Risk Management and Capital Structure: Information Costs and Agency Costs Effects

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Abstract

This paper seeks to encompass elements of both the Modigliani and Miller (1958, 1963) and Jensen and Meckling (1976) approaches to optimal capital structure within a unified framework with shadow costs of incomplete information. Making the most of the major work of Merton (1987), Leland (1996, 1998) and Bellalah (2001a), we put forward a model that reflects the interaction of financing decisions and investment risk strategies under incomplete information. In this context, the study shows that (i) leverage level and yield spread increase when firm operates with hedging strategy, (ii) hedging provides more benefits compared to the complete information case, (iii) short term debt is more incentive-compatible with hedging than long term debt. Another implication of the model is that the importance of agency costs is far less than tax advantages of debt.

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Two thoughts have deeply shaped the development of capital structure literature. First, the arbitrage argument of Modigliani et Miller (M-M) (1958, 1963) shows that, with fixed investment decisions, the optimal structure of the capital balances the tax deductions provided by interest payments against the external costs of potential default. Subsequently, Jensen and Meckling (J-M) (1976) extirpate the M-M assumption that investment decisions are independent of capital structure. Equityholders of levered firm can potentially extract value from debtholders by increasing investment risk after debt is in place: the asset substitution problem. Since this two famous work, subsequent progress was slow in finding analytical valuations for debt with realistic features. Particularly, the theories fail to offer quantitative guidance as to amount (and maturity) of debt a firm should issue in a range of environments. A main obstacle to developing quantitative models has been the valuation of corporate debt credit risk. The pricing of risky debt is a precondition for determining the optimal amount and maturity of debt. But risky debt is a complex instrument. Its value will depend on the amount issued, maturity, the determinants of default, default costs, taxes, dividend payouts. It will also depend on the risk strategy chosen by the firm, which in turn will depend on the amount and maturity of debt in the firm's capital structure.

Brennan and Schwartz (1978) devise the first quantitative analysis of risky debt valuation and capital structure, but their case study requires complex numerical techniques to find solutions for a few specific cases. Their formulation focuses on the special case in which default is triggered when the firm's asset value falls to the debt's firm value. Another limitation of the Brennan and Schwartz (1978) model is that it consider the changes in financial structure which last only until the bonds expiry. An expiration date is necessary for their numerical algorithm. Furthermore, Permanent capital structure changes are not explicitly analyzed. Leland (1994a) challenges the Brennan and Schwartz assumption that bankruptcy is triggered when the firm's asset value falls to the debt's firm value. He considers

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two possible bankruptcy determinants. The first results from the fact that the firm is unable to raise sufficient capital to meet its debt obligations when bankruptcy is triggered endogenously: *unprotected debt*. The second corresponds to a positive net-worth covenant in a *protected debt*. Leland (1994a) offers closed form results for the value of long-term risky debt and yield spreads. He shows that the optimal leverage is explicitly linked to firm risk, taxes, bankruptcy costs and bond covenants. These results provide an explanation for the different behavior of junk bonds against investment-grade bonds.

Another line of research uses numerical valuation techniques. Kim, Ramaswamy, and Sundaresan (1993)[3] and Longstaff and Schwartz (1995)[4] provide bond pricing with credit risk. Unfortunately, these works do not focus on the choice of optimal capital structure. Leland and Toft (1996) extend Leland's (1994a) results to examine the effect of debt maturity on bond prices, credit spreads, and the optimal amount of debt. They show that longer term debt better exploits tax advantages because bankruptcy tends to occur at lower asset values. But longer term debt also creates greater agency costs by providing incentives for equityholders to increase firm risk through asset substitution. This potential agency costs can be substantially reduced or eliminated by using shorter term debt. Leland (1998) argues that all claims must be jointly recognized in the determination of capital structure and investment risk. His model provides quantitative guidance on the amount and maturity of debt, on financial restructuring, and on the firm's optimal risk strategy. He analyzes both asset substitution and risk management.

This article prolongs this literature to explain the information cost's effects on the analysis of the firm's capital structure and on its risk management. Building on the results of Leland, this research aims to gather the arguments of both the M-M and J-M approaches within a unified framework with information costs. The model reflects the interaction of financing decisions and investment risk strategies in the presence of information uncertainty. When investment policies are chosen to maximize equity value after debt is in place (that is ex post), stockholders-bondholders conflicts will lead to agency costs as in J-M. The model supposes that the shareholders (respectively bondholders) do not invest on the actions (respectively obligations) of the firm that if they are informed on these titles. In other words, an information cost λ_F is required to be informed on the firm's claims. Symmetrically, an information cost λ_V is required to be informed on the firm's assets value. Therefore, the optimal capital structure will reflect both M-M and J-M concerns within a framework with shadow costs of incomplete information. We introduce information costs and examine optimal firm decisions. A quantitative advice on the amount and maturity of debt, on financial restructuring, and on the firm's optimal risk strategy is provided. The framework equally permits the study of potential decrease in risk: risk management. In particular, we analyze and compare the information costs' effects and the agency costs effects on the risk management strategies.

The remainder of the paper is organized as follows. Section I presents the model of asset value dynamics and capital structure. Section II investigates ex post selection of risk in the presence of both agency and information costs. Section III is dedicated to the comparative analysis of agency costs and information costs and shows how they affect risk flexibility and capital structure. In section IV we examine optimal risk management by taking into account the agency costs and information costs. Section V contains some concluding comments.

[3]Kim, Ramaswamy, and Sundaresan (1993) propose numerical solutions to show that yield spreads are sensitive to interest rate expectations, but not to the volatility of the interest rates.

[4]Longstaff and Schwartz (1995) find that the correlation between default risk and the interest rate has a significant effect on the properties of the credit spreads.

I. The Model

The introduction of information costs in the study of the firm's capital structure and in the valuation process of the corporate claims is done with reference to the context proposed in Merton (1987). The main results used in Merton's model are recently applied in Shleifer and Vishny (1997) and Orosel (1997). In Merton's model, the effect of incomplete information on the asset's equilibrium price is equivalent to applying an additional discount rate to this asset's future cash-flows.[5] Merton's Model, the CAPMI, is an extension of the capital asset pricing model, the CAPM, in a context of incomplete information. The model offers a general method for discounting future cash flows under uncertainty. When information is complete, this model is reduced to the Sharpe's (1964) standard CAPM.[6]

I.1. The Evolution of Asset Value

Consider a firm whose unlevered asset value V follows the process :

$$\frac{dV(t)}{V(t)} = (\mu - \delta)dt + \sigma dW(t) \tag{1}$$

where V represents the value of the net cash-flows generated by the firm's activities (and excludes cash-flows related to debt), $\mu(V,t)$ is the total expected rate of return, δ is the payout rate, σ is the risk (standard deviation) of the asset return, and W is a Brownian standard motion. It is assumed that cash-flows are measured by the cash-flows of issued securities. A riskless asset exists and broughts a constant continuously compounded interest rate r.

