Using Volatility Futures as Extreme Downside Hedges

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Abstract

"Long volatility" is thought to be an effective hedge against a long equity portfolio, especially during periods of extreme market movements. This study examines using volatility futures and variance futures as extreme downside hedges, and compares their effectiveness against traditional "long volatility" hedging instruments such as rolling series of 5% and 10% out-of-the-money put options. Using contract-rolling methodologies that are generally consistent with market practice, our results show that VIX volatility futures seem to be a more effective extreme downside hedge than traditional option rolling strategies with 5% and 10% out-of-the-money put options on the S&P 500 index as well as variance futures. In particular, using 1-month rolling VIX futures and a reasonable hedging model presents a cost-effective choice as a hedging instrument for extreme downside risk protection as well as for upside preservation. This observation is significant, since there are not yet obvious theoretical justifications as to why using VIX futures can be more efficient than using the underlying options when dealers are likely to charge defensive margins due to imperfect replication.

Keywords: VIX futures; Variance futures; VIX Term Structure; S&P 500 puts; Extreme Downside Risk; Hedging Effectiveness (*Classification Code: G12, G13, G14)*

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1. Introduction

Futures and options on VIX allow investors to buy or sell the VIX (which measures the SPX's implied volatility over 30 days), whereas VT allows investors to trade the difference between implied and realized variance of the SPX over three months. The distinction between variance futures and variance swaps is minimal, as the information contained in them is virtually identical. There are several problems associated with using the variance futures for empirical analysis: First, they are illiquid. The VT contracts are far less liquid than the VIX futures (Huang and Zhang, 2010 [2]). For example, on March 2, 2010, the trading volume of the VIX futures was 13,864 contracts, which was 4,621 times greater than 3 (2) contracts traded on the VT (VA). Second, VT has a maturity of three months. This means that there have been only 29 non-overlapping VT observations during the December 2004 to March 2012 period covered by this paper. Given the high volatility of returns on variance futures, there is not enough data to determine if the mean return is statistically significant.

Where data may be too sparse to be credible, this study uses the so-called "VIX squared" (which corresponds to the 1-month S&P500 variance swap rate) as and when necessary. Since the VIX goes back to the 1990s, this would give us over 250 non-overlapping monthly variance swap returns, making it possible to establish a variance risk premium with more meaningful precision.

From an investor's point of view, it seems attractive that the negative correlation between volatility and stock index returns is particularly pronounced in stock market downturns, thereby offering protection against stock market losses when it is needed most. Empirical studies, however, indicate that this kind of downside or crash protection might be expensive because of its constant negative carries. Practically speaking, however, it may be impossible to time the market to pay for protection only during a significant market downturn. Egloff, Leippold and Wu (2010) [1] have an extensive analysis of how variance swaps or volatility futures fit into optimal portfolios in dynamic context that takes into account how variance swaps, in addition to improving Sharpe ratios, improve the ability of the investor to hedge time-variations in investment opportunities. Moran and Dash (2007) [6] discuss the benefits of a long exposure to VIX futures and VIX call options. Szado (2009) [7] analyzes the diversification impacts of a long VIX exposure during the 2008 financial crisis. His results suggest that, dollar for dollar, VIX calls could have provided a more efficient means of diversification than provided by SPX puts.

This study begins by using volatility futures and variance swaps as extreme downside hedges. We apply hedging techniques as they are typically used in real-life trading, with rolling methods that are consistent with market practice – we approach this analysis from the perspective of real-life trading practices in order to come up with realistic estimates on true hedging effectiveness. Practically speaking, an investor often needs a dealer who is willing to take the other side of the trade on the exchange because of the lack of liquidity, while the dealers are simply replicating their VIX or variance futures exposures with options positions. In principle, it is not obvious how using the VIX and variance futures can be more efficient than using the underlying options, since dealers are likely to charge defensive margins due to imperfect replication, unless significant trading volume native to the VIX future or variance futures markets already exists. To objectively analyze the effectiveness of using VIX or variance futures in the time of market crisis, we use a long SPX portfolio and compare various hedging strategies using: (i) VIX futures; (ii) variance futures, (iii) 5% out-of-the-money (OTM) SPX put options, and (iv) 10% out-of-the-money (OTM) SPX put options.

The remainder of the paper is organized as follows: Section 2 describes the hedging strategies implemented. Section 3 provides an analysis of the hedging results. Section 4 concludes.

2. Methodology

This section will provide an in-depth discussion of the methodologies used in this paper: (i) the hedging schemes; (ii) the rolling methodology for the VIX futures; (iii) the rolling methodology for the variance futures and the creation of synthetic 1-month variance futures data where required; and (iii) the rolling methodology for OTM put options on SPX.

2.1. Hedging Schemes

Kuruc and Lee (1998) [3] describe the generalized delta-gamma hedging algorithm. In principle, this algorithm can be expanded to delta-gamma-vega by naming vega risk factors, but the author is not aware of any simple and practical solution to the vega mismatch problem: i.e. simply adding vegas corresponding to implied volatilities with different moneyness and maturities does not provide meaningful solutions.

The generalized delta solution using minimal Value-at-Risk (VaR) objective function is defined as follows: Let our objective function be $VaR\left(\vec{a}\right) = \sqrt{\vec{\alpha}^T \Theta \vec{\alpha}}$, where $\overrightarrow{a} = \left[\frac{\partial A}{\partial r_1}\ \frac{\partial A}{\partial r_2} \cdots \frac{\partial A}{\partial r_n}\right]^T$ is the vector of "delta equivalent cashflow" positions of our portfolio A as measured against a *m*-dimensional vector of nominated risk factors $\overrightarrow{r} = [r_1\ r_2 \cdots r_m]^T$, with $\alpha_j = \sigma_j\sqrt{\Delta t}(r_j a_j)$, and σ_j is the volatility of the log changes in the *j*-th risk factor multiplied by a scalar dependent on the confidence level, and Θ is the correlation matrix for the nominated risk factors in $\overrightarrow{r} = [r_1\ r_2 \cdots r_m]^T$. The corresponding variance/covariance matrix is given by $\Sigma = \mathrm{diag}_j\left(\sigma_j\sqrt{\Delta t}\right)\Theta\mathrm{diag}_j\left(\sigma_j\sqrt{\Delta t}\right)$. In the case of delta Value-at-Risk, the n-dimension vector $\hat{h}$ representing the hedging solution can be obtained from solving $\min_{\overrightarrow{h}} \sigma^2\left(\overrightarrow{a} + X\overrightarrow{h}\right)$, where $\sigma^2(\cdot)$ denotes the variance of the hedged portfolio $(\overrightarrow{a} + X\overrightarrow{h})$ and X is a $m \times n$-matrix with its *i*-th column being the corresponding *m*-dimensional delta equivalent cashflow mapping vector for the *i*-th hedging instrument. Its closed-form solution, in the absence of any trading constraints, is

$$\hat{h} = -\left[(GX)^T(GX)\right]^{-1}(GX)^T\left(G\overrightarrow{a}\right)$$

where G is the Cholesky decomposition of Σ, or $\Sigma = G^TG$.

The more general delta-gamma solution can be obtained by using a modified objective function $VaR(A) = \sqrt{\overrightarrow{\alpha}^T\Theta\overrightarrow{\alpha} + \frac{1}{2}trace\left(\beta\Theta\Delta\beta\Theta\right)}$, where (using δ_{ij} as the Kronecker delta notation):

$$\beta_{ij} = \sigma_i\sigma_j\Delta t\left(\frac{\partial^2 A}{\partial lnr_i\,\partial lnr_j}\right) = \sigma_i\sigma_j\Delta t\left(r_ir_j\frac{\partial^2 A}{\partial r_i\,\partial r_j} + \delta_{ij}r_j\frac{\partial A}{\partial r_j}\right)$$

In general, the hedging objective function above can be made arbitrarily complex, but doing so may result in non-unique boundary value solutions. Furthermore, highly complex hedging objective function may not be readily solvable in the context of real-world trading due to the presence of constraints: for instance, it is not uncommon for hedgers to be disallowed from "shorting" put options or other volatility instruments by their risk and compliance departments. For our empirical analysis piece to be impactful, this paper will focus on applying the general hedging scheme described above in ways that are relevant to practical real-world trading.

No matter how simple or complex the hedging methodology, hedging can almost always be translated into an equivalent optimization problem with different objective functions. To the best of the author' knowledge, the formulation by Kuruc and Lee (1998) [3] was one of the few original hedging formulations that do not assume a certain degree of resemblance between the portfolio and the chosen hedging instruments, as it is often the case when hedging problems are posed as statistical problems. Moreover, the general vector and matrix framework can still apply by customizing different objective functions for the specific hedging problem. For example, the value of a large fixed income portfolio is usually expressed in terms of "deltas" in order to consolidate a complex portfolio of many different bonds with relevant yield curve factors. For a simple portfolio of a few assets, one can simply nominate the asset itself as the risk factor. In this paper, we can simplify the problem by stating that a portfolio of one unit of the SPX index has an exposure to the dollar S&P 500 factor. These are the seven objective functions used in our study when applied to a single hedging instrument:

1. Minimum absolute residual hedge by minimizing the sum of absolute percentage changes in the market-to-market value of the hedged portfolio, $pherr(t)$, or

   $$\min_h \sum_{1 \le t \le T} |pherr\,(t)|$$

   where $pherr\,(t) = \frac{MTM(t)}{MTM(t-1)} - 1 = \frac{A(t)+hx_{cumP\&L}(t)}{A(t-1)+hx_{cumP\&L}(t-1)} - 1$; $A\,(t)$ is the day-t mark-to-market value of the unhedged portfolio; and $x_{cumP\&L}\,(t) = x\,(t) - x(0)$ is the cumulative P&L of the hedging instrument on day t, assuming $x(0)$= 0 in general.
2. Minimum variance hedge by minimizing the sum of squared percentage changes in mark-to-market value of the hedged portfolio, $pherr(\cdot)$, or

   $$\min_h \sum_{1 \le t \le T} (pherr\,(t))^2$$

3. Minimize the peak-to-trough maximum drawdown of the mark-to-market value of the hedged portfolio:

   $$\min_h MaxDD\left(T; \{A(t) + hx_{cumP\&L}(t)\}_{t=0}^{T}\right)$$

   The drawdown is the measure of the decline from a historical peak in some time series, typically representing the historical mark-to-market value of a financial asset. Let $MTM\,(t) = A\,(t) + hx_{cumP\&L}(t)$ be the dollar mark-to-market value of the brokerage account representing the hedged portfolio at the end of the period $[0, t]$, and $MTM_{cum,0\leq\tau<t}^{peak} = \max_{0\leq\tau\leq t}[MTM(\tau)]$ be the maximum cumulative dollar mark-to-market value in the $[0, t]$ period. The drawdown at any time, t, denoted $DD(t)$ is defined as

$$DD\left(t\right) = MTM_{0\leq\tau\leq t}^{peak} - MTM(t)$$

The Maximum Drawdown ($MaxDD$) from time 0 up to time T is the maximum of the drawdown over the history of the MTM. Formally,

$$MaxDD\left(T; \{MTM(t)\}_{t=0}^{T}\right) = \max_{0\leq t\leq T}\{DD(t)\}$$

Alternatively, $MaxDD\%$ is also used to describe the percentage drop from the peak to the trough as measured from the peak. Using $MaxDD$ as a measure of risk, the optimization procedure will find an estimate of the hedge ratio h that can achieve the lowest possible $MaxDD$.

