crash period, September 15, 2008 to December 31, 2008.

A total of 800 records of simultaneous strikes K_1 and K_2 of SPX options are used for the construction of the *forward-start* strangle over the sample period, consisting of 722 trading days. Average option quotes of K_1 range from 599.89 for DOTM short-term futures calls to 1,100.31 for ATM short-term futures calls, whereas average option quotes of K_2 range 1,066.82 for short-term DOTM futures calls to 1,464.49 for medium-term ATM futures calls.

[Table 1 about here]

This study uses SPX options to construct the *forward-start* strangle that expire in the February Quarterly Cycle as T_1-strangle and that expire in the March Quarterly Cycle as T_2-strangle. Therefore, the pair of (T_1, T_2) data must be February–March, May–June, August–September and November–December. The strike K_1 (K_2) is the minimum (maximum) strike of the SPX put (call) option, which expire in the February and March Quarterly Cycles simultaneously. To enhance the integrity of the study, less heavily traded SPX options are eliminated here. On the basis of an analysis of patterns of trading volumes on SPX options, four pairs of maturity-strike combinations are selected from the option contracts retained to form a *forward-start* strangle on each trading day: (T_1, K_1) and (T_2, K_1) of SPX puts, and (T_1, K_2) and (T_2, K_2) of SPX calls. There are in total 3,200 SPX option observations, consisting of 1,600 SPX calls and 1,600 SPX puts. Those option observations compose of 800 pairs of calls with strike K_2 and puts with strike K_1. The selected strikes K_1 of the SPX puts are on average 938.03 index points, whereas the strikes K_2 of the SPX calls are on average 1,377.21. Table 2 reports the average point of those SPX option and the observation for each moneyness-maturity category over the period, 3 July 2006 to 30 June 2009. *Moneyness* is defined as the current SPX value divided by the SPX option's strike, *S/K*. The average call prices range from 1.3716 points for short-term OTM call options to 59.4300 points for long-term near-the-money call options. The average put prices range from 0.4125 points for short-term OTM put options to 66.8500 points for long-term near-the-money put options.

[Table 2 about here]

The vector of structural parameters Φ for alternate processes is backed out by minimizing the sum of the squared pricing errors between option and futures model and market prices over the period, July 3, 2006 to May 14, 2009 (the last trading day of May-matured SPX futures call options in 2009). The minimization is given by

$$\min_{\Phi} \sum_{t=1}^{N_T} \sum_{n=1}^{N_t} [C_n - C_n^*(\Phi)]^2 \tag{25}$$

where N_T is the number of trading days in the estimation sample, N_t is the number of SPX futures call options, SPX options and VIX futures on day *t*, and C_n and C_n^*

are the observed and model option or futures prices, respectively. The parameters of the SV and the SVJ models are estimated separately each month and thus Φ are assumed to be constant over a month. The assumption that the structural parameters are constant over a month is justified by an appeal to parameter stability (Bates, 1996; Eraker, 2004; Zhang and Zhu, 2006). The estimation period is chosen because the settlement day of the SPX futures option is the third Friday of contract months. Hence, the month is defined as the period from the third Friday of the prior calendar month to the third Thursday of this calendar month. Table 3 reports the monthly average of each estimated parameter series and the monthly averaged in-sample mean squared errors for the SV and SVJ models, respectively.

[Table 3 about here]

Since SPX futures options are American, we follow Broadie et al.'s (2007) procedure to account for the early exercise feature. Our calibration procedure begins by converting American market prices to equivalent European market prices, and estimates model parameters based on European prices. This allows us to use computationally efficient European pricing routines that render the large-scale calibration procedure feasible. Using the observed price C we compute an American binomial-tree implied volatility, that is, a value σ^{BT} such that $C = BT^{A}(\sigma^{BT}, \Phi)$, where BT^A denotes the binomial-tree American option price. We then estimate that an equivalent European option would trade in the market at a price $BM^E(\sigma^{BT}, \Phi)$, where BM^E denotes the Black (1976) European option price.

5. Risk-Neutral Parameter Estimates and In-Sample Pricing Fit

Summary statistics for instantaneous volatility and risk-neutral parameter estimates from the SV and SVJ models implicit in SPX futures calls prices, VIX futures prices and SPX option prices along with in-sample mean squared errors are shown in Table 3. In 2008, a series of bank and insurance company failures triggered a financial crisis that effectively halted global credit markets and required unprecedented government intervention.[6] These failures caused a crisis of confidence that made banks reluctant to lend money amongst themselves, or for that matter, to anyone. The time-series plot of the VIX against the SPX in Figure 2 sheds light on these issues. Other than the entire sample period, we divide our data into four sub-periods: (i) the period prior to the Lehman Brothers bankruptcy: July 3, 2006 to August 14, 2008, and (ii) the 2008 crash fear periods: August 15, 2008 to December 18, 2008; August 15,

[6] In 2008, Fannie Mae (FNM) and Freddie Mac (FRE) were both taken over by the government. Lehman Brothers declared bankruptcy on September 14th after failing to find a buyer. Bank of America agreed to purchase Merrill Lynch, and American International Group was saved by an $85 billion capital injection by the government. Shortly after, on September 25th, J P Morgan Chase agreed to purchase the assets of Washington Mutual in what was the biggest bank failure in history. In fact, by September 17, 2008, more public corporations had filed for bankruptcy in the U.S. than in all of 2007.

2008 to June 18, 2009; and December 19, 2008 to June 18, 2009.[7]

Model-specific estimates of implicit distributions indicate extremely turbulent conditions in the joint markets of SPX futures, SPX futures options, SPX options and VIX futures over September 2008—December 2008, with somewhat quieter conditions over January 2009—June 2009 following the end of the Lehman Brothers bankruptcy. Indeed, Table 3 finds parameter estimates of the SV and SVJ models from their full July 2006−June 2009 sample diverge from estimates for a September 2008−June 2009 subsample. The estimates of jump-frequency intensity λ_J that capture outliers indicate the major shocks that affected the markets over September 2008−December 2008. The possibility of SPX price jumps occurs with an average size θ_J of -0.13, -0.12, and -0.11 (with jump size uncertainty σ_J^2 estimated at 0.35, 0.30, and 0.49) for the July 2006−August 2008, September 2008−December 2008, and January 2009−June 2009 periods, respectively.

The risk-neutral mean-reverting volatility process is controlled by three parameters: the long-term mean, $\sqrt{\theta_v/\kappa_v}$, to which variance reverts, the speed of reversion, κ_v, and the variation coefficient, σ_v. For instance, on the basis of mean

[7] The data split, for example, August 14, 2008 and December 18, 2008, is chosen because the estimation month is defined as the period from the third Friday of the prior calendar month to the third Thursday of this calendar month.

values of $\{\kappa_v, \sqrt{\theta_v/\kappa_v}, \sigma_v\} = \{5.77, 0.3458, 0.77\}$ for the SV model and $\{3.93, 0.5862, 1.80\}$ for the SVJ model over the crash period, volatility took 30 and 44 trading days to revert halfway toward a long-term mean of 34.58% for the SV model and 58.62% for the SVJ model, respectively.[8] The mean speed of reversion value of 5.77 and 3.19 for the SV model over the crash periods (Panels C and D) is higher than the values of 0.86 (Panel B) and 1.47 (Panel E) over the pre- and post-crash-relaxation periods. However, it is close to the value of 5.02 found by the SVJ model over the post-crash-relaxation period. The average long-term mean variance θ_v/κ_v of 34.36% for the SVJ model over the crash period compares with values of 3.91% in the pre-crash period and 13.33% in the post-crash-relaxation period. The variation coefficient, σ_v, determines how fat-tailed the distribution is and thereby the relative values of deep OTM options versus near-the-money options. The mean value of 1.80 found in the SVJ model over the crash period compares with values of 0.22 and 0.46 over the pre- and post-crash-relaxation periods. The consistently negative estimates of ρ indicate that implied volatility and index returns are negatively correlated. This means the implied distribution perceived by SPX option traders, SPX futures option traders and VIX futures traders is negatively skewed. The mean values of ρ for the

[8] The half-life of volatility shocks for the square-root process was calculated as $\frac{\ln(2)}{\kappa_v} \times 252$ trading days.