I.2. Initial Debt Structure

The firm chooses its initial capital structure at time $t = 0$. The choice of capital structure includes the amount of debt principal to be issued, coupon rate, debt's expiration date, and call policy. This structure remains fixed without time limit until either :

(i) the firm goes to default, if asset value falls to the default level ; or

(ii) the firm calls its debt and restructures with newly issued debt, if asset value rises to the call level.

[5] The Merton's (1987) model may be stated as follows :

$$\bar{R}_V - r = \beta_V[\bar{R}_m - r] + \lambda_V - \beta_V \lambda_m$$

where :

- $\bar{R}_V$: the equilibrium expected return on security V;
- r : the riskless rate of interest;
- $\beta_V = \frac{\text{cov}(\tilde{R}_V/\tilde{R}_m)}{\text{var}(\tilde{R}_m)}$: the beta of security V, that is the covariance of the return on that security with the return on the market portfolio, divided by the variance of market return;
- $\bar{R}_m$: the equilibrium expected return on the market portfolio;
- λ_V : the equilibrium aggregate "shadow cost" for the security V. It is of the same dimension as the expected rate of return on this security V;
- λ_m: the weighted average shadow cost of incomplete information over all securities.

[6] Merton's model was applied in various contexts for the evaluation of the firm and its assets like for the pricing of the real options. For a survey of this literature, the reader can refer to Bellalah (2001a) and Bellalah (2001b).

Debt is initially issued at time $t = 0$ with principal P and coupon payment rate C. Let us denote respectively by M the average maturity of debt, V_0 the initial asset value, and V_U the asset level at which debt will be called. Default happens if asset value brings down to a level V_B prior to the calling of debt. Different environments will conduct to alternative default-triggering asset values. A positive net worth covenant in the bond indenture triggers default when net worth falls to zero, or $V_B = P$. If net cash-flow is proportional to asset value, at a level ηV, a cash-flow triggered default involves $V_B = C/\eta$. Lastly, default may be initiated endogenously when shareholders are no longer willing to raise additional equity capital to meet net debt service requirements. This determines V_B by the smooth-pasting condition utilized in Black and Cox (1976), Leland (1994a), Leland and Toft (1996), and Leland (1998). It is the default condition assumed here.

If default occurs, bondholders receive all asset value less default costs, reflecting the absolute priority of debt claims. Default costs are assumed to be a proportion α of remaining asset value V_B. Following Leland (1994b), Ericsson (1997), Mauer and Ott (1996), and Leland (1998), we assume a finite average debt maturity. In other words, debt has no stated maturity but is continuously retired at par at a constant fractional rate m. At any time $t \succ 0$, a fraction e^{-mt} of the debt will remain outstanding, with principal $e^{-mt}P$ and coupon rate $e^{-mt}C$. Therefore, higher debt retirement lead to shorter average maturity.[7]

Between restructuring points (and prior to bankruptcy), retired debt is continuously replaced by the issuance of new debt with identical principal value, coupon rate, and maturity. The firm's total debt structure (C, P, m) remains constant through time until restructuring or bankruptcy, even though the amounts of previously debt are declining exponentially over time through retirement. New debt is issued at market value, which may diverge from par value. Higher retirement rates incur additional funding flows and raise the default value V_B.

I.3. Capital Restructuring

When $V(t)$ reaches V_U, debt will be retired at par value and a newly issued debt replaces it as in the word of Goldstein, Nengjiu, and Leland (1999).[8] The moment at which debt is called is termed a *capital restructuring debt.* At the first restructuring point, P, C, V_B and V_U will be scaled up by the same ρ that asset value has increased, where $\rho = V_U/V_0$. Subsequent restructurings will again scale up these variables by the same ratio. Initial debt issuance, and subsequent debt issuance at each restructuring point, incurs a fractional cost k_1 of the principal issued. Debt retirement and replacement incurs a fractional cost k_2 of the principal retired.

II. Ex Post Selection of Risk in the Presence of Agency Costs and Information Costs

Past studies of capital structure have assumed that risk σ and payout rate δ are exogenously fixed and remain constant through time. Following Leland (1998), this paper try to allow the firm to choose its risk strategy. The extension allows the analysis of three important and closely related topics: information costs, asset substitution, and risk management. It further permits an examination of the interaction between capital structure and risk choice.

[7] Leland (1994b), by neglecting calls or bankruptcy, shows that the average maturity of debt is $M = 1/m$.

[8] The authors show that even though the optimal strategy is carried out over an arbitrarily large number of restructurings-periods, a scaling feature inherent in the framework permits simple closed-form expressions to be obtained for equity and debt prices.

To capture the essential elements of agency, it is assumed that risk choices are made ex post (i.e., after debt is in place), and that the risk strategy followed by the firm cannot be precontracted in the debt covenants or otherwise precommitted. The analysis presumes rational expectations, in that both equityholders and the debtholders will correctly anticipate the effect of debt structure on the chosen risk strategy, and the effect of this strategy on security pricing. This environment with ex post risk choice can be contrasted with the hypothetical situation where the risk strategy as well as the debt structure can be contracted ex ante (i.e., before debt is in place). In this situation the firm simultaneously choose its risk strategy and its debt structure to maximize its initial value. The difference in maximal values between the ex ante and ex post cases serves as a measure of agency costs, because it reflects the loss in value that follows from the risk strategy maximizing equity value rather than firm value.

To keep the analysis as simple as possible, it is assumed that firms can choose continuously (and without cost) between a low and high risk level: σ_L and σ_H. Similar to Ross (1997) and Leland (1998), the risk strategy considered here determines a time independent switch point value V_S, such that when $V \prec V_S$, the firm chooses the high risk level σ_H, and when $V \succeq V_S$, the firm chooses the low risk level σ_L.