4. Minimize the Expected Shortfall or Conditional Value-at-Risk at 95% computed from the daily P&L of the hedged portfolio, or equivalently,

$$\min_{h} CVaR\left(A_{P\&L}\left(t\right) + hx_{P\&L}(t)\right)$$

with $A_{P\&L}\left(t\right) = A(t) - A(t-1)$ and $x_{P\&L}\left(t\right) = x(t) - x(t-1)$ using the Cornish-Fisher expansion (Lee and Lee, 2004 [5]), where

$$\begin{aligned} CVaR_{95\%}(\cdot) &= -\left[\mu(\cdot) + \sigma(\cdot)\mathbb{E}\left(z_{cf,1-\kappa} \mid \kappa > 95\%\right)\right] \\ &= -\mu(\cdot) - \sigma(\cdot)\mathbb{E}\left[\left.\begin{array}{l} z_{C(1-\kappa)} + \frac{1}{6}\left(z_{C(1-\kappa)}^2 - 1\right)S(\cdot) \\ +\frac{1}{24}\left(z_{C(1-\kappa)}^3 - 3z_{C(1-\kappa)}\right)K(\cdot) \\ -\frac{1}{36}\left(2z_{C(1-\kappa)}^3 - 5z_{C(1-\kappa)}\right)S(\cdot)^2 \end{array}\right| \kappa > 95\%\right] \end{aligned}$$

with $z_{C(1-\kappa)}$ being the critical value for probability $1-\kappa$ with standard normal distribution (e.g. $z_{C(1-\kappa)} = -1.64$ at $\kappa = 95\%$), while μ, σ, S and K following the standard definitions of mean, volatility, skewness and excess kurtosis, respectively, as computed from the daily P&L of the hedged portfolio. Practically, the expected value of the tail of $z_{cf,1-\kappa}$ at and above 95% estimated numerically by using the discrete average of $z_{C(1-\kappa)}$ taken at 95.5%, 96.5%, 97.5%, 98.5% and 99.5% (e.g. $z_{C(1-\kappa)} = -1.70$ at $\kappa = 95.5\%$, $z_{C(1-\kappa)} = -1.81$ at $\kappa = 96.5\%$, $z_{C(1-\kappa)} = -1.96$ at $\kappa = 97.5\%$,

$z_{C(1-\kappa)} = -2.17$ at $\kappa = 98.5\%$, and $z_{C(1-\kappa)} = -2.58$ at $\kappa = 99.5\%$, while their average is -2.04).

5. Minimize the Expected Shortfall or Conditional Value-at-Risk at 99%, or $CVaR_{99\%}$, by setting $\kappa = 99\%$.
6. All of the above are risk measures. It is not uncommon that minimizing risk measures will result in minimizing both the downside and the upside of the profit and loss stream. Since the specific problem is essentially one of combining two long assets, we will explore the possibility of maximizing an Omega-function-like measure (Omega functions are essentially functions based on ratios of a measure of upside cumulants to a measure of downside cumulants) known as the Alternative Sharpe Ratio (Lee and Lee, 2004 [5]). This is a more "balanced" approach of optimal hedging from the perspective of not only minimizing risk (which also tends to minimize returns) but also achieving an optimal balance between "upside moments" and "downside moments", and is generally consistent with real-world practice in that traders tend to underhedge to "preserve upside." The objective function to maximize is defined as:

$$ASR \equiv \frac{\sum_i e_i \pi_i}{z_\pi^- \sigma_\pi} + \frac{1}{2}\frac{\sum_i \pi_i \left(z_i^+ \sigma_i\right)^2}{z_\pi^- \sigma_\pi} - \frac{1}{2} z_\pi^- \sigma_\pi$$

where:
e_i = excess return rate of the i-th asset of the portfolio π; given that our study uses a single hedging instrument, the i-th asset is the portfolio itself, which is calculated as the percentage changes in the mark-to-market value of the hedged portfolio, $pherr(t)$
π_i = i-th position of the portfolio π
$z_i^+ = \frac{max\left(z_{cf}\left(z_C^+(i)\right),0\right)}{z_C^+}$ where z_C^+ is critical value for probability κ and
$z_\pi^- = \frac{min\left(z_{cf}\left(z_C^-(\pi)\right),0\right)}{z_C^-}$ where z_C^- is critical value for probability $1-\kappa$
(e.g. $z_C^+ = 2.33$ at 1%, $z_C^- = -2.33$ at 99%)
7. Maximize the traditional portfolio Sharpe Ratio, or $SR \equiv \frac{\sum_i e_i \pi_i}{\sigma_\pi}$.

Roughly consistent with real-world hedging practice, we use a minimum of 2 months of daily trading data prior to the hedging date in order to compute the hedging ratio. For objective function (2), all the residuals are exponentially weighted based on the industry-standard choice of $\lambda = 0.94$. At each rolling or hedge rebalancing date (3-month VT futures are rebalanced monthly), the hedging ratio is recomputed. An important factor affecting hedge effectiveness is the rolling assumption. That will be instrument specific and will be discussions in the next subsections.

2.2. VIX Futures

In order to test the various hedging strategies, our study uses the daily settlement prices on VIX futures from December 2004 to March 2012. The contract size of VIX futures is $1,000 times the value of the VIX Index. Price data on the VIX futures are obtained from the transaction records provided by the Chicago Futures Exchange (CFE).

According to the product specifications published by the CFE[2], the final settlement date for the VIX futures is the Wednesday which is 30 days before the third Friday of the calendar month immediately following the month in which the contract expires. This study chooses to roll on the fifth business day prior to the expiration date for the monthly VIX futures, in order to avoid well-known liquidity problems associated with the last week of trading. More specifically, on the first day of rolling to a contract, we want to take long positions on the second-nearby monthly VIX contracts based on closing price. The daily cumulative payoffs are calculated using daily settlement prices. The contracts are then closed at the closing prices. On the same day, we buy back the second-nearby contract at the closing price, and so on.

Since an investor does not pay upfront cash for the futures, his mark-to-market value (MTM) at the end of the day is the market value of his futures contract plus the cash balance of any financing required. The act of finally closing the futures in itself should create cash receivable/payable. The daily P&L should be computed based on a combination of the change in market values of the assets and in the balance of cash borrowed to finance any final settlement. For the purpose of this calculation, we have ignored the potential financing cost required to meet one's margin requirements. We then initiate a new contract on the next day to maintain the hedge. If the futures contracts close in the money, one should receive the exercise value of the contracts as cash, or pay cash if the contracts close out of the money. Any interest charges on a negative balance or interest accruals on a positive balance from the current period are treated as zero to simplify the analysis. The cumulative P&L as given below can be used as our mark-to-market value of the futures contracts starting from time t:

[2] See http://cfe.cboe.com/Products/Spec_VIX.aspx.

$$\begin{aligned}&Cumulative\ P\&L\,(t+l\triangle t)\ \ of\ \ VIX\ \ futures\ \ in\ \ the\ \ first\ \ rolling\ \ month\\&=\begin{cases}\$1000\ \times\ fvixfut_{cum}\,(t+l\triangle t)\,, & l=0,1,2,\ldots,M_1-1\\ cash\ \ (t+l\triangle t)\,, & l=M_1\end{cases}\end{aligned}$$

where $M_1 \equiv (T_1 - t)/\triangle t$ is the number of trading days between the current day t and position closing day T_1; $fvixfut_{cum}(t + l\triangle t) = vixfut_{settle}(t + l\triangle t) - vixfut_{open}(t)$ is the cumulative value of the futures contract at daily settlement on day $t + l\triangle t$, taking the difference between the daily settle futures price, $vixfut_{settle}(t + l\triangle t)$, and the day-$t$ futures price at the initiation of the contract, $vixfut_{open}(t)$.

On day T_1 we close out the first VIX futures and keep any resulting net cashflow in a cash account. Since the contract size of VIX futures is \$1,000 multiplied by the VIX Index points, the value of the day-T_1 cash account is:

$$cash\,(T_1) = \$1000\times fvixfut_{cum}\,(T_1) = \$1000\times\left[vixfut_{close}(T_1) - vixfut_{open}(t)\right]$$

In theory, the cumulative P&L for VIX futures initiated on day $(T_1 + \triangle t)$ in the second rolling month depends on whether interest charges from the first period become part of the P&L for the second period:

$$\begin{aligned}&Cumulative\ \ P\&L\ \ (T_1+\xi\triangle t)\ \ of\ \ VIX\ \ futures\ \ in\ \ the\ \ second\ \ rolling\ \ month\\&\quad = \$1000\times fvixfut_{cum}(T_1+\xi\triangle t) + cash(T_1+\xi\triangle t), \quad for\ \xi = 1,2,\ldots,M_2\end{aligned}$$

where $M_2 \equiv (T_2 - (T_1 + \triangle t))/\triangle t$; $fvixfut_{cum}(T_1 + \xi\triangle t) = vixfut_{settle}(T_1 + \xi\triangle t) - vixfut_{open}(T_1 + \triangle t)$ is the cumulative value of the contract on day $T_1 + \xi\triangle t$ with the opening price, $vixfut_{open}(T_1 + \xi\triangle t)$, of the second VIX futures initiated on day $T_1 + \triangle t$. The cash balance account, $cash(T_1 + \xi\triangle t)$, is given by

$$cash\ \ (T_1+\xi\triangle t)\ \ in\ \ second\ \ rolling\ \ month\ = cash\,(T_1)\times e^{R(T_1)\xi\Delta t}$$

where $R(T_1)$ is the continuously compounded zero-coupon interest rate on day

T_1. Similar cumulative P&L calculations are used for subsequent periods.

Typically, investors gain exposure to the SPX Index by trading ETF on the SPX.[3] Depositary receipts on the SPX, such as "SPDRs," represent ownership in unit trusts designed to replicate the underlying index. As such, SPDRs closely if not perfectly replicate movements in the underlying stock index. One of the most popular SPDRs, the SPY, is valued at 1/10th the value of the Index.[4] SPDRs typically tend to be transacted in 100-lot (or "round-lot") increments, like most other equities.[5] Further, the contract size of VIX futures is $1,000 times the index value of the VIX. In order to compute the number of VIX futures contracts required for one unit of the SPX index, we apply the appropriate multipliers for adjusting unit size and unit dollar values in the hedged portfolio.

In this study, we assume that a typical investor holds the long asset already, but it will be atypical for any fully-invested portfolio to set aside surplus cash to pay for the cost of hedging, except for realized P&L already captured by a cash account at time t. Any on-going margin funding requirement is assumed to be minimal. The total amount realized for the asset, when the profit or loss on the hedge is taken into account, is denoted by mark-to-market value (MTM), so that for , $\varsigma = 0, 1, 2, \ldots, M = (T - t)/\triangle t$,

$$\begin{aligned} MTM\,(t + \varsigma\Delta t) &= \$10 \times SPX\,(t + \varsigma\Delta t) + \\ &\quad h \times [\$1000 \times fvixfut_{cum}\,(t + \varsigma\Delta t)] + cash\,(t) \end{aligned}$$

The corresponding cumulative P&L of the portfolio from time t is given by

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[3] An ETF represents fractional ownership in an investment trust, or unit trusts, patterned after an underlying index, and is a mutual fund that is traded much like any other fund. Unlike most mutual funds, ETFs can be bought or sold throughout the trading day, not just at the closing price of the day.

[4] A single SPDR was quoted at $78.18, or approximately 1/10th the value of the S&P 500 at 778.12, on March 17, 2009.

[5] If a single unit of SPDRs was valued at $78.18 on March 17, 2009, it implies that a 100-lot unit of SPDRs was valued at $7,818 on that day.

$$
\begin{aligned}
MTM_{cumP\&L}\left(t+\varsigma\Delta t\right) &= MTM\left(t+\varsigma\Delta t\right)-MTM\left(t\right)\\
&= \$10\times SPX_{cumP\&L}\left(t+\varsigma\Delta t\right)+\\
&\quad h\times\left[\$1000\times fvixfut_{cum}\left(t+\varsigma\Delta t\right)\right]
\end{aligned}
$$

where $fvixfut_{cum}\left(t+\varsigma\Delta t\right)$ is the cumulative value of the futures contract on day $t+\varsigma\Delta t\ for\ \forall\varsigma$; $cash(t+\varsigma\Delta t)$ is the cash balance account; $MTM\left(t\right)=\$10\times SPX_{open}(t)$; and $SPX_{cumP\&L}(t+\varsigma\Delta t)=[SPX_{close}(t+\varsigma\Delta t)-SPX_{open}(t)]$.

When hedging is used, the hedger chooses a value for the hedge ratio h that minimizes an objective function of the value of the hedged portfolio, such as its variance. It is important to use the percentage changes in the cumulative P&L as input, i.e., $MTM_{cumP\&L}(t+\varsigma\Delta t)/MTM_{cumP\&L}(t+(\varsigma-1)\Delta t)-1$, because doing so avoids unstable and even non-sensical numerical values when there are massive market shocks in the market, and also because that is the most natural quantity to hedge against as seen from the investor's perspective. Figure 1 presents an example of the computed hedging ratios times 100 for one unit of the S&P index under all hedging models, against the specified numbers of round-lot VIX futures. All hedging ratios are lower-bounded by zero since in real-life trading traders are likely to face severe compliance restrictions against "shorting volatility".