SVJ model reached -0.91 in the crash period and -0.97 in the post-crash-relaxation period. An explanation for these extreme values is that a relatively continuous bear market in the SPX starting from September 15, 2008 caused investors to perceive the underlying distribution as being heavily left skewed. The SV model captures this effect by attaching a higher value to ρ. The mean value of ρ of -0.57 for the SV model in the pre-crash period compares with more extreme values of -0.75 in the crash period and -0.77 in the post-crash-relaxation period. The implied volatilities, $\sqrt{v_t}\%$, estimated by alternate models are able to capture the substantial 2008 crash. In particular, the SVJ model adds a price-jump component that directly captures the 2008 outliers. Table 3 also gives the t values for the parameter estimates of the index price process. These results indicate that all parameter estimates are significantly different from zero at the 90% confidence interval.

The parameter estimates in Table 3 are interesting in light of estimates obtained in prior studies. Bakshi et al. (1997) estimate the jump frequency for the SVJ model as a 0.59 annualized jump probability, and the jump-size parameter μ_J and σ_J are -5.37% and -7%, respectively. Their estimate of κ_v and σ_v are 2.03 and 0.38. Pan (2002), Bates (2000) and Eraker (2004) assume that the jump frequency depends on the spot volatility. The average jump intensity point estimates in Pan (2002) are in the range [0.07, -0.3%] across different model specifications, whereas Eraker (2004) indicates two to three jumps in a stretch of 1,000 trading days. Interestingly, Bates (2000) obtains quite different results, with an average jump intensity of 0.005. Bates (2000) also reports a jump size mean ranging from -5.4% to -9.5% and standard deviations of about 10% to 11%. Hence, Bates' estimates imply more frequent and more severe crashes than the parameter estimates reported in Bakshi et al. (1997) and Eraker (2004). Our estimates are in the same ballpark as those reported in Bates (1996).

Finally, the fact that allowing pricing jumps to occur enhances the SV model' fit is illustrated by the model's average of the squared pricing errors between the market price and the model price in an average month (MSE). The in-sample mean squared errors are considerably and consistently smaller for more complicated models. However, the resulting increase in MSE for the various specifications is significant under the crash period, suggesting that this jump risk assessment fails to capture the substantial 2008 crash.[9]

[9] Bates (2006) found the stochastic-intensity jump model fits S&P returns better than the constant-intensity specification, when jumps are drawn from a finite-activity normal distribution or mixture of normals.

6. Hedging Results

The need for a perfect *delta*-neutral hedge can arise in situations where not only is the underlying price risk present, but also are volatility and jump risks in our specifications. In conducting this exercise, however, we should first recognize that a *perfect* hedge may not be practically feasible in the presence of stochastic jump sizes (for example, for the SVJ model). The difficulty is seen from the existing work by Bakshi et al. (1997), Bates (1996), Cox and Ross (1976), and Merton (1976). For this reason, whenever jump risk is present, we follow Merton (1976) and only aim for a partial hedge in which diffusion risks are completely neutralized but jump risk is left uncontrolled for. We do this with the understanding that the overall impact on hedging effectiveness of not controlling for jump risk can be small or large, depending on whether the hedge is frequently rebalanced or not.

To examine the hedging effectiveness, at time t short the SPX futures call option and establish the hedge using two hedging schemes under three SPX price processes. The hedger will need a position in some $N_{1,t}$ shares of the underlying SPX futures (to control for price risk), and some $N_{2,t}$ units of either *forward-start* strangles or VIX futures to control for volatility risk ν_t. The study uses the *previous month's* structural parameters and the *current day's* (t) SPX, SPX futures, SPX futures options, SPX options, VIX futures and U.S. Treasury-bill rates to construct the hedged portfolio. Next, this study calculates the hedging error as of day $t+1$ if the hedge is rebalanced daily or as of day $t+5$ if the rebalancing takes place every five days. These steps are repeated for each futures call option that expires in February quarterly cycle and every trading day in the sample. Finally, the average absolute and the average dollar hedging errors for each moneyness-maturity category are then reported for each model in Tables 4 and 5.

In Table 4, under the SV model, the hedging errors of HS1 are from 1.09 points (DOTM short-term of five-day revision in the pre-crash period) to 33.97 points (ATM short-term of five-day revision in the crash period), whereas the hedging errors of HS2 range from 0.98 points (DOTM middle-term of one-day revision in the pre-crash period) to 38.86 points (ATM short-term of five-day revision in the crash period). For the SVJ model, HS1 has hedging errors from 1.13 points (DOTM middle-term of one-day revision in the pre-crash period) to 216.96 points (ITM middle-term of one-day revision in the post-crash-relaxation period), whereas HS2 has hedging errors from 1.15 points (DOTM short-term of one-day revision in the pre-crash period) to 33.46 points (DITM short-term of five-day revision in the crash period).

When the hedge revision frequency changes from daily to once every five days, the hedging errors increase, except for the HS1-SVJ short-term options in the post-crash-relaxation period. Though HS1 outperforms HS2 in some cases within short-term categories under five-day rebalancing, the HS1 performs poorly in daily rebalancing coupled with SVJ; for example, its absolute hedging errors are 141.85 for futures options with ATM1 and short-term, 216.96 with ITM and middle-term, and 140.00 with DITM and short-term during the post-crash-relaxation period (Panel E). The findings shed a light on instability issues of HS1's hedging performance.

Improvement by the stochastic-volatility models (SV and SVJ) is more evidence when the HS2 strategy is adopted, which indicates a consequence of better model specifications. A striking pattern emerging from this table is that, with respect to short-term options in the post-crash-relaxation period (Panel E) and short-term ITM and DITM options during the two in-crash periods (Panels C and D), the SVJ model coupled with HS2 has virtually smaller hedging errors. It represents the random price jump feature commonly exists in the SPX price process, especially in the in-crash periods. Noticeably, for the options in the other remaining moneyness-maturity categories, however, adding price jumps to the SV model does not improve its hedging performance. Thus, except for short-term DITM options, the result in the pre-crash period (Panel B), however, seems to be consistent with those of Bakshi et al. (1997)

and Bakshi and Kapadia (2003), which show the hedging superiority of the SV model relative to the SVJ model. Note that the parameter of jump-frequency intensity λ_J in Bakshi et al. (1997) is 0.59, i.e. one year and half for a price jump to occur, which is comparable to 0.60 of our empirical work only in the pre-crash period. Based on daily or every five-day rebalancing, Bakshi et al. (1997) conclude that the reason for the SV model dominates the SVJ model in terms of hedging performance is the chance for a price jump to occur is small in the daily or five-day rebalancing period. The estimated parameter λ_J in the crash (post-crash-relaxation) period, however, is 9.97 (1.02) larger than that of Bakshi et al. (1997). For this reason, whenever jump risk is present, the overall impact on short-term hedging effectiveness of not allowing for jump risk is significant. Therefore, Bakshi et al.'s (1997) reason does not completely hold for our empirical results. Another possible reason is that the SPX futures options used in this study are American-style, while SPX options are European-style for prior research. Since the traders with American-style options positions have early-exercise choice and thus can take caution to prevent any loss from the potential jump events than the ones with European-style options. Thus, American-style option buyers (sellers) may even favor (hate) volatility risk than the ones with European-style options. In addition, given the possibility of price jumps, the specification of SVJ can provide more accurate parameter estimates than SV (Bates, 1996). Thus, the *delta-vega*-neutral strategy could be constructed in a more effective way under SVJ than SV. It is thus not surprising for our hedging results for some short-term options showing that the SVJ model outperforms the SV model in the crash-related periods.

Further, the HS1 hedge of the SVJ model results in poor hedging errors comparable to that of the SV model during the post-crash-relaxation period, which suggests that model misspecification may have a significant effect on hedging when using the HS1 strategy through the crash periods. Instead, adopting HS2 hedge strategy has more robust hedging performance in magnitude across the SV and SVJ models. Out of 60 moneyness–maturity combinations within 1-day revision frequency reported in Table 4, absolute hedging error was lower for the HS2 strategy in 58 cases than the HS1 strategy. The improvement was generally indistinguishable between the crash-related periods and the pre-crash period. About the maturity impact on hedging performance, there is no pattern showing the difference between hedging schemes decreases with maturity and increases with maturity, as discovered by Psychoyios and Skiadopoulos (2006) for ITM and OTM target options. In terms of the moneyness effect, our results show that the absolute hedging errors of HS1 and HS2 strategies have the tendency to increase with moneyness. In contrast, Psychoyios and

Skiadopoulos (2006) find that the options perform best for ATM and worse for ITM, and the difference between hedging schemes is minimized for ATM and maximized for ITM. In sum, the results indicate that the *forward-start* strangle portfolio is in general a less efficient instrument to hedge *forward* volatility risk than VIX futures.