II.1. Debt Value D

Given constant risk σ over an interval of values $[V_1, V_2]$, Goldstein, Nengjiu, and Leland (1999) (following Merton (1974)) show that $D_0(V,t)$, the value of debt issued at time $t = 0$, will satisfy the partial differential equation :

$$\frac{1}{2}\sigma^2 V^2 D^0_{VV} + (r-\delta)V D^0_V - rD^0 + D^0_t + e^{-mt}(C+mP) = 0 \qquad V_1 \preceq V \preceq V_2 \qquad (2)$$

where subscripts indicate partial derivatives. This reflects the fact that the original debtholders receive a total payment rate (coupon plus principal's return) of $e^{-mt}(C+mP)$.

Following Modigliani and Miller (1958), Black and Cox (1976) and Leland (1994a), we assume that there is a riskless asset which pays a constant interest rate r. Consider any claim on the firm that pays continuously $C+mP$ per instant when the firm is solvent. When the firm finances the net cost of the coupon by issuing additional equity, then using the same approach both as Black and Scholes and as Bellalah and Jacquillat (1995) can lead to the valuation equations of derivatives assets.[9] In this context, the value $D_0(V,t)$ of the debt in the presence of shadow costs of incomplete information must satisfy the following partial differential equation :

$$\frac{1}{2}\sigma^2 V^2 D^0_{VV} + (r+\lambda_V-\delta)V D^0_V - (r+\lambda_F)D^0 + D^0_t + e^{-mt}(C+mP) = 0 \quad V_1 \preceq V \preceq V_2 \quad (3)$$

Note that the shadow costs of incomplete information regarding the claim λ_F affect the value of the claim while the shadow costs of incomplete information concerning the asset value λ_V affect the asset's value.

Define $D(V) = e^{mt}D^0(V,t)$. Observe that $D(V)$ is the value of total outstanding debt at any future time t prior to restructuring. Because D(V) receives a constant payment rate $(C+mP)$, it is independent of t. Substituting $e^{-mt}D(V)$ for $D_0(V,t)$ in equation (3), it

[9] When information costs are ignored, the Bellalah and Jacquillat (1995) model reduces to the Black and Sholes (1973) model for valuing derivatives asset.

follows that $D(V)$ satisfies the ordinary differential equation :

$$\frac{1}{2}\sigma^2 V^2 D_{VV} + (r + \lambda_V - \delta)V D_V - (r + \lambda_F + m)D + (C + mP) = 0 \tag{4}$$

with general solution :

$$D(V) = \frac{C + mP}{r + \lambda_F + m} + a_1 V^{y_1} + a_2 V^{y_2} \tag{5}$$

where :

$$y_1 = \frac{-(r + \lambda_V - \delta - \frac{\sigma^2}{2}) + \sqrt{(r + \lambda_V - \delta - \frac{\sigma^2}{2})^2 + 2\sigma^2(r + \lambda_F + m)}}{\sigma^2}$$

$$y_2 = \frac{-(r + \lambda_V - \delta - \frac{\sigma^2}{2}) - \sqrt{(r + \lambda_V - \delta - \frac{\sigma^2}{2})^2 + 2\sigma^2(r + \lambda_F + m)}}{\sigma^2} \tag{6}$$

and $a = (a_1, a_2)$ is determined by the boundary conditions at $V = V_1$ and $V = V_2$.

From equations (5) and (6), we see clearly that value of total outstanding debt, $D(V)$, is affected by the shadow costs of incomplete information regarding the cash-flows of the firm and the corporate claims.

The risk strategy characterized by V_S specifies $\sigma = \sigma_L$ when $V_S \preceq V \preceq V_U$ and $\sigma = \sigma_H$ when $V_B \preceq V \preceq V_S$. From equation (5), the solutions to this equation in the high and low regions are given by :

$$D(V) = DL(V) = \frac{C + mP}{r + \lambda_F + m} + a_{1L} V^{y_{1L}} + a_{2L} V^{y_{2L}} \qquad V_S \preceq V \preceq V_U$$

$$D(V) = DH(V) = \frac{C + mP}{r + \lambda_F + m} + a_{1H} V^{y_{1H}} + a_{2H} V^{y_{2H}} \quad V_B \preceq V \prec V_S \tag{7}$$

with (y_{1H}, y_{2H}) given by the equations (6) with $\sigma = \sigma_H$ and (y_{1L}, y_{2L}) given by the equations (6) with $\sigma = \sigma_L$.
The coefficients $a = (a_{1H}, a_{2H}, a_{1L}, a_{2L})$ are determined by four boundary conditions. At restructuring,

$$DL(V_U) = P \tag{8}$$

reflecting the fact that debt is called at par. At default,

$$DH(V_B) = (1 - \alpha)V_B \tag{9}$$

recognizing that debt receives asset value less the fractional default costs, α.

Value matching and smoothness conditions at $V = V_S$ are :

$$DH(V_S) = DL(V_S)$$

$$DH_V(V_S) = DL_V(V_S) \tag{10}$$

where subscripts of the functions indicate partial derivatives. A closed form expression for the coefficients a is provided in the mathematical appendix.

When the shadow costs of incomplete information on the firm's cash-flows λ_V and the claim λ_F are zero, we find the standard case in Leland (1998).

II.2. Firm Value, Equity Value, and Endogenous Bankruptcy

Total firm value $v(V)$ is the value of assets V, plus the value of tax benefits from debt $TB(V)$, less the value of potential default costs $BC(V)$ and costs of debt issuance $TC(V)$:

$$v(V) = V + TB(V) - BC(V) - TC(V) \tag{11}$$

These value functions include the benefits and costs in all future periods, and reflect possible future restructurings as well as possible default. They are time-independent because their cash-flows and boundary conditions are not functions of time. Again following Merton (1974), Merton (1987), and Bellalah and Jacquillat (1995), any time-independent value function $F(V)$ with volatility σ will satisfy the ordinary differential equation :

$$\frac{1}{2}\sigma^2 V^2 F_{VV} + (r + \lambda_V - \delta)VF_V - (r + \lambda_F)F + CF(V) = 0 \tag{12}$$

where $CF(V)$ is the time-independent rate of cash-flow paid to the security.