[Figure 1 about here]

2.3. Variance Futures

Our study uses the daily VT futures prices from December 2004 to March, 2012. The contract multiplier for the VT contract is \$50 per variance point. In the following, we describe the algorithm for the rolling strategies of variance futures at five business days prior to the expiration date. Where 3-month VT data may be too sparse to be credible, this study performs monthly rolls based on synthetic 1-month VT, replicated from using $VIXTerm$ observations. The contracts are rolled quarterly, but rehedging can occur monthly.

VT contracts are forward starting three-month variance swaps. Once a futures contract becomes the front-quarter contract, it enters the three-month window during which realized variance is calculated. Because VT is based on the realized variance of the SPX, the price of the front-month contract can be stated as two distinct components: the realized variance (RUG) and the implied forward variance (IUG). RUG indicates the realized variance of the SPX corresponding to the front-quarter VT contract. IUG represents the future variance of the SPX that is implied by the daily settlement price of the front-quarter VT contract.

Using martingale pricing theory with respect to a risk-neutral probability measure *Q*, the time-*t* VT price in terms of variance points is the annualized forward integrated variance, $F_t^{VT}(T) = \frac{1}{\tau_1}E_t^Q(\upsilon_{T-\tau_1,T})$ for τ_1=3 months = 1/4 year. The value of a forward-starting VT contract is composed of 100% implied forward variance ($IUG_{T-\tau_1,T}$), as given by

$$F_t^{VT,fs}(T) = \frac{1}{\tau_1}E_t^Q(\upsilon_{T-\tau_1,T}) = IUG_{T-\tau_1,T} \tag{1}$$

where $0 < t < T - \tau_1 < T$. The analytical pricing formula for front-month VT is given by

$$F_t^{VT,fm}(T) = \frac{1}{\tau_1}E_t^Q(\upsilon_{T-\tau_1,T})$$

$$= \left(1 - \frac{T-t}{\tau_1}\right) RUG_{T-\tau_1,t} + \left(\frac{T-t}{\tau_1}\right) IUG_{t,T}, \tag{2}$$

where τ_1 is the total number of business days in the original term to expiration of the VT contract, t is current time, T is the final expiration date of the VT contract, and $0 < \frac{T-t}{\tau} < 1$. The formula to calculate the annualized realized variance (RUG) is as follows:[6]

$$RUG = 252 \times \left(\sum_{i=1}^{N_a-1} R_i^2/(N_e-1)\right), \tag{3}$$

where $R_i = \ln(P_{i+1}/P_i)$ is daily return of the S&P 500 from P_i to P_{i+1}; P_{i+1} is the final value of the S&P500 used to calculate the daily return; and P_i is the initial value of the S&P 500 used to calculate the daily return. This definition is identical to the settlement price of a variance swap with N prices mapping to $N-1$ returns. N_a is the actual number of days in the observation period, and N_e is the expected number of days in the period. The actual and expected number of days may differ if a market disruption event results to the closure of relevant exchanges, such as

[6]See http://cfe.cboe.com/education/VT_info.aspx for the details. Our RUG in Eq. (2) multiplying 10,000 is the RUG data available in the Chicago Futures Exchange website.

September 11, 2001.

Because the square of VIX (denoted by $VIX_{t,T}^2$) is defined as the variance swap rate, we are able to evaluate $VIX_{t,T}^2$ by computing the conditional expectation under the risk-neutral measure *Q*, as follows:

$$VIX_{t,T}^2 \equiv \frac{1}{T-t} E_t^Q \left(\upsilon_{t,T}\right) \tag{4}$$

Based on Eqs. (1)−(4), the IUG portion of a front-quarter VT contract can be replicated by $VIX_{t,T}^2$ extracted from $VIXTerm$ with identical days to maturity. In other words, we can synthesize the front-quarter VT with the following using notations from above:

$$F_t^{VT,fm}\left(T\right) = \left(1 - \frac{T-t}{\tau_1}\right) \times RUG_{T-\tau_1,t} + \left(\frac{T-t}{\tau_1}\right) \times VIX_{t,T}^2 \tag{5}$$

Since the market price of a forward-starting VT future is completely attributable to IUG, this study takes the initial *forward* VIX (denoted $fVIX$) curve implicit in $VIXTerm$ to synthesize the forward-starting VT price, i.e., for $\forall t \in [0, T-\tau_1]$,

$$F_t^{VT,fs}\left(T\right) = fVIX_{T-\tau_1,T}^2(t) \tag{6}$$

Specifically, the following equation uses historic $VIXTerm$ observations to compute a time series history of *forward* VIX^2:

$$fVIX_{T-\tau_1,T}^2\left(t\right) = \frac{1}{\tau_1}\left[VIX_{t,T}^2 \times (T-t) - VIX_{t,T-\tau_1}^2 \times (T-\tau_1-t)\right] \tag{7}$$

where $0 < t < T - \tau_1 < T$.

The following steps are used to construct the monthly rolling of VT. On day *t*, we take a long position of the synthetic forward-starting 1-month variance futures. For forward-starting contracts, the daily cumulative payoffs are calculated using $fVIX_{T-\tau_1,T}^2\left(t\right)$ for $t < T - \tau_1$ based on Eq. (6), while for synthetic front-month contracts, we use $RUG_{T-\tau_0,t}$ and $VIX_{t,T}^2$ for $T - \tau_1 \leq t < T$ based on Eq. (5). The contracts are then closed on the second Friday of the contract month. On the same day, we buy back the next synthetic forward-starting 1-month variance futures. The primary reason to roll the synthetic 1-month variance futures one week before expiration (on the third Friday of the contract month) is to ensure consistency with other rolling strategies used in this study.

Given the growth of the futures and option markets on the VIX, the CBOE has calculated daily historical values for $VIXTerm$ dating back to 1992. $VIXTerm$ is a representation of implied volatility of SPX options, and its calculation involves applying the VIX formula to specific SPX options to construct a term structure for fairly-valued variance. The generalized VIX formula has been modified to reflect business days to expiration. As a result, investors will be able to use $VIXTerm$ to track the movement of the SPX option implied volatility in the listed contract months. $VIXTerm$ of various maturities allows one to infer a complete initial term structure of IUG that is contemporaneous with the prices of variance futures of various maturities.

2.4. Out-of-the-Money SPX Put Options

The monthly series of out-of-the-money (OTM) SPX put options [7] are created by purchasing 5% (or 10%) OTM SPX puts monthly one month prior to their expiration. Given good liquidity relative to the volatility derivatives market and the significant bid/ask in the options market, we will let any purchased options expire instead of trying to roll them forward. This is generally consistent with practice in real-world trading.

The monthly series of out-of-the-money (OTM) SPX put options are created by purchasing 5% (or 10%) OTM SPX puts one month prior to their expiration. In real-life trading, longer-dated options are usually rolled up (by paying additional premium) rolled down(i.e. monetizing earned premium) with significant market moves. However, given the average maturity of the options in the series is only about 10 trading days, it is unlikely that they will be rolled them up or down in real-life trading in view of the significant trading costs involved.

This study accounts for the option premia in SPX put options primarily by using the "burn rate" (which can be thought of as a form of daily theta) implied by the premia.[8] Although an investor pays upfront cash for the premium, his mark-to-market value (MTM) at the end of the day is his negative cash position paid plus

[7] Some may argue that the SPY options should be used instead of the CBOE SPX options, since the former is more liquid. However, the CBOE SPY options tend to be traded by institutions at larger sizes and therefore more consistent with the objective of this study.

[8] Suppose that the investor has a securities account. He has to account for both the asset and liability columns when computing his P&L. On day t he buys an option: the cash account is $-Put(t)$ while the asset account is $+Put(t)$. If he sells the option right away, the net account on day t is back at 0 P&L. On day $t+1$, if the underlying price has not changed, the cash account

the value of his option. The act of purchasing the option in itself should not create any P&L shock. The interest charges are also ignored to simply the analysis.[9] In general, the strategy is expected to maintain a negative cash balance until the option strategy generates enough profits to cover the outstanding debt. In other words, the P&L should be computed based on a combination of money borrowed to finance the option and the option itself. In a sense, one is not expected see any negative value representing the entire option premium, unless the option expires at less than the original premium paid plus any interest cost, or unless the option position has lost most of its intrinsic value.[10]

Suppose a put option is purchased at regular intervals of length $\triangle t$. As described above, at time t we short an instantaneously maturing risk-free bond $B(t)$ to raise cash, and then purchase a put option $Put(t, t + \triangle t)$ of maturity $\triangle t$, such that the net P&L at time t is zero. In other words, the combined position is a self-financed portfolio: The investor borrows cash in order to finance the purchase of the option, such that $B\left(t\right) = Put\left(t, t + \triangle t\right)$. Accordingly, interest based on a deterministic continuously compounded rate $R(t)$ should be paid when money is borrowed to purchase the option. At time $t + \triangle t$, the mark-to-market value (MTM) of the self-financed portfolio is given as follows:

$$MTM\left(t + \triangle t\right) = Put\left(t, t + \triangle t\right) - B\left(t\right) e^{R(t)\triangle t}$$

where $B\left(t\right) = Put(t, t + \triangle t)$. We can repeat this net P&L calculation at time $t + l\triangle t$, where $l = 1, 2, 3, \ldots$. The MTM value at time $(t + l\triangle t)$ of the $(l-1)$-th

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still remains at $-Put\ (t)$ and asset account at $Put\ (t+1)\ =\ Put\ (t)\ -\ one\ day\ of\ theta$. Thus, $net\ P\&L$ on day $t+1$ is equal to $one\ day\ of\ theta$. If the option expires worthless, his cumulative P&L become $-Put\,(t) - interest$ ONLY at the expiration day. In other words, while he has already paid upfront cash for the option on day t , the full negative P&L for the option premium usually does not manifest itself until the expiration day.

[9] While small initially, interest rate charges can become quite significant over time, thus one may argue that there is a need to account for them as part of the total costs in running a hedging strategy. However, both the negative carry in the volatility and variance futures and the premium in SPX options will run up identical financing costs if they have the same negative P&L, hedging effectiveness can be compared on an "apple-to-apple" basis without accounting for the interest rate charges.

[10] Some researchers treat the option premium as a negative P&L because "money is paid" upfront. Doing so results in a large P&L shock when the option is paid. Technically, that seems incorrect because one can buy the option in the morning and sell it in the afternoon. Thus, no P&L changes should be recorded for that day as long as the price of the option stays the same.

put strategy is given by

$$\begin{aligned}MTM(t+l\triangle t) =& Put\,(t+(l-1)\triangle t, t+l\triangle t) - \\ & Put\,(t+(l-1)\triangle t, t+(l-1)\triangle t)\, e^{R(t+(l-1)\triangle t)\times\triangle t}\end{aligned}$$

The study uses the formula above to estimate the P&L of a mechanical rolling strategy of buying one option and rolling it forward every month. The study then runs statistics to estimate the appropriate hedging ratio each month by minimizing residuals (or other relevant alternative objective functions).

Since bids and asks right before expiration often do not reflect actual tradable values of the option, it is more reliable to use the exercise value of the option at expiration date. Once a settlement price is published on a specific contract month, the movement of that put no longer reflects changes in the value of the underlying index; i.e., it is going into "settlement mode". Accordingly, we initiate a new contract on its expiration day to maintain the hedge. Typically, execution traders will be given at least one trading session to "build" a new position. To reflect real-world conditions, our study initiates a new 5% (or 10%) OTM put contract on its subsequent trading day. If the option expires in the money, one should include the exercise value into the cumulative P&L, i.e., one receives cash into the cash account if the option expires in the money. There is no value left in the option if it expires out of the money. Any interest receipts (charges) from the current period also become part of the positive (negative) P&L for the *next* period. Assuming that one put option expires on each $\triangle t$-interval, $Put\,(t+(l-1)\triangle t, t+l\triangle t) = (K(t+(l-1)\triangle t) - S(t+l\triangle t))^{+}$, where the final index settlement value is $S(t+l\triangle t)$ at expiration time $t+l\triangle t$ and the strike price is $K(t+(l-1)\triangle t)$.

The mark-to-market value for the put option on its first trading day $t+\triangle t+1$ of the second rolling month will depend on whether interest charges (surpluses) from the first period will become part of the negative (positive) P&L for the second period.

3. Hedging Performance

This section will discuss the empirical results from: (i) the hedging schemes as applied to the VIX futures; (ii) the hedging schemes as applied to the variance futures; (iii) the hedging schemes as applied to the 5% OTM SPX puts; and (iv) the hedging schemes as applied to the 10% OTM SPX puts.