[Table 4 about here]

Theoretically, if a portfolio is perfectly hedged, it should earn the risk-free rate of interest, and the average hedging errors should be close to zero. In this study, the hedging error is defined as the changes in the value of the hedged portfolio minus the risk-free return. Table 5 reports the average hedging errors. If the figure is greater (less) than zero, it means that the strategy gets more (less) profits than risk-free return.

During the in-crash period, the dollar hedging errors are relatively sensitive to revision frequency. Another pattern to note from Table 5 is that the HS1 and HS2's 1-day revision dollar hedging errors are always positive for short-term options in the post-crash-relaxation period, indicating that both HS1 and HS2 underhedge each target option, whereas the dollar hedging errors of the other periods are more random and can take either sign. Therefore, the HS1 and HS2 strategies exhibit a systematic hedging bias during the post-crash-relaxation period, while they do not in the other periods. Out of 60 moneyness–maturity combinations within 1-day revision frequency reported in

Table 5, dollar hedging errors were lower for the HS2 strategy in 49 cases than for the HS1 strategy, indicating VIX futures that are volatility sensitive as a superior hedge instrument.

[Table 5 about here]

7. Conclusion

This study has extended a parsimonious American futures option pricing model that admits stochastic volatility and random price jumps. It is shown that this closed-form pricing formula is practically implementable, leads to useful analytical hedge ratios, and contains the stochastic-volatility option formula as a special case. This last feature has made it relatively straightforward to study the relative empirical hedging performance of the three models. In particular, the study considers whether and by how much VIX futures improve option hedging by competitive risk-averse dealers or investors who have a short position on SPX futures call options. For the purpose of comparison, a *forward-start* strangle portfolio is constructed for managing *forward* volatility.

This study then couples two hedging schemes (HS1 and HS2) with two SPX price processes (SV and SVJ) to hedge the target asset. On the one hand, SPX futures and the *forward-start* strangle portfolio are used to construct two hedging strategies

(HS1−SV and HS1−SVJ). On the other hand, SPX futures and VIX futures are used to construct the other two hedging strategies (HS2−SV and HS2−SVJ).

There are some interesting empirical findings. First, HS2 in general dominates HS1, suggesting that the VIX futures contract is a more efficient instrument to hedge forward volatility risk than a *forward-start* strangle portfolio. This finding contributes to the existing literature documented by Psychoyios and Skiadopoulos (2006), who point out that volatility options are less powerful instruments to hedge a short position on a European call option than plain-vanilla options, and that the most naïve volatility option-pricing model can be reliably used for pricing and hedging purposes. Second, gauged by the absolute hedging errors, while the SVJ model performs slightly better for short-term options during the post-crash-relaxation period, the SV model coupled with HS2 achieves better performance for the remaining cases. This finding is slightly different from that obtained by Bakshi et al. (1997). Overall, our results support the claim that the VIX futures contract is a better alternative to the traditional *forward-start* strangle, if the target is a *futures call option*, or equivalently if the risk exposures to *forward* volatility randomness. This is because the VIX futures contract not only performs much better in a stochastic-volatility and/or price-jump economy but also is practically implementable.

The contributions of this paper are threefold. First, closed-form solutions to American futures options under three SPX price processes are derived. Second, the concept of *forward* volatility risk applied to VIX futures and a *forward-start* strangle portfolio is introduced. Third, this study derives the hedging weights of VIX futures and the *forward-start* strangle portfolio that will be convenient to practical participants for risk management purposes.

References

Aït-Sahalia, Y., Lo, A. W., 1998. Nonparametric estimation of state-price densities implicit in financial asset prices. Journal of Finance 53, 499−548.

Bakshi, G., Cao, C., Chen, Z., 1997. Empirical performance of alternative option pricing models. Journal of Finance 52, 5, 2003−2049.

Bakshi, G., Kapadia, N., 2003. Delta-hedged gains and the Negative market volatility risk premium. Review of Financial Studies 16, 527−566.

Barone-Adesi, G., Whaley, R., 1987. Efficient analytic approximation of American option values. Journal of Finance 42, 301−320.

Bates, D.S., 1991. The crash of '87: Was it expected? The evidence from options markets. Journal of Finance 46, 1009−1044.

Bates, D.S., 1996. Jumps and stochastic volatility: exchange rate processes implicit in Deutsche Mark options. Review of Financial Studies 9, 69−107.

Bates, D.S., 2000. Post-'87 crash fears in the S&P 500 futures option market. Journal of Econometrics 94, 181−238.

Bates, D.S., 2006. Maximum likelihood estimation of latent affine processes. Review of Financial Studies 19, 909−965.

Black, F., 1976. The pricing of commodity contracts. Journal of Financial Economics 3, 167−179.

Brenner, M., Galai, D., 1989. New financial instruments for hedging changes in volatility. Financial Analysts Journal 45, 61−65.

Brenner, M., Ou, E. Y., Zhang, J. E., 2006. Hedging volatility risk. Journal of Banking and Finance 30, 811−821.

Broadie, M., Chernov, M., Johannes, M., 2007. Model specification and risk premia: Evidence from futures options. Journal of Finance 62, 1453−1490.

Broadie, M., Detemple, J., Ghysels, E., Torrés, O., 2000. American options with stochastic dividends and volatility: A nonparametric investigation. Journal of Econometrics 94, 53−92.

Carr, P., Madan, D., 1998. Towards a theory of volatility trading, Chapter in Volatility, in: Jarrow, R. (Ed.), Volatility, Risk Publications, pp. 417−427.

Chaput, J. S., Ederington, L. H., 2005. Volatility trade design. Journal of Futures Markets 25, 243−279.

Cox, J., Ross, S., 1976. The valuation of options for alternative stochastic processes. Journal of Financial Economics 3, 145−166.

Eraker, B., 2004. Do equity prices and volatility jump? Reconciling evidence from spot and option prices. Journal of Finance 59, 1367−1403.

Gârleanu, N., Pedersen, L. H., Poteshman, A. M., 2009. Demand-based option pricing. Review of Financial Studies 22, 4259–4299.

Grant, M., Gregory, K., Lui, J., 2007. Considering all options. Working paper, Goldman Sachs Global Investment Research.

Heston, S., 1993. A closed-form solution for options with stochastic volatility with applications to bond and currency options. Review of Financial Studies 6, 2, 327–343.

Lin, Y. N., 2007. Pricing VIX futures: Evidence from integrated physical and risk-neutral probability measures. Journal of Futures Markets 27, 1175−1217.

MacMillan, L. W., 1987. Analytic approximation for the American put options. Advances in Futures and Options Research 1A, 119−139.

Merton, R., 1976. Option pricing when the underlying stock returns are discontinuous. Journal of Financial Economics 4, 125−144.

Moran, M. T., Dash, S., 2007. VIX futures and options: Pricing and using volatility products to manage downside risk and improve efficiency in equity portfolios. Journal of Trading 2, 96−105.

Neuberger, A., 1994. The log contract. Journal of portfolio management 20, 2, 74–80.

Pan, J., 2002. The jump-risk premia implicit in options: Evidence from an integrated time-series study. Journal of Financial Economics 63, 3−50.

Psychoyios, D., Skiadopoulos, G., 2006. Volatility options: Hedging effectiveness, pricing, and model error. Journal of Futures Markets 26, 1, 1−31.

Rebonato, R., 1999. Volatility and Correlation: In the Pricing of Equity, FX, and Interest-Rate Options. John Wiley, New York.

Szado, E., 2009. VIX futures and options—A case study of portfolio diversification during the 2008 financial crisis. Journal of Alternative Investments, forthcoming.

Vishnevskaya, A., 2004. Essays in option pricing: Vega-hedged gains and the volatility risk premium. Working paper, University of Virginia.

Zhang, J. E., Zhu, Y., 2006. VIX futures. Journal of Futures Markets 26, 521−531.