- If the cash-flow rate is a constant CF, equation (12) has solution :

$$F(V) = \frac{CF}{r + \lambda_F} + c_1 V^{x_1} + c_2 V^{x_2} \tag{13}$$

where :

$$x_1 = \frac{-(r + \lambda_V - \delta - \frac{\sigma^2}{2}) + \sqrt{(r + \lambda_V - \delta - \frac{\sigma^2}{2})^2 + 2\sigma^2(r + \lambda_F)}}{\sigma^2}$$

$$x_2 = \frac{-(r + \lambda_V - \delta - \frac{\sigma^2}{2}) - \sqrt{(r + \lambda_V - \delta - \frac{\sigma^2}{2})^2 + 2\sigma^2(r + \lambda_F)}}{\sigma^2} \tag{14}$$

and c_1 and c_2 are constants determined by boundary conditions.

- If the cash-flow $CF(V) = kV$, equation (12) has solution :

$$F(V) = \frac{kV}{r + \lambda_F - \lambda_V} + c_1 V^{x_1} + c_2 V^{x_2} \tag{15}$$

II.2.1. The Value of Tax Benefits TB

Two simplifications permit closed-form results: that $EBIT = \eta V$ (earnings before interest and taxes are proportional to asset value), and that losses cannot carried forward. Under these assumptions, the cash-flows associated with tax benefits are :

$$CF = \tau C \qquad V_T \preceq V \preceq V_U$$

$$CF = \tau \eta V \qquad V_B \preceq V \preceq V_T$$

where $V_T = C/\eta$ is the asset value below which the interest payments exceed $EBIT$, and full tax benefits will not be received.

There are several possible regimes for the value of tax benefits, depending on the ordering of the values V_T, V_S, and V_0. Here (following Leland 1998), it is assumed that :

$$V_B \prec V_T \prec V_S \prec V_0 \prec V_U$$

Using equations (13) and (15),

$$TB(V) = TBL(V) = \frac{\tau C}{r + \lambda_F} + b_{1L} V^{x_{1L}} + b_{2L} V^{x_{2L}} \qquad V_S \preceq V \preceq V_U$$

$$= TBH(V) = \frac{\tau C}{r+\lambda_F} + b_{1H}V^{x_{1H}} + b_{2H}V^{x_{2H}} \qquad V_T \preceq V \prec V_S$$

$$= TBT(V) = \frac{\tau\eta V}{\delta+\lambda_F-\lambda_V} + b_{1T}V^{x_{1H}} + b_{2T}V^{x_{2H}} \quad V_B \preceq V \prec V_T \qquad (16)$$

where (x_{1H}, x_{2H}) and (x_{1L}, x_{2L}) are given by equation (14) with $\sigma = \sigma_H$ and $\sigma = \sigma_L$, respectively.

Boundary conditions are $TBL(V_U) = \rho TBL(V_0)$, reflecting the scaling property of the valuation functions at V_U and $TBT(V_B) = 0$, reflecting the loss of tax benefits at bankruptcy. In addition, there are value-matching and smoothness requirements at V_S and V_T. These six conditions determine the coefficient vector $(b_{1L}, b_{2L}, b_{1H}, b_{2H}, b_{1T}, b_{2T})$. A closed form expression for b is provided in the mathematical appendix.

In the context of complete information regarding the firm, its assets and cash-flows, there are no shadow costs of incomplete information and the tax benefits equations collapse to the one in Leland (1998).

II.2.2. The Value of Bankruptcy Costs BC

There is no continuous cash-flow associated with default costs, and $CF = 0$ in equation (13). It follows that :

$$BC(V) = BCL(V) = c_{1L}V^{x_{1L}} + c_{2L}V^{x_{2L}} \qquad V_S \preceq V \preceq V_U$$

$$= BCH(V) = c_{1H}V^{x_{1H}} + c_{2H}V^{x_{2H}} \qquad V_B \preceq V \preceq V_S \qquad (17)$$

Boundary conditions are $BCL(V_U) = \rho BCL(V_0)$, $BCH(V_B) = \alpha V_B$, and the value matching and smoothness conditions at V_S. The mathematical appendix provides a closed-form solution for the coefficients $c = (c_{1L}, c_{2L}, c_{1H}, c_{2H})$.

II.2.3 The Value of Debt Issuance Costs TC

It is obvious that debt issuance is costly. It is presumed that k_1 and k_2 represent the after-tax costs of debt issuance. Following Goldstein, Nengjiu, and Leland (1999), consider function $T\hat{C}(V)$, the value of transaction costs exclusive of the initial issuance at time $t = 0$. Noting that the flow of transactions costs associated with continuous debt retirement and replacement is $CF = k_2 mP$, and using equation (13) yields the function :

$$T\hat{C}(V) = T\hat{C}L(V) = \frac{k_2 mP}{r+\lambda_F} + d_{1L}V^{x_{1L}} + d_{2L}V^{x_{2L}} \qquad V_S \preceq V \preceq V_U$$

$$= T\hat{C}H(V) = \frac{k_2 mP}{r+\lambda_F} + d_{1H}V^{x_{1H}} + d_{2H}V^{x_{2H}} \quad V_B \preceq V \preceq V_S \qquad (18)$$

with boundary conditions $T\hat{C}L(V_U) = \rho(T\hat{C}L(V_0) + k_1 P)$, $T\hat{C}H(V_B) = 0$, and the value matching and smoothness conditions at V_S. The coefficients $d = (d_{1L}, d_{2L}, d_{1H}, d_{2H})$ are derived in the mathematical appendix.

Debt issuance costs $TC(V)$ are the sum of $T\hat{C}(V)$ and initial issuance costs $k_1 P$:

$$TC(V) = T\hat{C}L(V) + k_1 P$$

$$TCL(V) = k_1 P + \frac{k_2 mP}{r+\lambda_F} + d_{1L}V^{x_{1L}} + d_{2L}V^{x_{2L}} \qquad V_S \preceq V \preceq V_U$$

$$TCH(V) = k_1 P + \frac{k_2 mP}{r+\lambda_F} + d_{1H}V^{x_{1H}} + d_{2H}V^{x_{2H}} \qquad V_B \preceq V \preceq V_S \qquad (19)$$

The value of debt issuance costs depends on information costs which reflect the asymmetric information. Again, in the absence of these costs, the formula collapses to that in Leland (1998).