[Table 1 about here]

3.1. 1-month VIX Futures

The study conducts the empirical hedging analysis based on the seven different hedging methodologies as described in Section 2.2, by using VIX futures as a hedge to one unit of the SPX index. The rebalancing, done every month, takes place five business days prior to the expiration of VIX futures to avoid well-known liquidity problems in the last week of trading of futures contracts. The study focuses on a one-month out-of-sample hedging horizon, using data for the period December 2004 through March 2010. Hedge effectiveness is measured based on the magnitude of percentage drawdown reduction from before the hedge to after the hedge:

$$MaxDD\%\left(T; MTM^{before\ hedge}\right) - MaxDD\%\left(T; MTM^{after\ hedge}\right)$$

where $MTM^{before\ hedge} = \$10 \times SPX$; $MTM^{after\ hedge} = \$10 \times SPX + h \times \$1000 \times fvixfut_{cumP\&L}$; and $MaxDD\%(T;\cdot)$ is defined as the maximum sustained percentage decline (peak to trough) for period $[0, T]$, which provides an intuitive and industry-standard empirical measure of the loss arising from potential extreme events. We use the percentage Maximum Drawdown in this case, which is calculated as the percentage drop from the peak to the trough as measured from the peak:

$$MaxDD\%\left(T; \{MTM\}_{t=0}^{T}\right) = \max_{0 \le t \le T}\left[\frac{MTM_{0\le\tau\le t}^{peak} - MTM\left(t\right)}{MTM_{0\le\tau\le t}^{peak}}\right]$$

The graphical results are plotted in Figures 2 and 3. Descriptive statistics on both the unhedged and hedged profits and losses (P&Ls) are also reports in Panel A of Table 1. Note the following technical details: First, all mark-to-market value (MTM) time series are starting at at the *unhedged* value of one unit of the SPX index at the beginning of the empirical analysis, from December 2004 to March 2011, covering a period of extreme volatility due to the bankruptcy of Lehman Brothers. The in-sample data period allows for the use of roughly 2 months of data to estimate the first out-of-sample hedging ratio. Second, summary statistics are computed based on raw daily P&Ls, without any time scaling. Third, maximum drawdown is computed based on the percentage drop from the peak to the trough as measured from the peak. Finally, in this specific analysis, we have computed the Cornish-Fisher CVaR at 95% and 99%, but have noticed minimal differences between the two choices. One may conclude from the statistical results that:

1. Minimizing Cornish-Fisher CVaR is not effective in minimizing maximum drawdown. In addition, "second-order techniques" such as minimizing absolute residuals and squared residuals have not shown particularly effective performance as extreme downside hedges.
2. Minimizing maximum drawdown is effective in reducing maximum drawdown, its overall performance in terms of offering protection during a drawdown scenario without incurring unreasonable cost is unimpressive. This is not too surprising considering the "look back" nature of maximum drawdown as a measure. It is generally believed that, under this method, one can only compute the correct hedging ratio after a significant drawdown has already happened. By then, the hedge is put on only when it is no longer needed, while incurring heavy losses when the hedging instrument is "recoiling" in its P&L.
3. Both the Sharpe Ratio and Alternative Sharpe Ratio perform reasonably as an objective function for extreme downside hedges.

[Figures 2 and 3 about here]

3.2. 1-Month Variance Futures

This subsection conducts the empirical hedging analysis based on the seven different hedging methodologies as described above, by using VT futures as a hedge to one unit of the SPX index. The algorithm for creating the monthly rolls of synthetic 1-month variance futures has been described in Section 2.3. The graphical results are shown in Figures 4 and 5.

The standard descriptive statistics on both the unhedged and hedged profits and losses (P&Ls) are reported in Panel B of Table 1. Note the following technical details: First, all mark-to-market value (MTM) time series are starting at the *unhedged* value of one unit of the SPX index at the beginning of the empirical analysis, from December 2004 to March 2011, covering a period of extreme volatility due to the bankruptcy of Lehman Brothers. The in-sample data period allows for the use of roughly 2 months of data to estimate the first out-of-sample hedging ratio. Second, summary statistics are computed based on raw daily P&Ls, without any time scaling. Third, maximum drawdown is computed based on the percentage drop from the peak to the trough as measured from the peak. Finally, in this specific analysis, we have computed the Cornish-Fisher CVaR at 95% and 99%, but have noticed minimal differences between the two choices. One may conclude from the statistical results that:

1. Minimizing Cornish-Fisher CVaR is not effective in minimizing maximum drawdown. In addition, "second-order techniques" such as minimizing absolute residuals and squared residuals have not shown particularly effective performance as extreme downside hedges.
2. Minimizing maximum drawdown, maximizing Sharpe Ratio as well as maximizing the Alternative Sharpe Ratio are roughly as effective in reducing maximum drawdown. However, in this case minimizing maximum drawdown and maximizing Sharpe Ratio both give better performance in terms of preserving upside.
3. In all cases, the trader ends up being better off by not hedging.

The practical issue with using the VT is that its implied negative carry costs can be very high. This implied negative carry is caused by the significant burn rate on the premia of options used to replicate the variance futures. Because VT is the square of the VIX, when the strategy benefits from the upside of VT, there can be a dramatic improvement to portfolio performance. In fact, the impressive surge followed by the expected "recoil" has resulted in a potential increase in maximum drawdown for some hedging models tested. Such an extreme swing is likely to deter real-life traders from using such an instrument as a practical hedging solution.

[Figures 4 and 5 about here]

3.3. 1-Month 5% (or 10%) Out-of-the-Money SPX Put Options

This section conducts the out-of-sample hedging analysis based on the five different hedging methodologies as described above, by using 5% (or 10%) OTM SPX puts as a hedge to one 100-lot unit of long S&P500 ETF. The algorithm for creating the monthly rolls of synthetic 1-month SPX puts has been described in Section 2.4. The graphical results are plotted in Figures 6 and 7 (as well as Figures 8 and 9).

The standard descriptive statistics on both the unhedged and hedged profits and losses (P&Ls) in Panels C and D of Table 1. Note the following technical details: First, all mark-to-market value (MTM) time series are starting at the *un-hedged* value of one unit of the SPX index at the beginning of the empirical analysis, from December 2004 to March 2011, covering a period of extreme volatility due to the bankruptcy of Lehman Brothers. The in-sample data period allows for the use of roughly 2 months of data to estimate the first out-of-sample hedging ratio. Second, summary statistics are computed based on raw daily P&Ls, without any time scaling. Third, maximum drawdown is computed based on the percentage drop from the peak to the trough as measured from the peak. Finally, in this specific analysis, we have computed the Cornish-Fisher CVaR at 95% and 99%, but have noticed minimal differences between the two choices. One may conclude from the statistical results that:

1. In general, all hedging schemes produce low hedging ratios in an up market and provide very reasonable protection during extreme events.
2. After an extreme event, all hedging schemes produce significantly higher hedging ratios, resulting in rather poor performance when market recovers.
3. In all cases, the trader ends up being better off by not hedging.

Monthly 10% OTM SPX puts go into the money about once every decade. Very often, they become costly propositions as extreme downside hedges due to the constant need to pay premia without benefiting from the payoff, especially when market volatility shoots up (thereby increasing the costs of the options), but the net return to the hedger (even after the option goes into money) is still far from sufficient to cover the cumulative option premia over time. Monthly 5% OTM SPX puts go into the money more often but they are also more expensive. Because of the steep premia of OTM puts, many hedgers either (i) underhedge with a smaller than suitable notional amount or (ii) use options further out of the money, potentially lowering the payoff when protection is needed. In this case, the observed negative carry is likely deter to any real-life traders from using such an instrument for hedging.

[Figures 6 and 7 about here]

[Figures 8 and 9 about here]

3.4. Overall Comparison on Choices of Hedging Instruments

The following P&L graphs presents examples of combining the choices of hedging instruments under different hedging models, to provide traders a more visual way to choose an appropriate hedging strategy:

[Figures 10, 11, 12 and 13 about here]

Our key observations are as follows:

1. As noted earlier, the negative carries from using VT futures and OTM SPX puts are way too high for them to be deployed as practical hedging solutions. The trader ends up being better off by not hedging. As expected, one can get worse performance by using 10% OTM SPX puts than using 5% OTM SPX puts, but futures have done better than options. This is surprising considering that volatility and variance futures are created from a series of SPX options in theory. This observation is significant, since there are not yet obvious theoretical justifications as to why using a synthesized product can be more efficient than using the relevant raw materials used to synthetically replicate the synthesized product. This does not appear to be caused by the "smile" of the volatility curve being so steep that it becomes cost-ineffective to use way out-of-the-money put options, since the observation is consistent between 10% OTM SPX puts and 5% OTM SPX puts. Lee et. al. 2010 [4] have shown by sophisticated simulation that inability to carry out frictionless hedging due to liquidity conditions may result in significant deviation from theoretical option pricing. This may be another piece of empirical evidence supporting such a result, and the likely mechanism giving plausible explanations to this observation will be an interesting topic for future research.
2. The overall winner in our empirical analysis is the VIX futures, which has provided both extreme downside protection as well as upside preservation under reasonable choices of hedging models. The pragmatic issue faced by real-world hedgers is whether they can execute any such hedging trades with the extended tenor in reasonable size. That is an empirical question that can only be satisfactorily answered by placing large trades directly in the VIX futures market.

4. Conclusions

This paper attempts to address whether "long volatility" is an effective hedge against a long equity portfolio, especially during periods of extreme market movements. Our study examines using volatility futures and variance futures as extreme downside hedges, and compares their effectiveness against traditional "long volatility" hedging instruments such as 5% and 10% OTM put options on the SPX.

In each case, out-of-sample hedging ratios are calculated based on seven reasonable choices of objective functions, and the hypothetical performance of the hedge is computed again a long portfolio consisted of one unit of the SPX index.

Our empirical results show that the CBOE VIX futures can potentially be more effective extreme downside hedges than out-of-the-money put options on the S&P 500 index, especially after applying contract-rolling methodologies that are generally consistent with market practice. In particular, using 1-month rolling VIX futures presents a cost-effective choice as hedging instruments for extreme downside risk protection as well as for upside preservation.

The pragmatic issue faced by real-world hedgers is whether they can execute any such hedging trades with the desirable tenor in reasonable size, which is an empirical question that can only be satisfactorily answered by placing large trades directly in the VIX futures market.

By replicating hedging techniques used by real-life traders, our findings support the following conclusions and recommendations:

1. First, using volatility instruments as extreme downside hedges, especially when combined with an appropriate model to estimate the hedging ratios, can be a viable alternative to buying a series of out-of-the-money put options on SPX.
2. Second, the volatility and variance future markets may be more efficient than suggested by the general perception among market participants .
3. Third, there is a business case supporting that volatility/variance instruments can be made more widely available as extreme downside hedging instruments. For instance, in the Asia region where the author is currently located, this can be accomplished by: (i) creating daily benchmark volatility indices on the Hang Seng Index (HSI) and the Straits Times Index (STI), in a fashion similar to the Volatility Index Japan (VXJ), the Volatility Index of TAIFEX index options, or the Volatility Index of the KOSPI 200 (VKOSPI); and (ii) creating exchange-traded volatility/variance instruments once such reference indices gain wider acceptance by the OTC market.