Table 1 Sample Properties of S&P 500 Futures Call Options, S&P 500 Futures, VIX Futures, VIX and Selected SPX Options

The average points of futures call options ($250 per point), the average points of underlying futures ($250 per point), the average points of VIX futures ($1,000 per point), the average points of VIX, the average strikes K_1 and K_2 of selected SPX options and the total number of futures call options are presented in each moneyness-maturity category. Contract months of both futures call options and VIX futures are in the February Quarterly Cycle, whereas contract months of SPX futures are in the March Quarterly Cycle. K_1 and K_2 are selected strikes of SPX put and call options to construct *forward-start* strangles, both of which expire on the maturity dates of the SPX futures call and the SPX futures simultaneously. We divide the futures option data and contemporaneous futures levels, VIX futures, VIX, K_1 and K_2 into several categories according to either moneyness or term to expiration. Define $F_t(T_2)/K$ as the time-t intrinsic value of a futures call. A futures call is then said to be *deep out-of-the-money* (DOTM) if its $F_t(T_2)/K<0.94$; *out-of-the-money* (OTM) if $F_t(T_2)/K \in [0.94,0.97)$; *at-the-money* (ATM) if $F_t(T_2)/K \in [0.97,1.03)$; *in-the-money* (ITM) if $F_t(T_2)/K \in [1.03,1.06)$; and *deep in-the-money* (DITM) if $F_t(T_2)/K \geqq 1.06$. By the term to expiration, an option contract can be classified as (i) short-term (<30 days); (ii) medium-term (30－60 days); and (iii) long-term (>60 days). The data period is from July 3, 2006 to June 30, 2009.

Moneyness			Days to Maturity <30	30–60	>60	Subtotal
		SPX futures call	5.5696	6.8933	8.4643	
		SPX futures	947.8534	1078.1347	1167.9128	
		VIX futures	46.2295	34.2556	30.4921	
DOTM	<0.94	VIX	50.8742	38.7459	32.2216	
		K_1	599.8907	720.3157	793.8094	
		K_2	1066.8182	1255.1761	1340.5847	
		Observation	869	2,867	3,053	6,789
		SPX futures call	7.7356	12.3137	17.0321	
		SPX futures	1231.1835	1323.0393	1328.2712	
		VIX futures	29.7906	23.3573	21.4009	
OTM	0.94–0.97	VIX	30.5768	23.3098	20.9756	
		K_1	972.7829	1005.5325	1002.6713	
		K_2	1345.6457	1456.6037	1464.8980	
		Observation	875	1,615	1,226	3,716
		SPX futures call	13.8811	22.9812	33.3377	
		SPX futures	1311.4333	1345.6390	1320.7551	
		VIX futures	23.3768	21.3924	21.7445	
ATM1	0.97–1.00	VIX	23.6443	20.9470	21.1742	
		K_1	1100.3112	1046.4877	998.3112	
		K_2	1400.5785	1464.4924	1458.7431	
		Observation	1,478	1,704	903	4,085
		SPX futures call	33.7082	44.3218	55.6945	
		SPX futures	1306.8449	1347.8856	1305.5544	
		VIX futures	23.2381	21.0243	22.9012	
ATM2	1.00–1.03	VIX	23.3762	20.6828	22.1135	
		K_1	1100.1285	1050.9091	974.7164	
		K_2	1394.2252	1463.4335	1445.6805	
		Observation	1,323	1,331	529	3,183
		SPX futures call	62.0779	72.8389	84.7926	
		SPX futures	1225.0017	1292.5961	1217.1777	
		VIX futures	28.9174	25.3501	27.0426	
ITM	1.03–1.06	VIX	29.3179	25.5828	27.1678	
		K_1	987.9844	960.6031	850.1330	
		K_2	1328.0622	1421.9455	1364.6277	
		Observation	707	514	188	1,409
		SPX futures call	109.8420	118.5584	116.0519	
		SPX futures	1060.5809	1070.8000	1041.5613	
		VIX futures	35.8835	34.9171	35.6560	
DITM	≧1.06	VIX	36.5378	36.1897	36.7596	
		K_1	806.5996	695.8935	634.9057	
		K_2	1165.8359	1213.4278	1156.9811	
		Observation	969	582	106	1,657
Subtotal			6,221	8,613	6,005	20,839

Table 2 Sample Characteristics of S&P 500 Index Options

Out-of-the-money and near at-the-money SPX call and put options are selected in this study to construct *forward-start* strangles, both of which expire on the maturity dates of the SPX futures call and the SPX futures simultaneously. The average points of the SPX options ($100 per point) and the total number of the SPX options are presented in each moneyness-maturity category. Contract months of SPX options are in the February and March quarterly Cycles. This study classifies those observations into short-term (<30 days), medium-term (30–60 days), and long-term (>60 days). *Moneyness* is defined as *S*/*K* where *S* is the price of the SPX and *K* is the strike price of SPX options. DOTM (DITM), OTM (ITM), ATM1 (ATM2), ATM2 (ATM1), ITM (OTM) and DITM (DOTM) for calls (puts) are defined by *Moneyness* <0.94, 0.94–0.97, 0.97–1.00, 1.00–1.03, 1.03–1.06, and >1.06, respectively. The data period is from July 3, 2006 to June 30, 2009.

		Days to Expiration							
		Call				Put			
Moneyness		<30	30–60	>60	Subtotal	<30	30–60	>60	Subtotal
DOTM	Option price	1.5607	3.8948	8.2840		0.5031	2.1075	3.0217	
	Observation	105	293	812	1,210	192	452	930	1,574
OTM	Option price	1.3716	6.2070	15.9739		0.4125	8.4417	53.9000	
	Observation	74	146	110	330	8	9	1	18
ATM1	Option price	4.9698	16.5250	59.4300		12.5417	31.3625	66.8500	
	Observation	24	26	10	60	3	4	1	8
ATM2	Option price	NA	NA	NA		NA	NA	NA	
	Observation	0	0	0	0	0	0	0	0
ITM	Option price	NA	NA	NA		NA	NA	NA	
	Observation	0	0	0	0	0	0	0	0
DITM	Option price	NA	NA	NA		NA	NA	NA	
	Observation	0	0	0	0	0	0	0	0
Subtotal		203	465	932	1,600	203	465	932	1,600

Table 3 Implied Parameter Estimation

The in-sample mean squared pricing errors (MSE) and estimated structural parameters reported in Panel A are their averages over 36 nonoverlapping estimation months from the whole sample period, July 3, 2006 to June 18, 2009, with a total of 20,839 SPX futures calls, 722 VIX futures, and 3,200 SPX options. Panel B presents estimation results that obtained preceding the Lehman Brothers bankruptcy on September 15, 2008. Panels C, D and E report parameter estimates and in-sample mean squared errors that occurred covering the stock market crash on September 15, 2008. We divide the post-'08 crash fears in the S&P 500 market into three periods: August 15, 2008 to December 18, 2008, August 15, 2008 to June 18, 2009, and December 19, 2008 to June 18, 2009. The instantaneous variance of the S&P 500 index, v_t, is replaced by VIX and structural parameters, given by

$$v_t \equiv [\tau_0(\text{VIX}_t^2 - \zeta_2) - b_{\tau_0}]/a_{\tau_0}$$

where $\tau_0 = 30/365$, $\zeta_2 = 2\lambda_J(\mu_J - \theta_J)$, $b_{\tau_0} = \theta_v(\tau_0 - a_{\tau_0})/\kappa_v$, $a_{\tau_0} = (1 - e^{-\kappa_v \tau_0})/\kappa_v$, and $\mu_J = e^{\theta_J + \sigma_J^2/2} - 1$. The figures within parentheses are the t-statistics of parameter estimates. The symbols of ***, ** and * indicate significance of t-statistics at the 1%, 5% and 10% levels, respectively.