II.2.4. Firm Value v

From equation (11), firm value can be written as follows :

$$
\begin{aligned}
v(V) &= V + TB(V) - BC(V) - TC(V) \\
vL(V) &= V + TBL(V) - BCL(V) - TCL(V) && V_S \preceq V \preceq V_U \\
vH(V) &= V + TBH(V) - BCH(V) - TCH(V) && V_T \preceq V \preceq V_S \\
vT(V) &= V + TBT(V) - BCH(V) - TCH(V) && V_B \preceq V \preceq V_T
\end{aligned} \tag{20}
$$

where $TBL(V)$, $TBH(V)$ and $TBT(V)$ are given in equation (16), $BCL(V)$ and $BCH(V)$ are given in equation (17), and $TCL(V)$ and $TCH(V)$ are given in equation (19).

II.2.5. Equity Value and Endogenous Bankruptcy

Since the equity value $E(V)$ is given by the difference between the total firm value $v(V)$ (from equation (20)) and the debt value $D(V)$ (from equation (7)), we have :

$$
\begin{aligned}
E(V) &= v(V) - D(V) \\
EL(V) &= vL(V) - DL(V) && V_S \preceq V \preceq V_U \\
EH(V) &= vH(V) - DH(V) && V_T \preceq V \preceq V_S \\
ET(V) &= vT(V) - DH(V) && V_B \preceq V \preceq V_T
\end{aligned} \tag{21}
$$

All security values are now expressed in closed form as functions of the debt choice parameters $X = (C, P, m, V_U)$, the default value V_B, the risk-switching point V_S, and the exogenous parameters $(V_0, \lambda_F, \lambda_V, \alpha, \delta, \eta, r, \sigma_L, \sigma_H, \tau)$.

The default V_B is chosen endogenously ex post to maximize the value of equity at $V = V_B$, given the limited liability of equity and the debt structure $X = (C, P, m, V_U)$ in place. This requires the smooth pasting condition :

$$
h(X, V_B, V_S) \equiv \left. \frac{\partial ET(V, V_S)}{\partial V} \right|_{V=V_B} = 0 \tag{22}
$$

While $h(X, V_B, V_S)$ can be expressed in closed form, a closed form solution for V_B satisfying condition (22) remains a crucial challenge for futures research. However, root finding algorithms can find V_B, given V_S and X.

II.3. The Choice of the Optimal Risk Switching Value V_S

Time homogeneity ensures that optimal switching point between low and high volatility, V_S, will not change through time until restructuring, at which point the scaling property implies V_S will be increased by the factor ρ. V_S will depend on whether it can be contracted ex ante or will be determined ex post, after debt is already in place. The difference in maximal firm value between these two cases will be taken as a measure of agency costs.

When the risk switching point can be committed ex ante, the firm will choose its capital structure $X = (C, P, m, V_U)$, default value V_B, and risk switching point V_S to maximize its initial value :

$$\max_{X, V_B, V_S} v(V, X, V_B, V_S)\Big|_{V=V_0} \tag{23}$$

subject to :

$$h(X, V_B, V_S) = 0 \tag{24}$$

$$P = D(V_0) \tag{25}$$

where equation (24) is the required smooth pasting condition at $V = V_B$ and equation (25) is the requirement that debt sells at par.

When the risk switching point V_S cannot be precommited, it will be chosen ex post to maximize equity value E given the debt structure X that is in place. To measure the change in equity value that would result from a small change of switch point at $V = V_S$, consider the derivative :

$$\begin{aligned} z((V_S, V_B, X) &= \frac{dEL}{dV_S}\Big|_{V=V_S} \\ &= \frac{dEL}{dV_S}\Big|_{V=V_S} + \frac{dEL}{dV_B}\Big|_{V=V_S} \frac{\partial V_B}{\partial V_S} \end{aligned} \tag{26}$$

where :

$$\frac{\partial V_B}{\partial V_S} = \frac{-\partial h/\partial V_S}{\partial h/\partial V_B}$$

recognizing that V_B will change with V_S but the capital structure X will not.[10] So that V_S to be ex post optimal, the following condition is necessary :

$$z(V_S, V_B, X) = 0 \tag{27}$$

Agency costs are measured by the difference in firm value between the ex ante optimal case, the maximum of equation (23) subject to constraints (24) and (25), and the ex post optimal case, the maximum of equation (23) subject to constraints (24), (25), and (27).

Note that condition (27) is a necessary but not a sufficient condition. Indeed, when several locally optimal solutions exist, the solution with the larger initial firm value is chosen. The capital structure of that solution will induce its associated risk switching point.

II.4. The Expected Maturity of Debt EM

Expected debt maturity EM depends on the retirement rate m, and the possible calling of debt if V reaches V_U or default if V falls to V_B. Because there are two volatility levels, analytic measures of expected maturity are difficult to obtain. To circumvent this difficulty, Leland (1998) computes approximate bounds for expected debt maturity supposing that default can be ignored, and risk is a constant σ.[11] These two assumptions are justified

[10] Due to smoothness at risk switching point, V_S, equation (26) does not change to whether we use EL or EH.

[11] Leland (1998) shows that expected maturity of the debt is given by

$$EM = \frac{1}{m}\left(1 - (\frac{V_U}{V_0})^{\xi}\right)$$

by the fact that for most examples considered below, the likelihood of restructuring far exceeds the likelihood of default, so ignoring the latter may not be a significant problem. Furthermore, although risk is not constant, average risk is bounded above by σ_H and below by σ_L. Expected debt maturity $EM(\sigma)$ is monotonic in risk σ for the range of parameters considered. Therefore the computed bounds on expected maturity are given by $EM_{max} = Max[EM(\sigma_L), EM(\sigma_H)]$ and $EM_{min} = Min[EM(\sigma_L), EM(\sigma_H)]$.