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Table 1: Descriptive Statistics on Hedging – The first row gives the descriptive statistics on the unhedged SPX ETF. The remaining rows give the descriptive statistics on the hedged P&L under the seven different models used, ranked by the maximum drawdown reduction starting from the most effective hedging model. Panels A, B, C and D provide summary statistics on hedging with 1-month rolling VIX futures, 1-month rolling VT futures, 1-month rolling 5% OTM SPX puts and 1-month rolling 10% OTM SPX puts, respectively

	N	Mean	Median	Max	Min	Stddev	Skew	E Kurt	%MaxDD	MaxDD %Red
Panel A. 1-Month Rolling VIX Futures										
Unhedged Daily P&L of SPX	1832	33.46	54.91	370.50	-518.12	175.15	-0.60	0.17	37.19	NA
Hedged P&L of CVaR (99%)	1832	90.14	69.85	397.94	-120.10	118.96	0.55	-0.37	33.31	3.88
Hedged P&L of Squared Residuals	1832	105.39	106.32	347.04	-184.19	108.10	0.06	-0.58	29.05	8.14
Hedged P&L of CVaR (95%)	1832	100.67	67.23	338.42	-135.87	117.01	0.25	-1.17	28.33	8.86
Hedged P&L of Absolute Residuals	1832	-6.03	-67.93	327.91	-421.79	172.36	0.04	-0.96	27.45	9.74
Hedged P&L of SR	1832	50.29	42.43	305.80	-264.12	108.40	-0.03	0.04	25.60	11.59
Hedged P&L of ASR	1832	55.76	44.00	303.52	-220.05	99.91	0.14	0.07	25.41	11.78
Hedged P&L of Max Drawdown	1832	-32.16	-19.66	168.02	-371.56	106.84	-0.71	0.22	14.06	23.13
Panel B. 1-Month Rolling VT Futures										
Unhedged Daily P&L of SPX	1832	33.46	54.91	370.50	-518.12	175.15	-0.60	0.17	37.19	NA
Hedged P&L of CVaR (95%)	1832	115.92	75.18	873.64	-143.36	159.83	1.55	4.01	73.13	-35.94
Hedged P&L of CVaR (99%)	1832	105.39	75.75	873.64	-122.26	152.81	1.85	5.61	73.13	-35.94
Hedged P&L of Squared Residuals	1832	108.40	86.31	733.45	-206.83	159.03	0.80	0.93	61.39	-24.20
Hedged P&L of Absolute Residuals	1832	63.68	62.45	488.34	-311.08	146.54	0.09	-0.16	40.88	-3.69
Hedged P&L of Max Drawdown	1832	75.78	58.84	354.95	-243.23	109.82	0.24	-0.13	29.71	7.48
Hedged P&L of ASR	1832	45.33	39.50	334.25	-193.98	72.49	0.19	0.30	27.98	9.21
Hedged P&L of SR	1832	73.06	54.37	320.48	-151.42	95.39	0.55	-0.36	26.83	10.36

	N	Mean	Median	Max	Min	Stddev	Skew	E Kurt	%MaxDD	MaxDD %Red
Panel C. 1-Month Rolling 5% OTM SPX Puts										
Unhedged Daily P&L of SPX	1832	33.46	54.91	370.50	-518.12	175.15	-0.60	0.17	37.19	NA
Hedged P&L of CVaR (99%)	1832	85.46	69.01	1110.54	-548.11	254.82	1.01	2.42	92.96	-55.77
Hedged P&L of CVaR (95%)	1832	146.96	115.07	961.18	-181.31	151.30	2.01	5.82	80.46	-43.27
Hedged P&L of Absolute Residuals	1832	-86.37	-18.75	862.64	-917.53	260.25	-0.08	-0.12	72.21	-35.02
Hedged P&L of Max Drawdown	1832	-151.14	3.45	604.82	-803.49	358.36	-0.39	-1.35	50.63	-13.44
Hedged P&L of SR	1832	-154.67	-13.75	454.75	-575.29	246.69	-0.47	-0.98	38.07	-0.87
Hedged P&L of ASR	1832	-145.23	6.93	374.19	-713.71	337.55	-0.26	-1.56	31.32	5.87
Hedged P&L of Squared Residuals	1832	-170.63	-41.38	274.76	-791.70	302.60	-0.38	-1.20	23.00	14.19
Panel D. 1-Month Rolling 10% OTM SPX Puts										
Unhedged Daily P&L of SPX	1832	33.46	54.91	370.50	-518.12	175.15	-0.60	0.17	37.19	NA
Hedged P&L of Absolute Residuals	1832	29.87	10.24	728.15	-303.48	162.39	1.11	2.14	60.95	-23.76
Hedged P&L of SR	1832	-141.68	0.94	387.59	-674.25	315.51	-0.15	-1.60	32.44	4.75
Hedged P&L of ASR	1832	-111.57	-17.61	350.83	-677.41	272.78	-0.19	-1.26	29.37	7.82
Hedged P&L of CVaR (95%)	1832	-143.18	-26.06	339.28	-759.07	288.44	-0.24	-1.20	28.40	8.79
Hedged P&L of CVaR (99%)	1832	-117.71	-16.57	334.62	-601.68	259.89	-0.18	-1.33	28.01	9.18
Hedged P&L of Max Drawdown	1832	-69.54	-7.95	334.36	-470.63	222.06	0.00	-1.35	27.99	9.20
Hedged P&L of Squared Residuals	1832	-14.12	-15.78	316.22	-316.79	150.17	0.29	-0.90	26.47	10.72

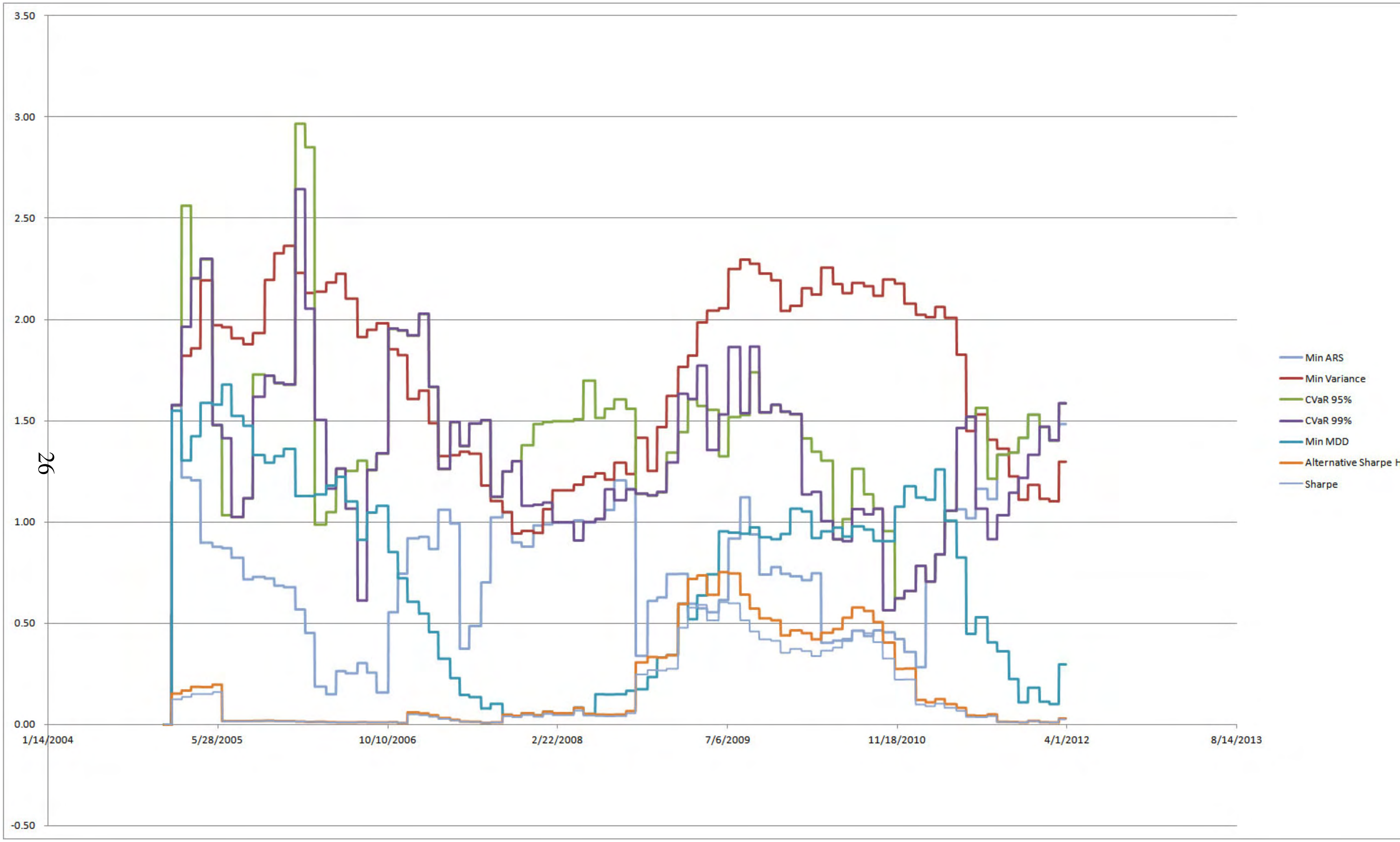
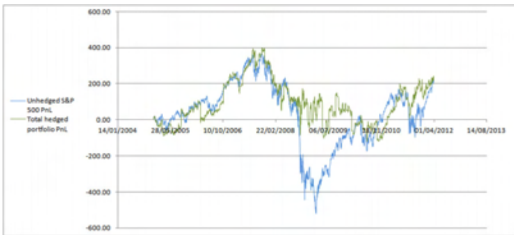
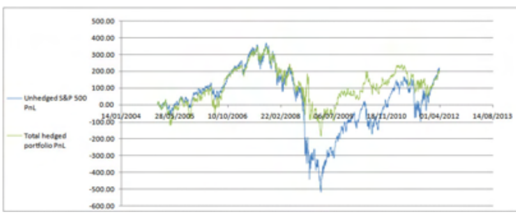


Figure 1: Hedging Ratios for VIX Futures 1-Month Rolling Contracts (Dec 2004 – Mar 2011) under All Hedging Models



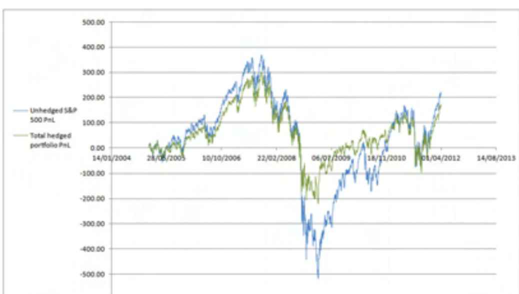
(a) Hedging Model = CVAR(99%)

(b) Hedging Model = CVAR(95%)

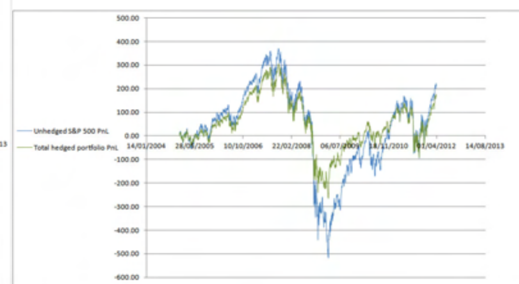


(c) Hedging Model = Squared Residuals

(d) Hedging Model = Absolute Residuals



(e) Hedging Model = Alternative Sharpe Ratio



(f) Hedging Model = Sharpe Ratio

(g) Hedging Model = Maximum Drawdown

Figure 2: VIX Futures 1-Month Rolling Contracts (Dec 2004 – Mar 2011) by Different Hedging Models