Model Fit	SV	SVJ
A. July 3, 2006－June 18, 2009		
In-sample pricing error (MSE)	67.7801	17.5971
Adjustment speed of v (κ_v)	1.5071*** (4.90)	3.8388*** (8.30)
Long-run mean of v (θ_v/κ_v)	0.1838*** (6.26)	0.0886*** (3.91)
Total variation of v (σ_v)	0.7548*** (12.54)	0.4363*** (3.18)
Correlation between diffusion Brownian motions (ρ)	-0.6254*** (-12.90)	-0.7844*** (-13.22)
Mean jump intensity (λ_J)		1.7078** (1.95)
Mean of price jump-size innovations (θ_J)		-0.1248*** (-6.35)
Variance of price jump-size innovations (σ_J^2)		0.3698*** (6.51)
Implied volatility (%) ($\sqrt{v_t}$%)	26.1041	27.0852
B. July 3, 2006－August 14, 2008		
In-sample pricing error (MSE)	51.5359	13.4391
Adjustment speed of v (κ_v)	0.8602*** (4.62)	3.5516*** (7.05)
Long-run mean of v (θ_v/κ_v)	0.1718*** (5.95)	0.0391*** (5.80)
Total variation of v (σ_v)	0.7294*** (11.30)	0.2197*** (4.98)
Correlation between diffusion Brownian motions (ρ)	-0.5723*** (-10.15)	-0.7211*** (-9.26)
Mean jump intensity (λ_J)		0.5961** (2.34)
Mean of price jump-size innovations (θ_J)		-0.1301*** (-5.17)
Variance of price jump-size innovations (σ_J^2)		0.3520*** (6.20)
Implied volatility (%) ($\sqrt{v_t}$%)	18.2347	19.2580
C. August 15, 2008－December 18, 2008		
In-sample pricing error (MSE)	161.0398	31.9449
Adjustment speed of v (κ_v)	5.7651*** (7.80)	3.9328* (2.20)
Long-run mean of v (θ_v/κ_v)	0.1196* (1.99)	0.3436** (2.36)
Total variation of v (σ_v)	0.7664*** (8.50)	1.8034* (1.72)
Correlation between diffusion Brownian motions (ρ)	-0.7467** (-4.37)	-0.9127*** (-10.46)

Mean jump intensity (λ_J)		9.9650*
		(1.64)
Mean of price jump-size innovations (θ_J)		-0.1201**
		(-2.74)
Variance of price jump-size innovations (σ_J^2)		0.3043*
		(1.65)
Implied volatility (%) ($\sqrt{\nu_t}$%)	54.4009	52.1165
D. August 15, 2008－June 18, 2009		
In-sample pricing error (MSE)	110.0147	28.408
Adjustment speed of v (κ_v)	3.1893***	4.5854***
	(3.98)	(4.42)
Long-run mean of v (θ_v/κ_v)	0.2150**	0.2174***
	(2.80)	(3.32)
Total variation of v (σ_v)	0.8210***	0.9996**
	(5.80)	(2.23)
Correlation between diffusion Brownian motions (ρ)	-0.7633***	-0.9491***
	(-9.09)	(-25.85)
Mean jump intensity (λ_J)		4.5983*
		(1.54)
Mean of price jump-size innovations (θ_J)		-0.1112***
		(-3.92)
Variance of price jump-size innovations (σ_J^2)		0.4161***
		(2.84)
Implied volatility (%) ($\sqrt{\nu_t}$%)	46.5644	47.4362
E. December 19, 2008－June 18, 2009		
In-sample pricing error (MSE)	75.9980	26.0501
Adjustment speed of v (κ_v)	1.4721**	5.0204***
	(3.06)	(3.68)
Long-run mean of v (θ_v/κ_v)	0.2787**	0.1333***
	(2.33)	(4.63)
Total variation of v (σ_v)	0.8574***	0.4636**
	(3.62)	(3.28)
Correlation between diffusion Brownian motions (ρ)	-0.7743***	-0.9734***
	(-7.97)	(-36.61)
Mean jump intensity (λ_J)		1.0205*
		(1.68)
Mean of price jump-size innovations (θ_J)		-0.1053**
		(-2.61)
Variance of price jump-size innovations (σ_J^2)		0.4906**
		(2.86)
Implied volatility (%) ($\sqrt{\nu_t}$%)	41.3401	44.316

Table 4 Absolute Hedging Errors

The figures in this table denote the average points of absolute hedging errors ($250 per point): $\sum_{l=1}^{M} | HE_{t+(l-1)\Delta t}(t + l\Delta t) e^{r\Delta t(M-l)} | / M$ where $M = (T_1 - t)/\Delta t$ and T_1 is the maturity date of SPX futures call options. The hedging error between time t and time $t + \Delta t$ is defined as $HE_t(t + \Delta t)$. The instrument portfolio of hedging scheme 1 (HS1) consists of $N_{1,t}$ shares of T_2-matured SPX futures, and $N_{2,t}$ shares of *forward-start* strangle portfolios. The *forward-start* strangle portfolio consists of a short position on a T_1-matured strangle and a long position on a T_2-matured strangle. The instrument portfolio of hedging scheme 2 (HS2) consists of $N_{1,t}$ shares of T_2-matured SPX futures, and $N_{2,t}$ shares of the VIX futures $\mathrm{F}_t^{\mathrm{VIX}}(T_1)$ with expiry T_1. This study classifies hedging errors into moneyness-maturity categories. Define $F_t(T_2)/K$ as the time-t intrinsic value of a futures call. A futures call is then said to be *deep out-of-the-money* (DOTM) if its $F_t(T_2)/K$<0.94; *out-of-the-money* (OTM) if $F_t(T_2)/K \in$[0.94,0.97); *at-the-money* (ATM) if $F_t(T_2)/K \in$[0.97,1.03); *in-the-money* (ITM) if $F_t(T_2)/K \in$[1.03,1.06); and *deep in-the-money* (DITM) if $F_t(T_2)/K \geqq$1.06. By the term to expiration, an option contract can be classified as (i) short-term (<60 days); (ii) medium-term (60−180 days); and (iii) long-term (>180 days). The out-of-sample hedging period is from 21 July 2006 to 18 June 2009.

			1-Day Revision			5-Day Revision		
	Hedging		Days-to-Expiration			Days-to-Expiration		
Moneyness	Scheme	Model	<60	60−180	>180	<60	60−180	>180
A. Jul 21, 2006−Jun 18, 2009								
DOTM	HS1	SV	6.3632	5.6973	NA	9.5021	10.3761	NA
		SVJ	33.4637	8.1925	NA	24.1823	9.2254	NA
	HS2	SV	4.8084	1.9527	NA	8.6915	3.3650	NA
		SVJ	5.3711	2.2702	NA	9.2961	3.8778	NA
OTM	HS1	SV	5.1153	NA	NA	9.5244	NA	NA
		SVJ	27.9749	NA	NA	20.6359	NA	NA
	HS2	SV	3.8156	NA	NA	9.5799	NA	NA
		SVJ	4.2278	NA	NA	10.3038	NA	NA
ATM1	HS1	SV	5.9481	7.8628	NA	11.3570	10.7725	NA
		SVJ	32.5398	11.1507	NA	22.4562	14.2753	NA
	HS2	SV	4.6932	4.0339	NA	11.4669	7.2320	NA
		SVJ	4.9902	5.1210	NA	11.7750	9.0196	NA
ATM2	HS1	SV	8.6628	8.8071	NA	16.8057	13.1141	NA
		SVJ	23.1278	12.8809	NA	23.5359	18.5878	NA
	HS2	SV	7.3054	5.6731	NA	17.1326	10.9915	NA
		SVJ	7.5161	7.1724	NA	17.3088	13.6157	NA
ITM	HS1	SV	9.2380	10.8061	5.2657	18.7035	17.9655	6.7988
		SVJ	25.5432	60.2912	7.9828	23.0510	43.0296	9.6633
	HS2	SV	8.9247	7.3065	2.3606	21.5786	14.4162	3.9708
		SVJ	8.7227	9.7977	3.2370	20.9328	19.1125	5.3365
DITM	HS1	SV	12.5563	10.4566	NA	27.3971	17.2110	NA
		SVJ	77.5575	20.3055	NA	49.8613	31.2482	NA
	HS2	SV	13.8977	8.5752	NA	31.0028	14.4759	NA