III. The Significance of Information Costs and Agency Costs

This section applies the formulas of the previous section to examine properties of the optimal capital structure and the optimal risk strategy, and to compare agency costs effect with shadow costs of incomplete information effect. Base case parameters are :

Initial asset value :	$V_0 = 100$
Bankruptcy costs :	$\alpha = 0.25$
Payout rate :	$\delta = 0.05$
Cash-flow rate :	$\eta = 0.10$
Tax rate[12] :	$\tau = 0.20$
Riskless interest rate :	$r = 0.06$
Restructuring cost :	$k_1 = 0.01$
Continuous issuance cost :	$k_2 = 0.005$
Low risk level :	$\sigma_L = 0.20$
High risk level :	$\sigma_L = 0.30$

Table 1 : Choice of Risk Strategy and Capital Structure : with and without Information Costs Cases

This table shows the optimal capital structure and risk switch points for with and without information costs cases determination of the risk switching point V_S. σ_L and σ_H are low and high risk levels. v denotes firm value. V_B stands for the asset value at which default occurs and V_U is the asset value at which the debt is called. EM stands for expected debt maturity. LR, YS, and AC denote respectively the optimal leverage, yield spread, and agency costs. λ_F stands for shadow costs of incomplete information regarding to firm claims value and λ_V represents shadow costs of incomplete information concerning firm asset value. The values of base case parameters are defined in the text.

	v	V_S	V_U	EM_{max} (yrs)	EM_{min}	V_B	LR (%)	YS (bp)
Base Case ex ante : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	108.6	44.94	201	5.66	5.54	33.31	49.0	69
Base Case ex ante : $\lambda_F = 1\%$ and $\lambda_V = 0\%$	106.2	43.95	202	5.68	5.55	32.03	42.2	76
Base Case ex ante : $\lambda_F = 0\%$ and $\lambda_V = 1\%$	108.0	49.60	184	5.16	5.04	35.03	42.2	63
Base Case ex ante : $\lambda_F = 1\%$ and $\lambda_V = 1\%$	108.5	45.00	202	5.67	5.54	35.16	48.1	75
Base Case ex post : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	107.2	79.10	187	5.26	5.14	29.90	45.8	108
$\sigma_L = \sigma_H = 0.20$: $\lambda_F = 0\%$ and $\lambda_V = 0\%$	107.4	-	195	5.50	5.50	30.69	42.6	44
$\sigma_L = \sigma_H = 0.20$: $\lambda_F = 1\%$ and $\lambda_V = 0\%$	103.0	-	192	5.41	5.41	34.28	28.9	41
$\sigma_L = \sigma_H = 0.20$: $\lambda_F = 0\%$ and $\lambda_V = 1\%$	104.7	-	192	5.40	5.40	33.84	28.4	13
$\sigma_L = \sigma_H = 0.20$: $\lambda_F = 1\%$ and $\lambda_V = 1\%$	106.5	-	195	5.49	5.49	33.63	38.2	51

Note that the debt retirement rate is a choice variable.[13] It is assumed that at least 10 percent of debt principal must be retired per year, implying $M \preceq 10$ years (that is $m \succeq 10\%$).

where

$$\xi = \frac{(\mu - \delta - 0.5\sigma^2) - ((\mu - \delta - 0.5\sigma^2)^2 + 2m\sigma^2)^{1/2}}{\sigma^2}$$

[13]For computing expected maturity bounds, EM_{max} and EM_{min}, the expected asset total rate of return, μ, is needed. This is why an annual risk premium of 7 percent above the risk-free rate is assumed.

The effects of relaxing this constraint are examined in Table 3 below. Table 1 describes the optimal capital structure and risk switch points for the base case, for ex ante and ex post determination of the risk switching point V_S, and for both complete and incomplete information cases. The case where the firm has no risk flexibility ($\sigma_L = \sigma_H = 0.20$) is also examined like a means of comparison.

When the firm's risk policy can be committed ex ante to maximize firm value (line 1 in Table 1), it nonetheless will increase risk when asset value is low. For asset values between $V_B = 33.31$ and $V_S = 44.94$, the high risk strategy is chosen. Increasing risk exploits the firm's option to continue the realization of potential tax benefits and avoid default. What confirms the predictions of Smith and Stulz (1985) concerning the function convexity of tax benefits.

When the firm's risk policy is determined ex post to maximize equity value, the firm will switch to the high-risk level at a much greater asset value: V_S increases to 79.1. Higher V_S implies that the firm operates with higher average risk, and reflects the asset substitution problem. Nevertheless, amount of agency costs remains moderate: 1.37 percent, less than one-fifth of the tax benefits associated with debt. Thus covenants that restrict the firm from adopting the high risk strategy will have very little value in the environment considered.

In the presence of the shadow costs of incomplete information regarding corporate claims, line 2 in Table 1 shows that yield spread increases. In the case where information costs concern the firm's asset value, line 3 in Table 1 shows that yield spread drops instead of increasing. The taking into account of information costs both regarding to corporate claims and asset value is represented in line 4. It is convenient to note that negative effect on the yield spread due to information costs on the asset value, and the positive effect due to information costs on corporate claims are compensated. But, on the whole, the presence of two types of information costs increases the yield spread compared to its level in the complete information case.

The behavior of yield spread in the presence of shadow costs of incomplete information can be interpreted as follows. When the investors pay a rate $\lambda_F = 1\%$ to be informed on the firm claims, and/or a rate $\lambda_V = 1\%$ to be informed on the asset value, they require a larger yield on the risky debt than what should be claimed in absence of information uncertainty.

Surprisingly, while the shadow costs of incomplete information on corporate claims (described in line 2 of Table 1) and the information costs on the firm's assets (described in line 3 of Table 1) bring down the optimal leverage ratio, the presence of the two types of costs simultaneously leaves the optimal leverage ratio almost unchanged, as described in line 4 of Table 1.

While results in lines 7 and 8 of Table 1 are consistent with what is expected, those of line 9 merit comment. When the firm has no risk flexibility ($\sigma_L = \sigma_H = 0.20$), the taking into account of information costs both regarding to corporate claims and asset value engenders a large increase in the optimal leverage ratio, in the yield spread as well as an significant increase in the firm value.

Note that both agency costs and information costs increase the yield spread by a significant amount. Thus, the agency costs, even when small, may have a significant effect on the yield of corporate debt. The effect of information costs can be interpreted in the same way, above all in the presence of the two types of incomplete information.

Comparative Statics of Financial Variables : Complete and Incomplete Information Cases

Figure 1 illustrates ex ante firm value v, yield spreads YS, asset value at which the debt is called V_U, asset value at which default occurs V_B, risk switching point V_S, and optimal leverage LR as functions of the high risk level σ_H when we locate in a world without information costs ($\lambda_F = \lambda_V = 0$). All other parameters remain as in the base case.