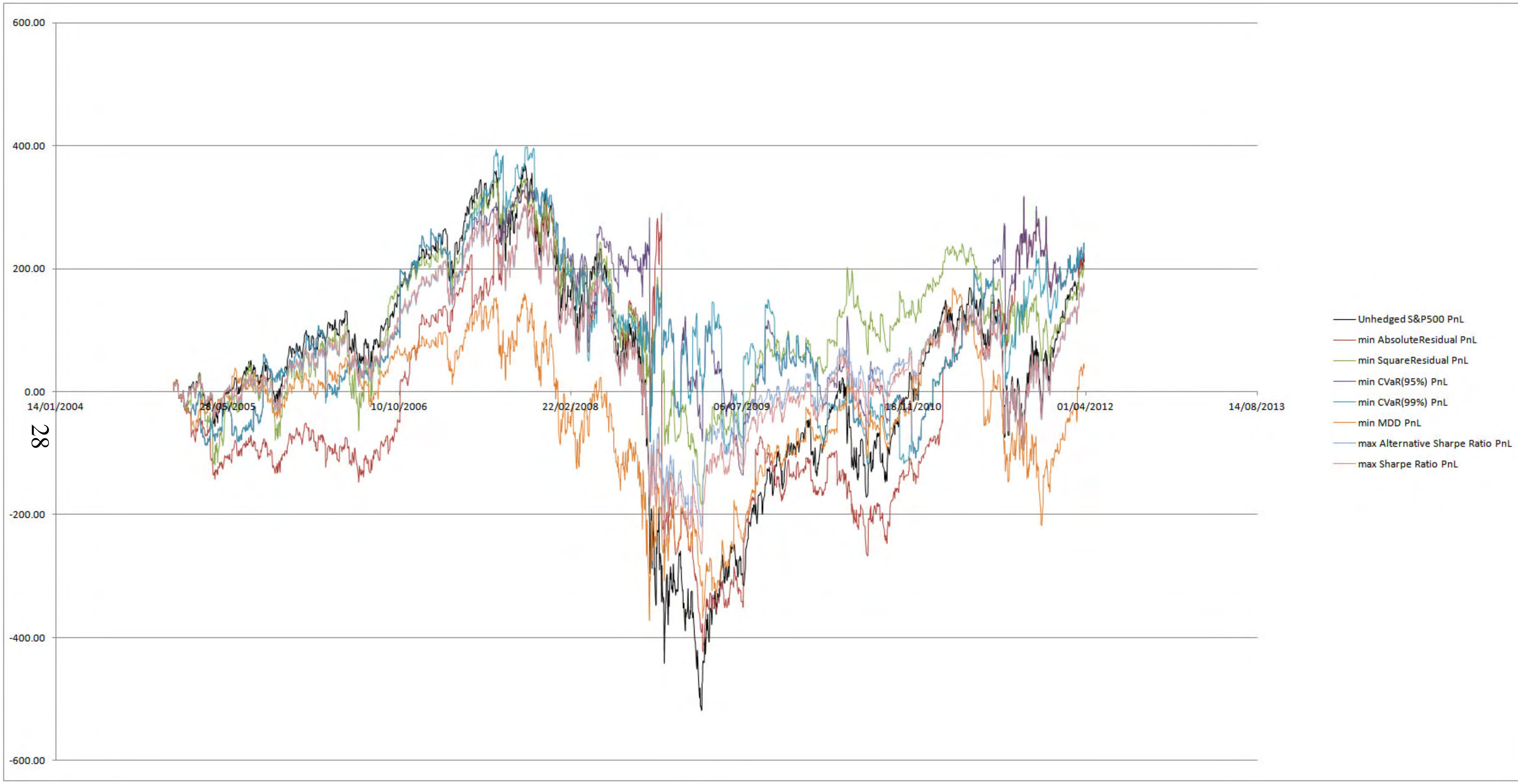
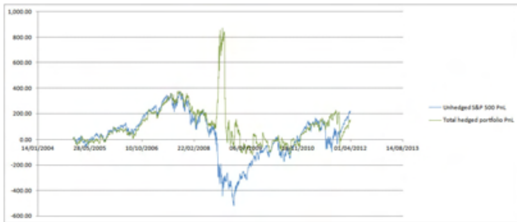
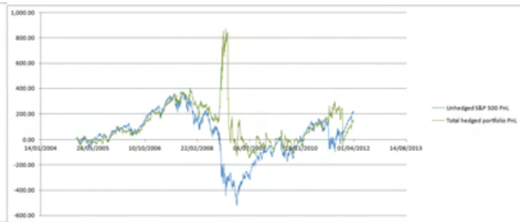


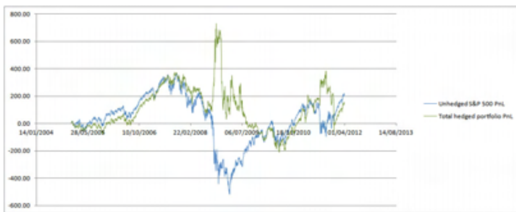
Figure 3: VIX Futures 1-Month Rolling Contracts (Dec 2004 – Mar 2011) under All Hedging Models



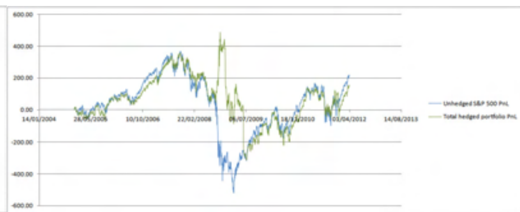
(a) Hedging Model = CVAR(99%)



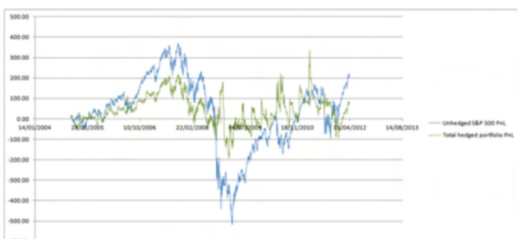
(b) Hedging Model = CVAR(95%)



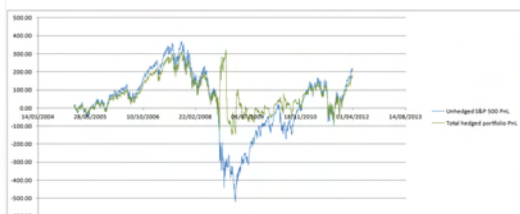
(c) Hedging Model = Squared Residuals



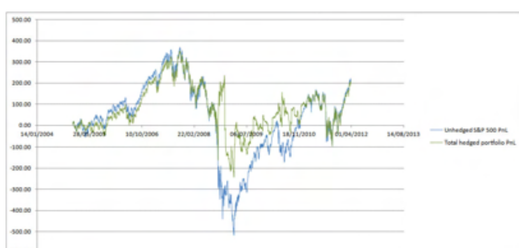
(d) Hedging Model = Absolute Residuals



(e) Hedging Model = Alternative Sharpe Ratio



(f) Hedging Model = Sharpe Ratio



(g) Hedging Model = Maximum Drawdown

Figure 4: VT Futures 1-Month Rolling Contracts (Dec 2004 – Mar 2011) by Different Hedging Models

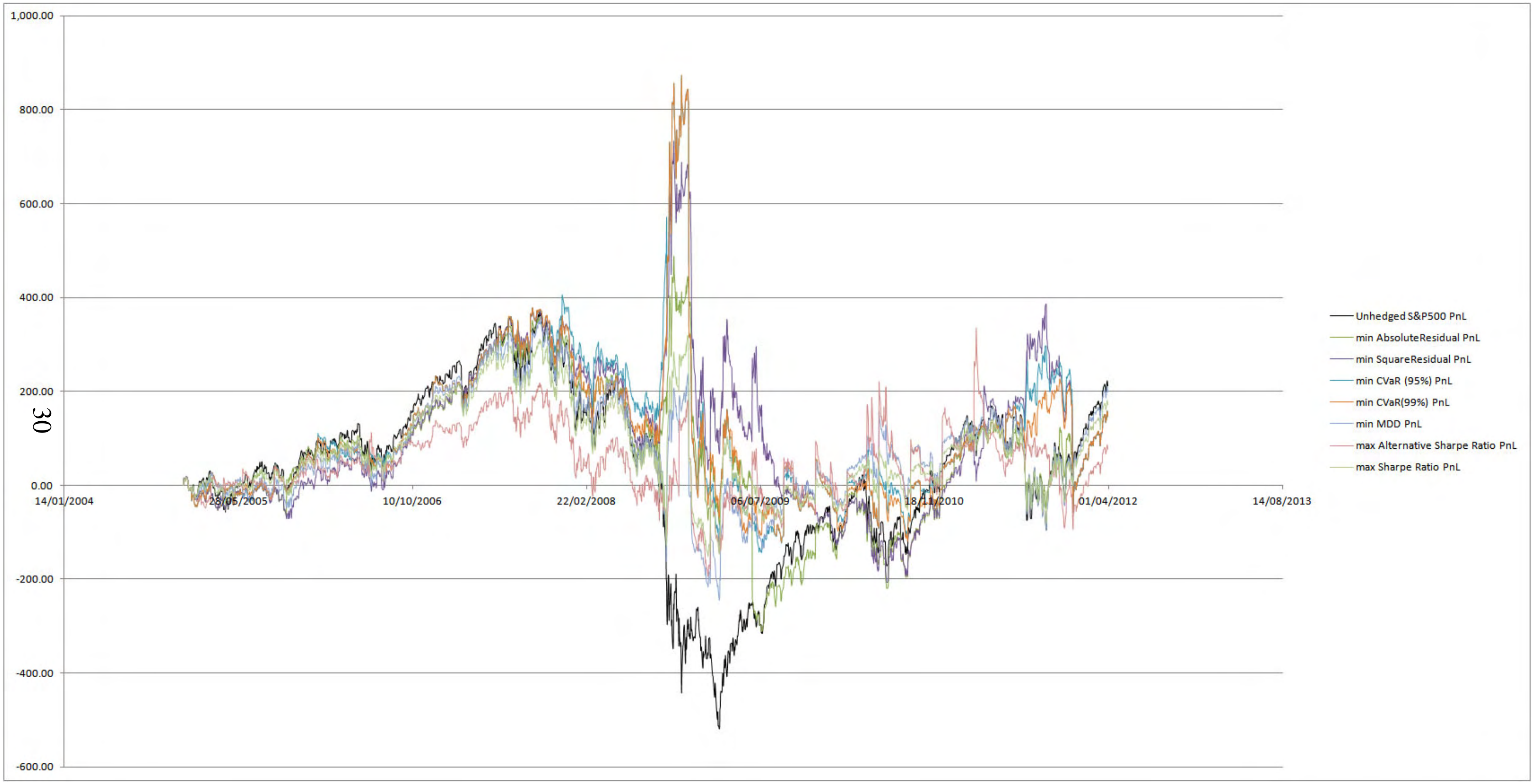
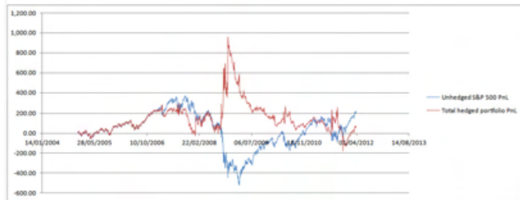
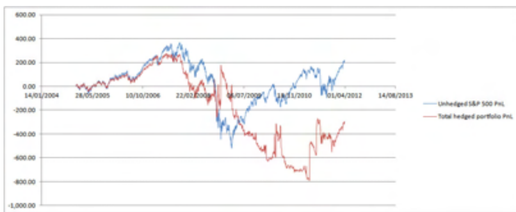


Figure 5: VT Futures 1-Month Rolling Contracts (Dec 2004 – Mar 2011) under All Hedging Models

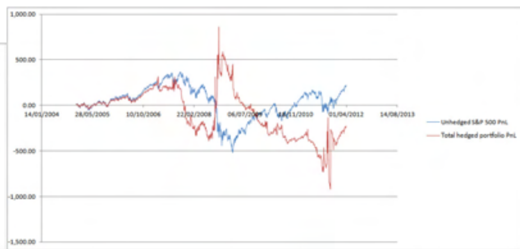
(a) Hedging Model = CVAR(99%)



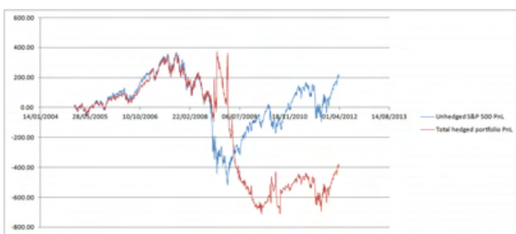
(b) Hedging Model = CVAR(95%)



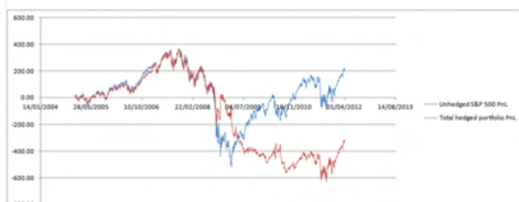
(c) Hedging Model = Squared Residuals



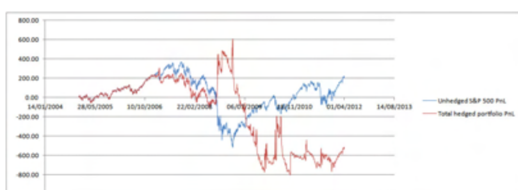
(d) Hedging Model = Absolute Residuals



(e) Hedging Model = Alternative Sharpe Ratio



(f) Hedging Model = Sharpe Ratio



(g) Hedging Model = Maximum Drawdown

Figure 6: 5% OTM SPX Put Options 1-Month Rolling Contracts (Dec 2004 – Mar 2011) by Different Hedging Models