		SVJ	12.4760	14.2575	NA	28.8017	25.3368	NA
B. Jul 21, 2006−Aug 14, 2008								
DOTM	HS1	SV	1.6271	1.2178	NA	1.0857	1.6810	NA
		SVJ	1.2366	1.1325	NA	1.1520	1.4655	NA
	HS2	SV	1.1006	0.9832	NA	1.9167	1.6103	NA
		SVJ	1.1500	1.3442	NA	1.2992	2.1122	NA
OTM	HS1	SV	1.9532	NA	NA	2.2629	NA	NA
		SVJ	2.0454	NA	NA	2.8458	NA	NA
	HS2	SV	1.2986	NA	NA	2.2892	NA	NA
		SVJ	1.4350	NA	NA	2.3661	NA	NA
ATM1	HS1	SV	3.9689	5.9950	NA	5.8632	8.1794	NA
		SVJ	4.2744	5.4708	NA	6.5958	7.9208	NA
	HS2	SV	3.0985	3.0499	NA	5.6791	6.2113	NA
		SVJ	3.3586	3.6063	NA	5.6930	7.4192	NA
ATM2	HS1	SV	6.7815	6.5657	NA	10.5221	10.2949	NA
		SVJ	7.1242	6.7447	NA	11.2855	10.8328	NA
	HS2	SV	5.4907	4.5320	NA	10.2085	9.8554	NA
		SVJ	5.9103	5.3767	NA	10.1996	11.5051	NA
ITM	HS1	SV	6.7833	6.4763	3.2729	12.7463	14.1034	3.8647
		SVJ	6.8680	6.8006	2.2594	12.3273	14.4697	3.1666
	HS2	SV	6.5618	6.2657	1.4700	14.5355	15.4006	2.6745
		SVJ	7.3731	7.3726	1.8959	13.6645	18.2209	3.5430
DITM	HS1	SV	12.3294	8.0027	NA	13.0597	13.3174	NA
		SVJ	11.7860	9.5670	NA	12.5780	13.6326	NA
	HS2	SV	13.8723	7.9045	NA	17.2277	15.7472	NA
		SVJ	12.3444	10.9772	NA	13.5022	19.9988	NA
C. Aug 15, 2008−Dec 18, 2008								
DOTM	HS1	SV	10.2786	10.7445	NA	15.5854	21.1990	NA
		SVJ	11.2412	3.4474	NA	16.7245	5.2536	NA
	HS2	SV	8.6603	3.1539	NA	15.0282	4.7203	NA
		SVJ	10.0824	3.0954	NA	16.2191	4.6470	NA
OTM	HS1	SV	11.3648	NA	NA	22.1098	NA	NA
		SVJ	12.4328	NA	NA	23.6604	NA	NA
	HS2	SV	8.1884	NA	NA	22.8581	NA	NA
		SVJ	9.5830	NA	NA	23.8533	NA	NA
ATM1	HS1	SV	12.3432	13.8284	NA	26.0009	18.7455	NA
		SVJ	13.6087	12.3066	NA	27.1859	13.8196	NA
	HS2	SV	9.1173	9.2577	NA	27.9439	11.4061	NA
		SVJ	9.9005	10.6086	NA	28.5429	12.5987	NA
ATM2	HS1	SV	15.3990	14.9844	NA	33.9719	20.0752	NA
		SVJ	16.1772	15.0249	NA	35.2682	18.5841	NA
	HS2	SV	13.4468	11.2431	NA	38.8582	14.7783	NA
		SVJ	13.7320	12.6910	NA	18.3527	17.0005	NA
ITM	HS1	SV	13.2632	18.0270	10.4035	25.5016	25.9797	14.3131
		SVJ	13.9370	16.7372	8.8769	24.8437	22.4414	10.4882
	HS2	SV	12.4548	12.6016	6.6167	31.9561	17.6768	8.8610
		SVJ	11.1111	13.7276	7.5838	30.7140	20.0581	9.6245
DITM	HS1	SV	14.1390	11.6558	NA	27.9134	17.7928	NA
		SVJ	14.2067	23.5612	NA	26.3868	29.3993	NA
	HS2	SV	17.1347	10.9146	NA	35.8170	14.5072	NA
		SVJ	15.3151	17.4858	NA	33.4585	27.7136	NA
D. Aug 15, 2008−Jun 18, 2009								
DOTM	HS1	SV	6.8930	9.0560	NA	10.4435	16.8957	NA

		SVJ	37.0684	13.4860	NA	26.7583	15.0437	NA
	HS2	SV	5.2231	2.6797	NA	9.4492	4.6807	NA
		SVJ	5.8432	2.9646	NA	10.1906	5.2016	NA
OTM	HS1	SV	8.2775	NA	NA	16.7859	NA	NA
		SVJ	53.9044	NA	NA	38.4259	NA	NA
	HS2	SV	6.3326	NA	NA	16.8706	NA	NA
		SVJ	7.0206	NA	NA	18.2415	NA	NA
ATM1	HS1	SV	9.2950	12.8060	NA	20.6474	17.6350	NA
		SVJ	80.3381	26.1824	NA	49.2772	31.0923	NA
	HS2	SV	7.3899	6.6382	NA	21.2546	9.9331	NA
		SVJ	7.7493	9.1297	NA	22.0601	13.2551	NA
ATM2	HS1	SV	11.6729	13.8663	NA	26.8594	19.4776	NA
		SVJ	48.7336	26.7312	NA	43.1366	36.0919	NA
	HS2	SV	10.2091	8.2489	NA	28.2111	13.5557	NA
		SVJ	10.0855	11.2256	NA	28.6836	18.3797	NA
ITM	HS1	SV	10.9667	15.2887	10.6587	22.8986	21.9639	14.7396
		SVJ	38.6948	115.6698	23.4721	30.6029	72.5975	27.2455
	HS2	SV	10.5887	8.3841	4.7710	26.5385	13.3971	7.4790
		SVJ	9.6730	12.3084	6.8665	26.0513	20.0356	10.1905
DITM	HS1	SV	12.5792	11.0456	NA	28.8393	18.1454	NA
		SVJ	84.1736	22.8828	NA	53.6117	35.4760	NA
	HS2	SV	13.9003	8.7362	NA	32.3885	14.1708	NA
		SVJ	12.4892	15.0447	NA	30.3407	26.6179	NA
E. Dec 19, 2008−Jun 18, 2009								
DOTM	HS1	SV	3.7577	6.4093	NA	5.6818	10.1500	NA
		SVJ	60.9855	29.2222	NA	36.0501	30.3903	NA
	HS2	SV	2.0401	1.9362	NA	4.2829	4.6185	NA
		SVJ	1.9175	2.7597	NA	4.6079	6.0710	NA
OTM	HS1	SV	5.0210	NA	NA	11.1703	NA	NA
		SVJ	97.6484	NA	NA	54.0004	NA	NA
	HS2	SV	4.3751	NA	NA	10.5551	NA	NA
		SVJ	4.3178	NA	NA	12.3222	NA	NA
ATM1	HS1	SV	6.4850	11.3596	NA	15.7121	16.0640	NA
		SVJ	141.8543	45.8117	NA	69.6426	55.5268	NA
	HS2	SV	5.7975	2.9326	NA	15.0879	7.8492	NA
		SVJ	5.7661	7.0376	NA	16.0837	14.1836	NA
ATM2	HS1	SV	8.0172	12.2382	NA	19.8811	18.6074	NA
		SVJ	80.6758	43.7772	NA	50.8566	61.5858	NA
	HS2	SV	7.0324	3.8890	NA	17.7650	11.7753	NA
		SVJ	6.5077	9.0916	NA	18.0233	20.3881	NA
ITM	HS1	SV	8.7341	12.4852	11.0451	20.3680	17.8525	15.3854
		SVJ	62.7648	216.9580	45.5696	36.2021	123.9478	52.6163
	HS2	SV	8.7745	4.0662	1.9765	21.2714	9.0154	5.3865
		SVJ	8.2749	10.8555	5.7805	21.5181	20.0125	11.0474
DITM	HS1	SV	11.3346	10.0777	NA	29.5781	18.7047	NA
		SVJ	139.9982	21.8067	NA	75.3337	45.1149	NA
	HS2	SV	11.3197	5.2808	NA	29.6530	13.6371	NA
		SVJ	10.2345	11.1726	NA	27.8531	24.8800	NA

Table 5 Average Hedging Errors

The figures in this table denote the average points of hedging errors ($250 per point): $\sum_{l=1}^{M} HE_{t+(l-1)\Delta t}(t+l\Delta t)e^{r\Delta t(M-l)}/M$ where $M=(T_1-t)/\Delta t$ and T_1 is the maturity date of SPX futures options. The hedging error between time t and time $t+\Delta t$ is defined as $HE_t(t+\Delta t)$. The instrument portfolio of hedging scheme 1 (HS1) consists of $N_{1,t}$ shares of T_2-matured SPX futures, and $N_{2,t}$ shares of *forward-start* strangle portfolios. The *forward-start* strangle portfolio consists of a short position on a T_1-matured strangle and a long position on a T_2-matured strangle. The instrument portfolio of hedging scheme 2 (HS2) consists of $N_{1,t}$ shares of T_2-matured SPX futures, and $N_{2,t}$ shares of the VIX futures $\mathrm{F}_t^{\mathrm{VIX}}(T_1)$ with expiry T_1. This study classifies hedging errors into moneyness-maturity categories. Define $F_t(T_2)/K$ as the time-t intrinsic value of a futures call. A futures call is then said to be *deep out-of-the-money* (DOTM) if its $F_t(T_2)/K$<0.94; *out-of-the-money* (OTM) if $F_t(T_2)/K\in$[0.94,0.97); *at-the-money* (ATM) if $F_t(T_2)/K\in$[0.97,1.03); *in-the-money* (ITM) if $F_t(T_2)/K\in$[1.03,1.06); and *deep in-the-money* (DITM) if $F_t(T_2)/K\geqq$1.06. By the term to expiration, an option contract can be classified as (i) short-term (<60 days); (ii) medium-term (60−180 days); and (iii) long-term (>180 days). The out-of-sample hedging period is from 21 July 2006 to 18 June 2009.