All things being equal, and as expected, larger σ_H can be associated with a greater yield spread. Optimal leverage ratio increases from 44.8 for $\sigma_H = 0.2$ to 57.5 for $\sigma_H = 0.4$. Maximal firm value goes up by 4.15 points in the same interval of high risk level variation. Asset value at which the debt is called rises by 5.3 points. Less expected is that the risk switching point, and the asset value at which default occurs do not change significantly. Thus, the high risk level generates an increase in the yield spread; it encourages the firm to be more involved in debt; it also increases the firm value, v, and asset value at which the debt is called, V_U. Nevertheless, it leaves unchanged risk switching point, V_S, and asset value at wich default occurs V_B in the environment considered.

Figure 1 : Variation of optimal corporate financial structure with high risk level when information is complete.

The curves plot the firm value v, yield spreads YS, asset value at which the debt is called V_U, asset value at which default occurs V_B, risk switching point V_S, and optimal leverage LR for varying high risk levels σ_H. It is assumed that $m = 0.1$, $\delta = 0.05$, $\tau = 0.2$, $\eta = 0.1$, $r = 0.06$, $\sigma_L = 0.2$, $\alpha = 0.25$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$.

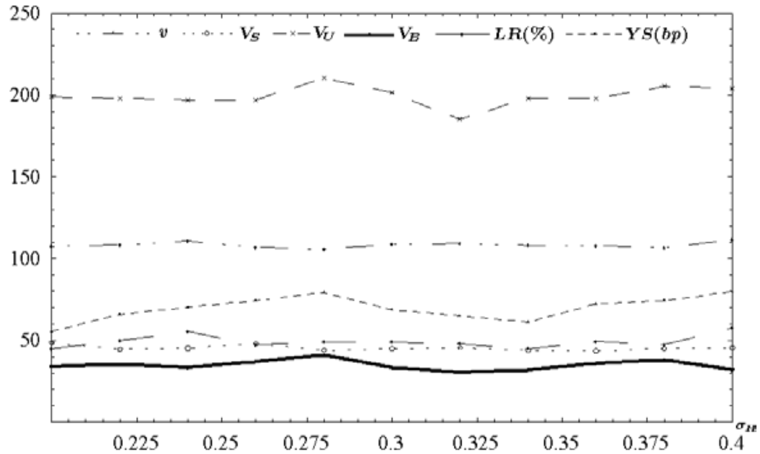


Figure 2 charts the effect of high risk level changes in a word where information is costly ($\lambda_F = \lambda_V = 1\%$). Within this framework, the increase in asset value at which the debt is called becomes more consistent throughout the interval. Yield spreads increase rapidly, reflecting the rise in average risk. An investor informed on the firm's assets and corporate claims will naturally require a higher yield on the risky debt when firm operates with higher high risk level. Relative to its level without information costs, optimal leverage ratio increases. However, risk switching point, asset value at which default occurs, and the maximal firm value are relatively flat.

Figure 2 : Variation of optimal corporate financial structure with high risk level in the presence of incomplete information.

The curves plot the firm value v, yield spreads YS, asset value at which the debt is called, V_U, asset value at which default occurs V_B, risk switching point V_S, and optimal leverage LR for varying high risk levels σ_H when an information cost regarding firm claims, $\lambda_F = 1\%$, is to taking into account and an information cost concerning firm assets, $\lambda_V = 1\%$, also. It is assumed that $m = 0.1$, $\delta = 0.05$, $\tau = 0.2$, $\eta = 0.1$, $r = 0.06$, $\sigma_L = 0.2$, $\alpha = 0.25$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$.

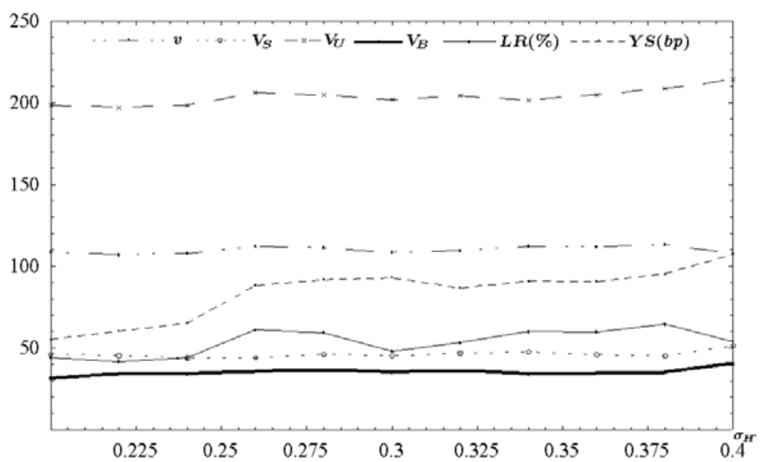
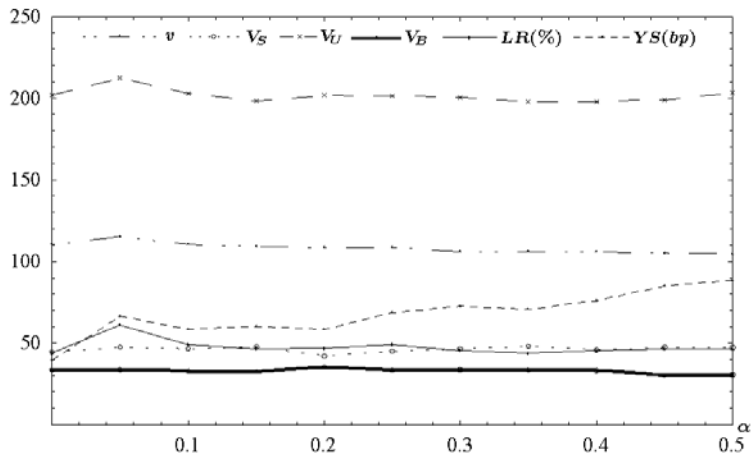


Figure 3 : Variation of optimal corporate financial structure with bankruptcy costs when information is complete.

The curves plot the firm value v, yield spreads YS, asset value at which the debt is called, V_U, asset value at which default occurs V_B, risk switching point V_S, and optimal leverage LR for varying bankruptcy costs levels α. It is assumed that $m = 0.1$, $\delta = 0.05$, $\tau = 0.2$, $\eta = 0.1$, $r = 0.06$, $\sigma_L = 0.2$, $\sigma_H = 0.3$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$.