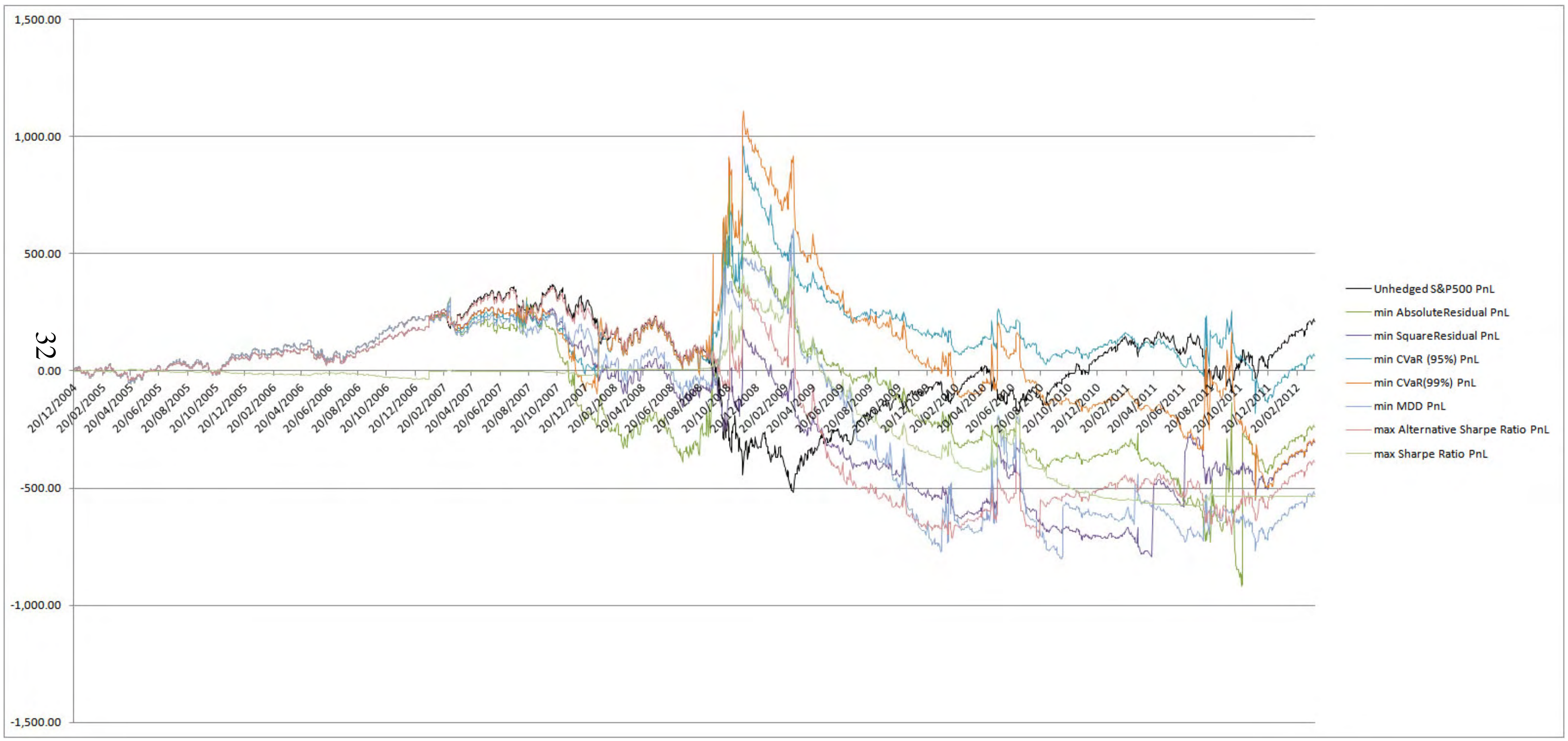
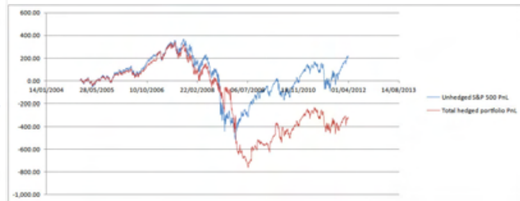


Figure 7: 5% OTM SPX Put Options 1-Month Rolling Contracts (Dec 2004 – Mar 2011) under All Hedging Models

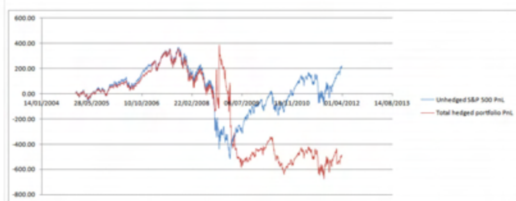


(a) Hedging Model = CVAR(99%)

(b) Hedging Model = CVAR(95%)

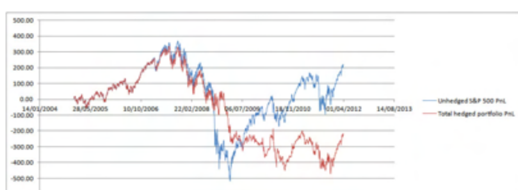
(c) Hedging Model = Squared Residuals

(d) Hedging Model = Absolute Residuals



(e) Hedging Model = Alternative Sharpe Ratio

(f) Hedging Model = Sharpe Ratio



(g) Hedging Model = Maximum Drawdown

Figure 8: 10% OTM SPX Put Options 1-Month Rolling Contracts (Dec 2004 – Mar 2011) by Different Hedging Models

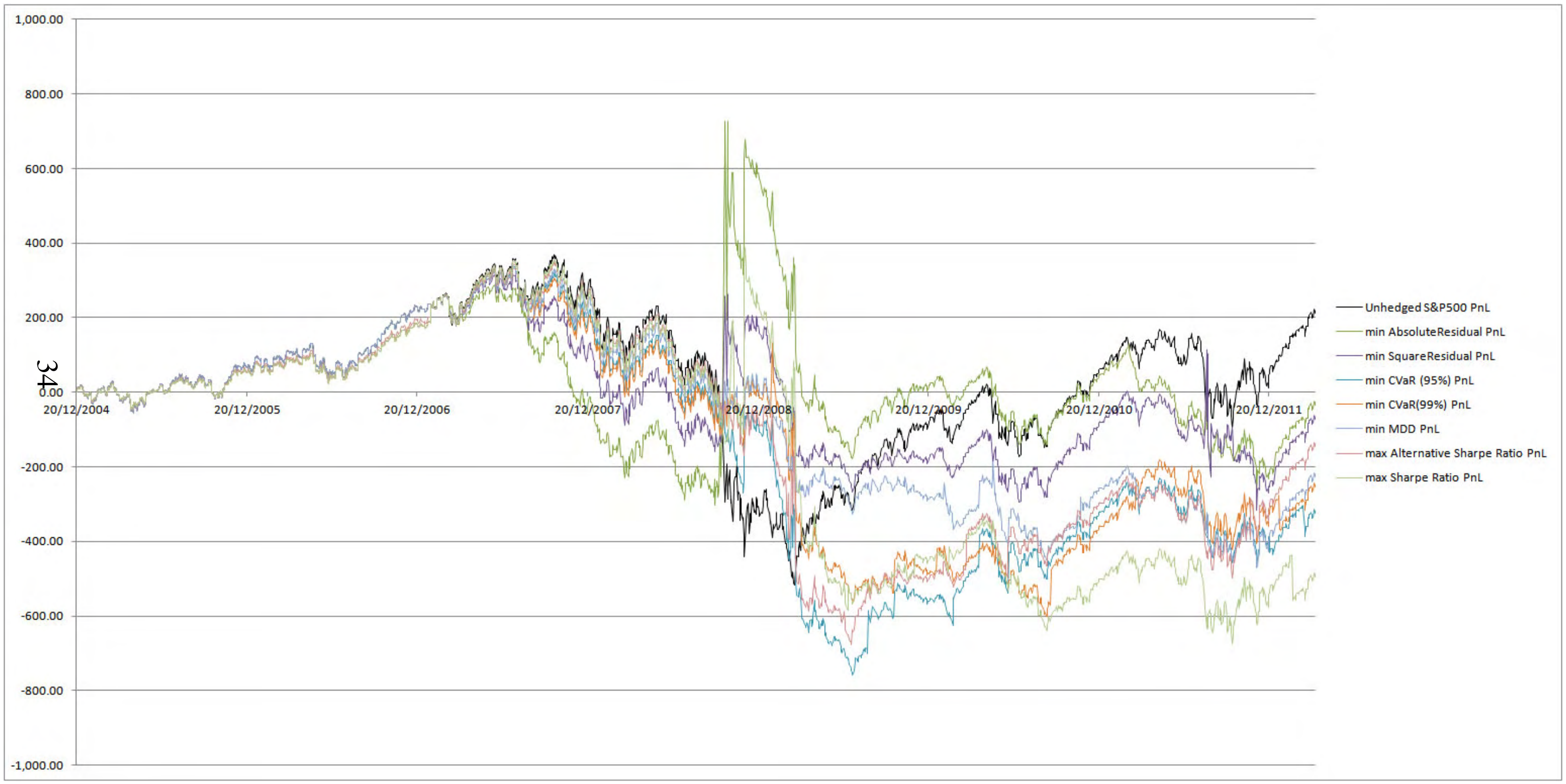


Figure 9: 10% OTM SPX Put Options 1-Month Rolling Contracts (Dec 2004 – Mar 2011) under All Hedging Models

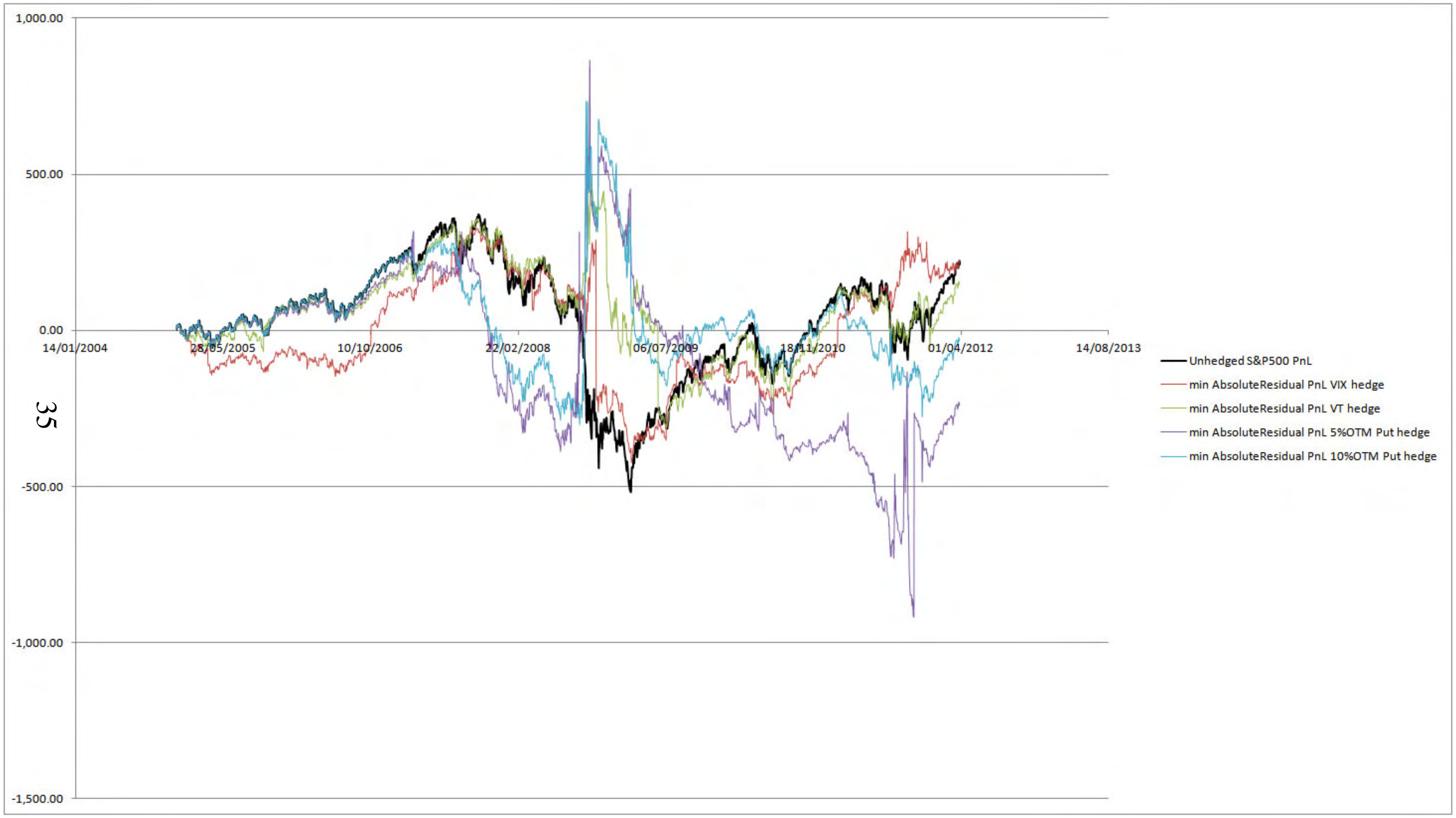


Figure 10: 10% OTM SPX Put Options 1-Month Rolling Contracts (Dec 2004 – Mar 2011) under All Hedging Models