			1-Day Revision Days-to-Expiration			5-Day Revision Days-to-Expiration		
Moneyness	Hedging Scheme	Model	<60	60−180	>180	<60	60−180	>180
A. Jul 21, 2006−Jun 18, 2009								
DOTM	HS1	SV	2.6751	1.7914	NA	5.8495	0.4414	NA
		SVJ	27.0928	-0.7557	NA	11.2146	1.0824	NA
	HS2	SV	1.5824	0.0343	NA	6.4765	1.0221	NA
		SVJ	1.2730	-0.2707	NA	6.0054	0.3283	NA
OTM	HS1	SV	1.8377	NA	NA	4.3552	NA	NA
		SVJ	22.9151	NA	NA	9.7183	NA	NA
	HS2	SV	1.7310	NA	NA	6.1668	NA	NA
		SVJ	1.5719	NA	NA	5.9076	NA	NA
ATM1	HS1	SV	1.0652	-1.3493	NA	4.2041	-0.4061	NA
		SVJ	25.8742	-0.0613	NA	10.7589	1.1783	NA
	HS2	SV	1.3532	-0.1526	NA	6.1106	0.8160	NA
		SVJ	1.0854	-0.2912	NA	5.5098	0.3016	NA
ATM2	HS1	SV	2.1109	-0.2209	NA	7.4830	1.4061	NA
		SVJ	14.8455	-0.2544	NA	10.4667	1.7994	NA
	HS2	SV	2.2409	0.4255	NA	8.9910	2.3051	NA
		SVJ	1.7644	0.1888	NA	7.9154	1.7746	NA
ITM	HS1	SV	5.2660	2.1356	-0.3057	13.6082	4.2484	0.6765
		SVJ	20.1153	-44.2970	1.1750	15.4093	-16.7687	2.1988
	HS2	SV	4.6870	1.0610	0.0092	16.1043	4.4351	0.8643
		SVJ	3.8976	1.3676	-0.0946	13.7040	4.9788	0.5375
DITM	HS1	SV	8.4072	7.0410	NA	17.2502	10.4616	NA
		SVJ	71.6119	1.8917	NA	34.5850	7.1923	NA
	HS2	SV	8.2722	2.9026	NA	21.1030	6.4221	NA

		SVJ	7.5708	4.0142	NA	18.4119	5.9572	NA
B. Jul 21, 2006−Aug 14, 2008								
DOTM	HS1	SV	1.1005	-0.1551	NA	0.4717	0.3662	NA
		SVJ	0.1498	-0.2512	NA	0.5120	-0.0138	NA
	HS2	SV	0.3383	-0.0488	NA	1.9032	0.5607	NA
		SVJ	-0.1463	-0.2531	NA	0.9262	0.0290	NA
OTM	HS1	SV	-0.1757	NA	NA	0.1411	NA	NA
		SVJ	-0.7473	NA	NA	-0.0734	NA	NA
	HS2	SV	-0.0272	NA	NA	0.9624	NA	NA
		SVJ	-0.2122	NA	NA	0.4965	NA	NA
ATM1	HS1	SV	-1.4282	-3.9936	NA	-1.1171	-4.0615	NA
		SVJ	-2.2169	-3.4203	NA	-1.7715	-3.5659	NA
	HS2	SV	-0.8878	-0.6871	NA	0.0114	-1.0542	NA
		SVJ	-1.2743	-0.8452	NA	-0.9385	-1.1887	NA
ATM2	HS1	SV	-1.5623	-2.7536	NA	-1.5626	-1.6550	NA
		SVJ	-2.7302	-2.9284	NA	-2.8810	-1.7831	NA
	HS2	SV	-0.7772	-0.3025	NA	-0.4029	0.5217	NA
		SVJ	-1.5495	-0.5879	NA	-2.1998	0.4280	NA
ITM	HS1	SV	1.1713	-0.5817	-2.1437	4.2390	0.0866	-2.1977
		SVJ	-0.7676	-0.7356	-1.1146	1.9703	0.1899	-1.2827
	HS2	SV	0.0445	-0.3196	-0.2534	5.0249	1.0550	-0.4425
		SVJ	-1.6006	-0.3830	-0.3994	0.8304	1.6909	-0.6580
DITM	HS1	SV	5.8529	-0.0518	NA	-0.4809	-3.1088	NA
		SVJ	1.7059	0.3941	NA	-1.1598	-1.8185	NA
	HS2	SV	2.1414	0.3231	NA	3.8632	-2.2608	NA
		SVJ	1.1414	1.3946	NA	0.3446	2.5661	NA
C. Aug 15, 2008−Dec 18, 2008								
DOTM	HS1	SV	3.4756	2.2566	NA	12.5453	-2.3971	NA
		SVJ	3.4942	-0.1377	NA	12.6128	1.3987	NA
	HS2	SV	2.1859	0.2285	NA	13.7370	2.6005	NA
		SVJ	1.9603	-0.3251	NA	13.0335	1.4649	NA
OTM	HS1	SV	4.6537	NA	NA	15.7165	NA	NA
		SVJ	4.6785	NA	NA	15.7687	NA	NA
	HS2	SV	4.8896	NA	NA	20.2839	NA	NA
		SVJ	4.6017	NA	NA	19.7219	NA	NA
ATM1	HS1	SV	6.2796	3.0750	NA	22.1032	8.0260	NA
		SVJ	6.1910	3.8531	NA	22.0878	9.6287	NA
	HS2	SV	7.1605	2.0326	NA	27.6385	9.2923	NA
		SVJ	6.7503	1.6120	NA	27.1801	6.0380	NA
ATM2	HS1	SV	10.8362	2.3677	NA	33.3233	7.6202	NA
		SVJ	10.9938	6.4346	NA	33.9861	12.8222	NA
	HS2	SV	11.4165	3.5637	NA	38.8582	11.8325	NA
		SVJ	11.3499	3.4508	NA	39.4183	8.7791	NA
ITM	HS1	SV	10.2128	0.6728	1.4199	25.5016	4.2004	6.4732
		SVJ	10.1136	7.4511	2.0230	24.6431	12.6709	6.4332
	HS2	SV	10.6790	5.1764	1.2534	31.9293	13.3154	6.9852
		SVJ	10.0371	5.2605	0.9014	30.0599	10.6106	5.1834
DITM	HS1	SV	10.7169	9.5972	NA	27.9062	16.1542	NA
		SVJ	10.1716	14.2991	NA	26.3868	21.0737	NA
	HS2	SV	12.3570	4.9257	NA	35.1664	10.9297	NA
		SVJ	11.2316	5.0179	NA	32.2531	3.8648	NA
D. Aug 15, 2008−Jun 18, 2009								
DOTM	HS1	SV	2.8513	3.2510	NA	6.4510	0.4977	NA
		SVJ	30.1065	-1.1340	NA	12.4117	1.9043	NA