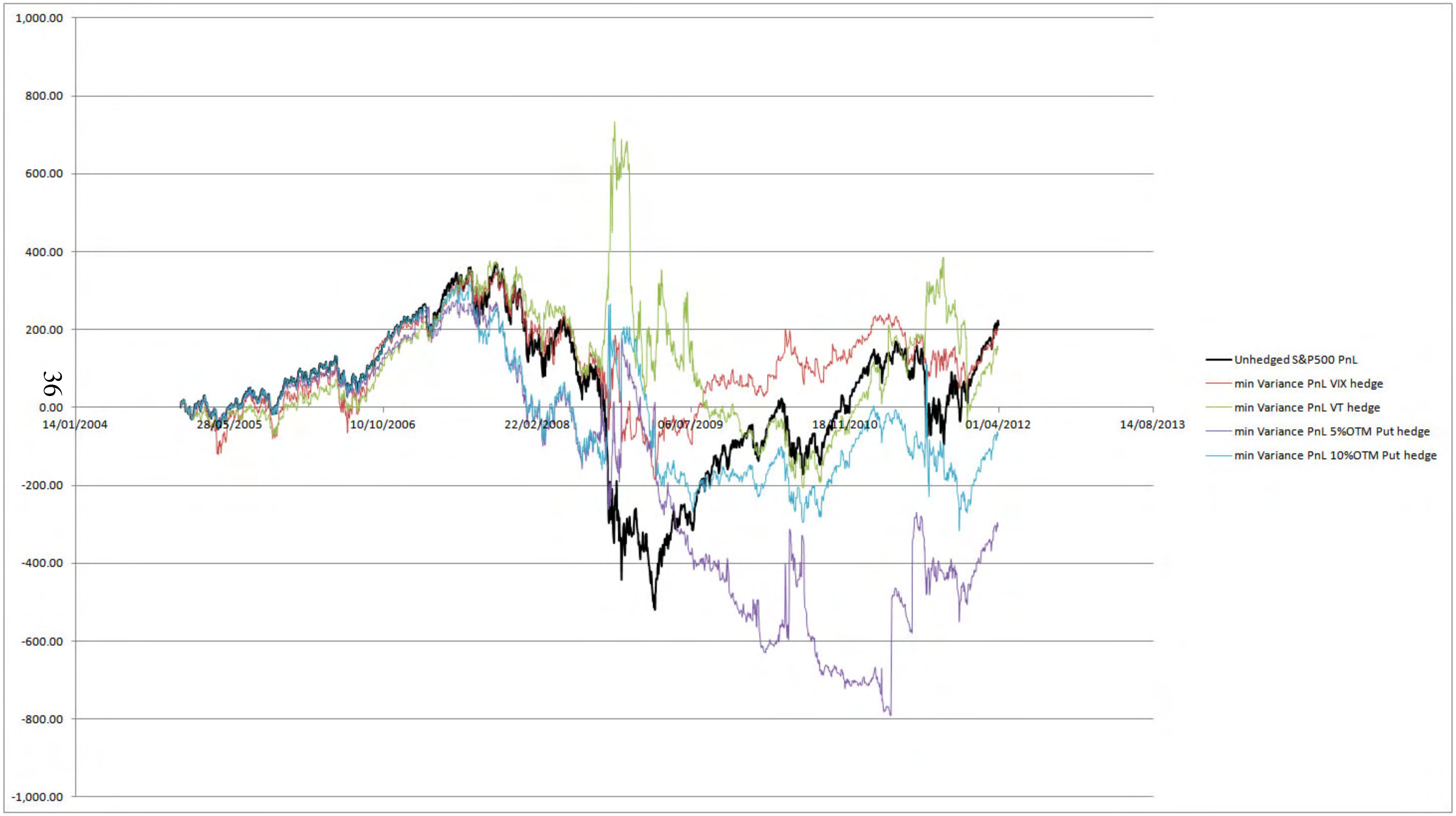


Figure 11: All Hedging Instruments (Dec 2004 – Mar 2011) under Minimum Variance

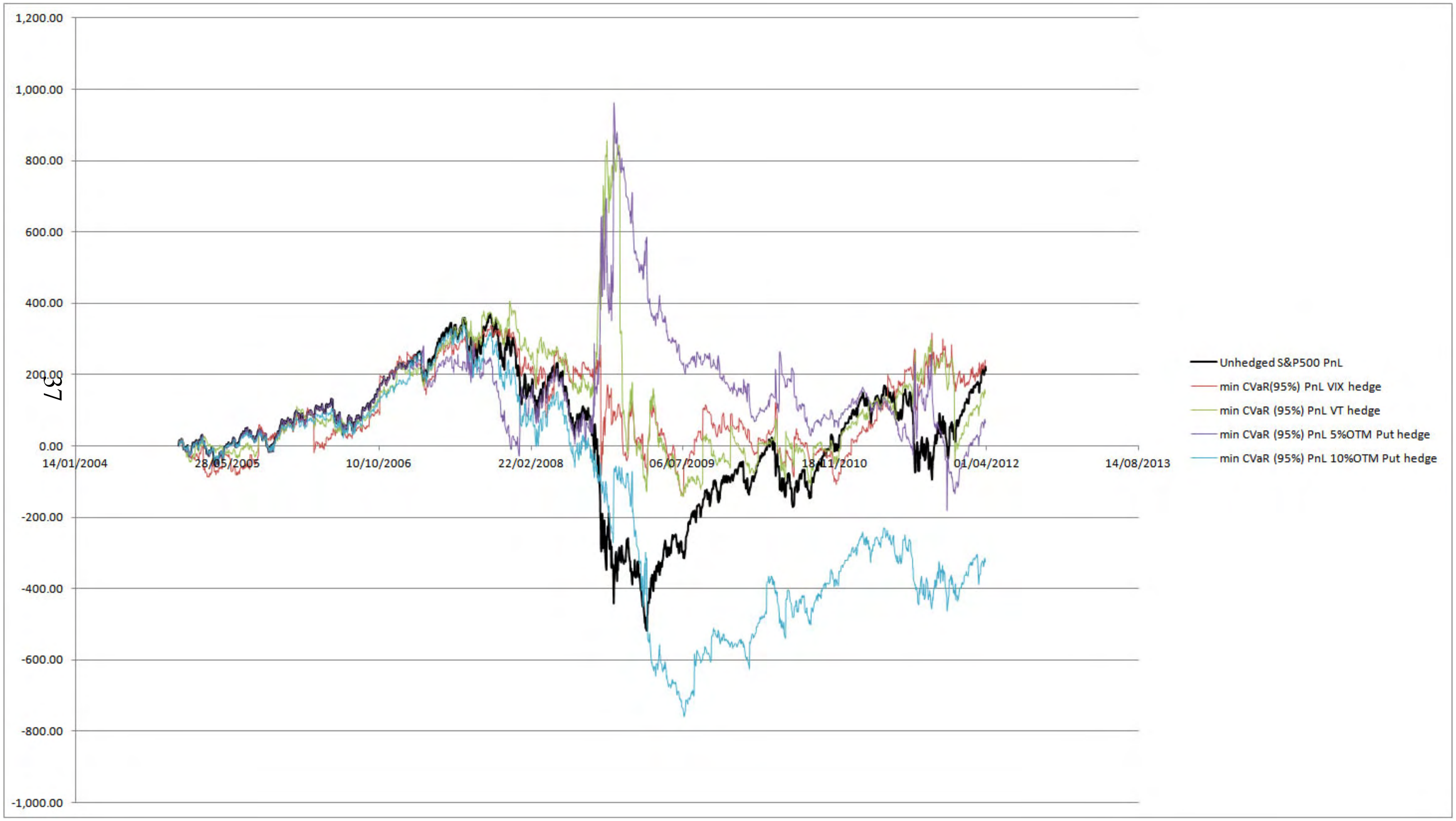


Figure 12: All Hedging Instruments (Dec 2004 – Mar 2011) under CVaR(95%)

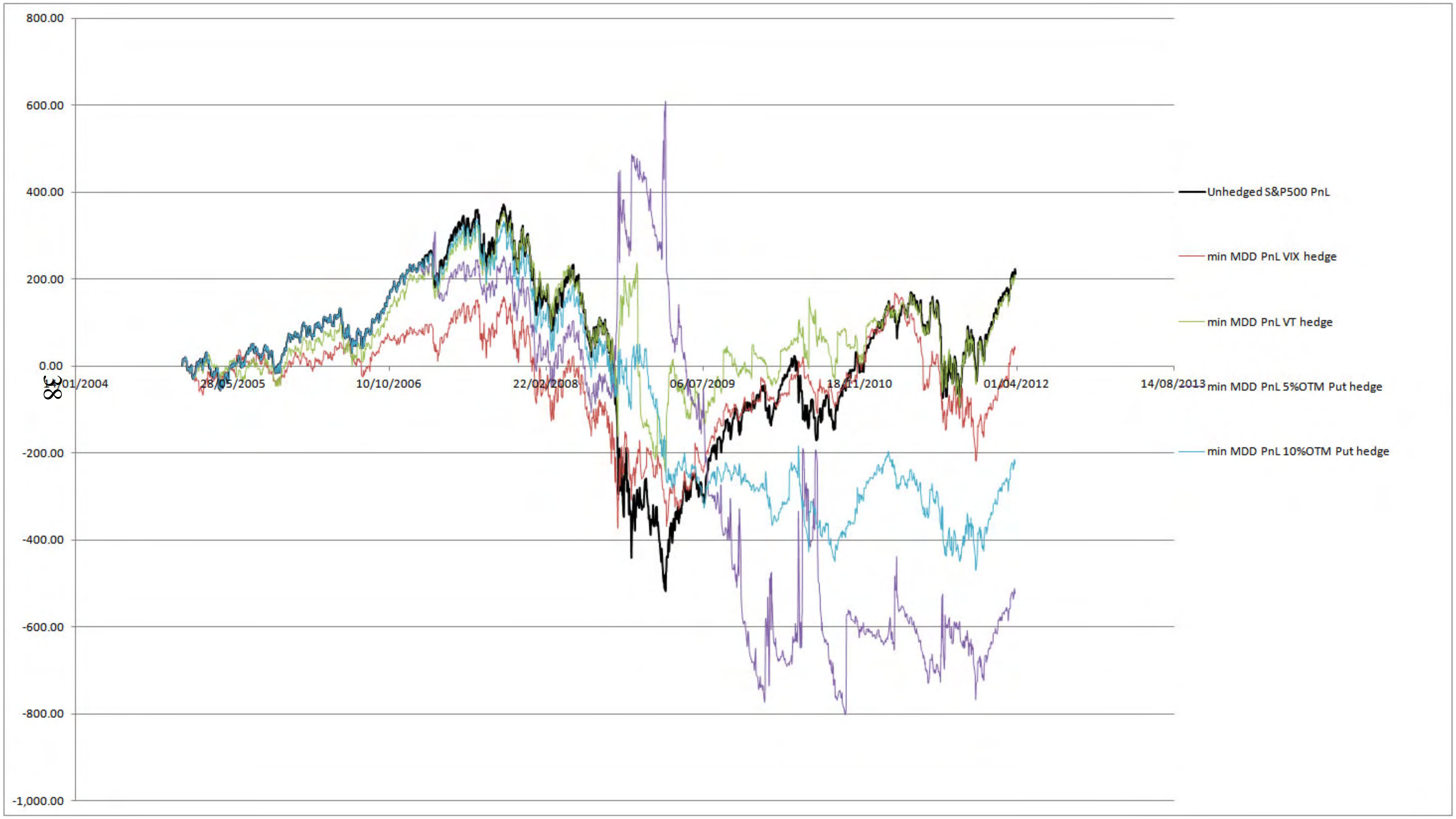


Figure 13: All Hedging Instruments (Dec 2004 – Mar 2011) under Maximum Drawdown