	HS2	SV	1.7215	0.0966	NA	6.9881	1.3680	NA
		SVJ	1.4317	-0.2839	NA	6.5735	0.5527	NA
OTM	HS1	SV	3.8510	NA	NA	8.5692	NA	NA
		SVJ	46.5776	NA	NA	19.5099	NA	NA
	HS2	SV	3.4891	NA	NA	11.3712	NA	NA
		SVJ	3.3560	NA	NA	11.3187	NA	NA
ATM1	HS1	SV	5.2817	5.6485	NA	13.2025	9.2676	NA
		SVJ	73.3778	8.8283	NA	31.9484	13.7338	NA
	HS2	SV	5.1429	1.2620	NA	16.4245	5.7655	NA
		SVJ	5.0757	1.1751	NA	16.4142	4.2457	NA
ATM2	HS1	SV	7.9881	5.4960	NA	21.9560	8.3154	NA
		SVJ	42.9668	5.7811	NA	31.8231	9.8858	NA
	HS2	SV	7.0698	2.0688	NA	24.0212	6.3305	NA
		SVJ	7.0665	1.9420	NA	24.0999	4.8140	NA
ITM	HS1	SV	8.1497	4.9488	4.6685	20.2063	8.5570	8.4548
		SVJ	34.8215	-89.3959	7.3716	24.8733	-34.3259	11.6207
	HS2	SV	7.9563	2.4904	0.7199	23.9067	7.9345	4.4007
		SVJ	7.7696	3.1800	0.7302	22.7700	8.3827	3.7729
DITM	HS1	SV	8.6641	8.7433	NA	19.0337	13.7186	NA
		SVJ	78.6438	2.2511	NA	38.1806	9.3548	NA
	HS2	SV	8.8889	3.5217	NA	22.8372	8.5060	NA
		SVJ	8.2175	4.6429	NA	20.2293	6.7711	NA
E. Dec 19, 2008−Jun 18, 2009								
DOTM	HS1	SV	2.2731	4.8096	NA	0.8075	5.0356	NA
		SVJ	54.7506	-2.6957	NA	12.2255	2.6968	NA
	HS2	SV	1.2915	-0.1103	NA	0.7383	-0.5641	NA
		SVJ	0.9423	-0.2194	NA	0.5913	-0.8772	NA
OTM	HS1	SV	3.0043	NA	NA	1.0303	NA	NA
		SVJ	90.7725	NA	NA	23.4561	NA	NA
	HS2	SV	2.0119	NA	NA	1.9700	NA	NA
		SVJ	2.0420	NA	NA	2.4550	NA	NA
ATM1	HS1	SV	4.3617	9.2891	NA	4.9972	11.0240	NA
		SVJ	135.3156	15.8665	NA	41.0387	19.5409	NA
	HS2	SV	3.2829	0.1719	NA	6.0867	0.7763	NA
		SVJ	3.5319	0.5571	NA	6.4895	1.7102	NA
ATM2	HS1	SV	5.1936	10.0511	NA	10.8031	9.3275	NA
		SVJ	74.3365	4.8295	NA	29.7009	5.6100	NA
	HS2	SV	2.8052	-0.1080	NA	9.4641	-1.6811	NA
		SVJ	2.8640	-0.2551	NA	9.0706	-0.9598	NA
ITM	HS1	SV	6.1438	9.3266	9.5870	15.0581	13.0174	11.4549
		SVJ	58.8430	-188.5487	15.4696	25.0971	-82.4417	19.4747
	HS2	SV	5.3093	-0.2594	-0.0879	16.1070	2.4255	0.4878
		SVJ	5.5652	1.0500	0.4711	15.6825	6.1017	1.6373
DITM	HS1	SV	7.0263	7.3888	NA	11.9546	9.8550	NA
		SVJ	133.2760	-16.8594	NA	47.5906	-9.2337	NA
	HS2	SV	6.1218	1.2948	NA	13.0001	4.6614	NA
		SVJ	5.8127	4.0481	NA	10.6358	11.3811	NA

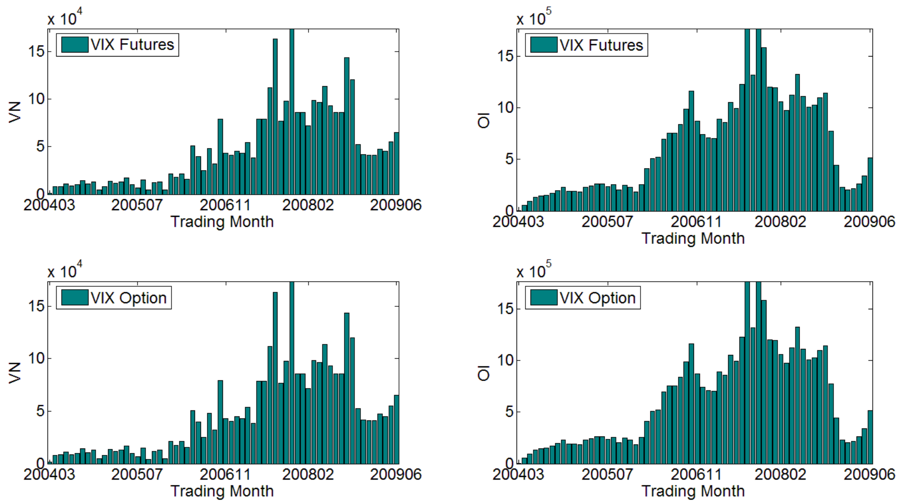
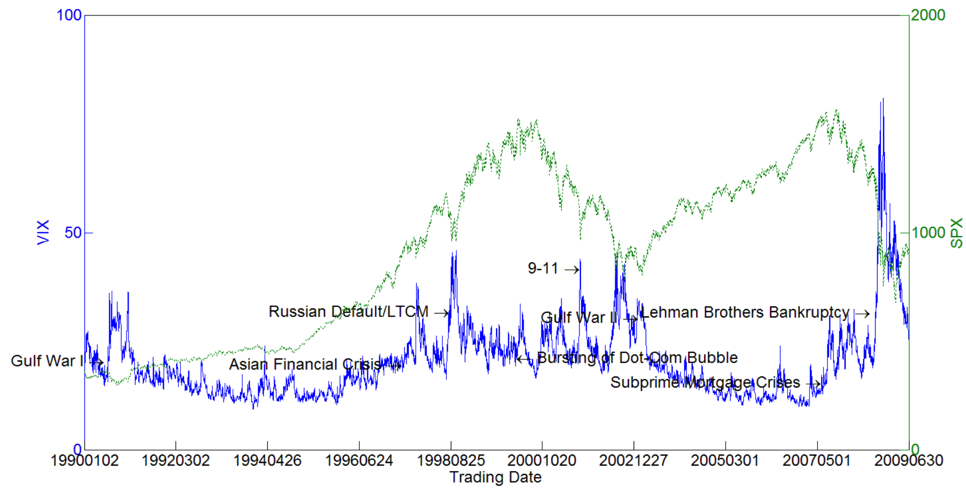


Figure 1 Trading volume (VN) and open interest (OI) of VIX futures and VIX options across trading months, March 2004 – June 2009 and February 2006 – June 2009, respectively.

Panel A: VIX (solid line) and SPX (dot line) across trading dates, January 2, 1990 – June 30, 2009



Panel B: VIX (solid line) and SPX (dot line) across trading dates, July 3, 2006 – June 30, 2009

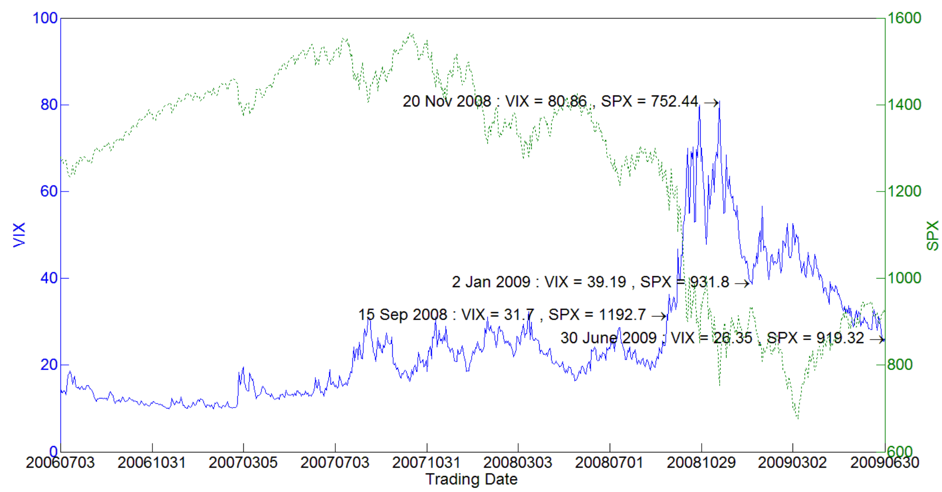


Figure 2 Time-series plot of the VIX (solid line) against the S&P 500 index (SPX) (dot line) across trading dates