When bankruptcy costs are going to go up, Figure 3 shows the changes in financial variables with complete information. Yield spreads increase significantly, but they increase even more in the presence of incomplete information, as illustrated in Figure 4. While the optimal leverage ratio remains almost flat in the case without information costs, its level relatively goes up when we take into account the information problems. Maximal firm value slides from 110.39 to 104.66 when information is complete and comes down from 114.27 to 107.11 when information is incomplete. Asset value at which default occurs, asset value at which the debt is called, and risk switching point are relatively stable in the two cases.

Figure 4 : Variation of optimal corporate financial structure with bankruptcy costs in the presence of incomplete information.

The curves plot the firm value v, yield spreads YS, asset value at which the debt is called V_U, asset value at which default occurs V_B, risk switching point V_S, and optimal leverage LR for varying bankruptcy costs levels α when an information cost regarding firm claims, $\lambda_F = 1\%$, is to taking into account and an information cost concerning firm assets, $\lambda_V = 1\%$, also. It is assumed that $m = 0.1$, $\delta = 0.05$, $\tau = 0.2$, $\eta = 0.1$, $r = 0.06$, $\sigma_L = 0.2$, $\sigma_H = 0.3$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$.

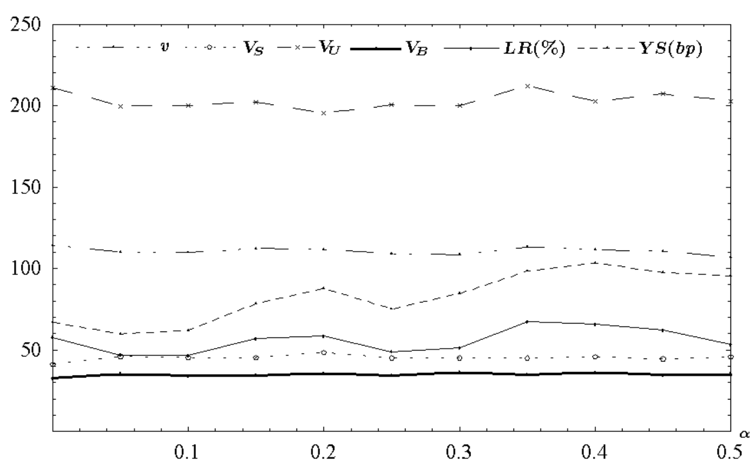
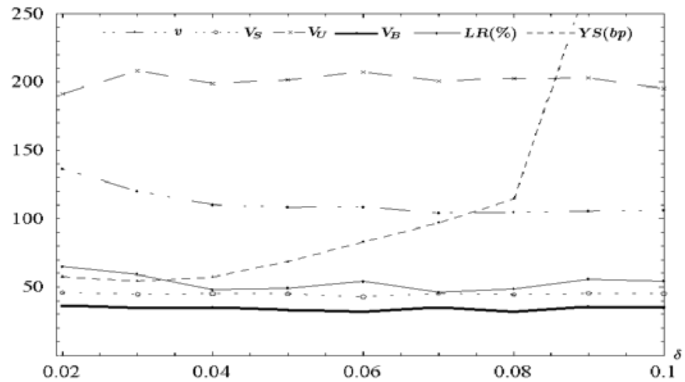


Figure 5 : Variation of optimal corporate financial structure with payout rate when information is complete.

The curves plot the firm value v, yield spreads YS, asset value at which the debt is called V_U, asset value at which default occurs V_B, risk switching point V_S, and optimal leverage LR for varying payout rate levels δ. It is assumed that $m = 0.1$, $\tau = 0.2$, $\eta = 0.1$, $r = 0.06$, $\sigma_L = 0.2$, $\sigma_H = 0.3$, $\alpha = 0.25$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$.



Figures 5 and 6 consider changes in the payout rate δ. For $\delta \prec 0.07$, yield spreads YS increase more quickly in the presence of incomplete information. On the other hand, when payout rate is large enough, $\delta \succ 0.07$, we notice a reversal on the YS behavior: while yield spreads increase quickly in Figure 5, they fall in figure 6. Lower payouts produce higher firm value v in the two cases with and without shadow costs of incomplete information. This can be justified by the fact that a higher leverage ratio can be supported when more assets remain in the firm. Leverage ratio is more volatile when $\lambda_F = \lambda_V = 1\%$.

Figure 6 : Variation of optimal corporate financial structure with payout rate in the presence of incomplete information.

The curves plot the firm value v, yield spreads YS, asset value at which the debt is called V_U, asset value at which default occurs V_B, risk switching point V_S, and optimal leverage LR for varying payout rate levels δ when an information cost regarding firm claims, $\lambda_F = 1\%$, is to taking into account and an information cost concerning firm assets, $\lambda_V = 1\%$, also. It is assumed that $m = 0.1$, $\tau = 0.2$, $\eta = 0.1$, $r = 0.06$, $\sigma_L = 0.2$, $\sigma_H = 0.3$, $\alpha = 0.25$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$.

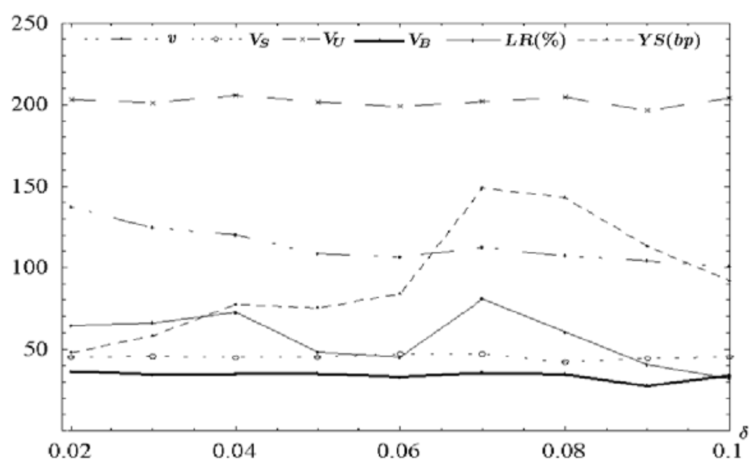


Figure 7 : Variation of optimal corporate financial structure with debt retirement rate when information is complete.

The curves plot the firm value v, yield spreads YS, asset value at which the debt is called V_U, asset value at which default occurs V_B, risk switching point V_S, and optimal leverage LR for varying retirement rate levels m. It is assumed that $\delta = 0.05$, $\tau = 0.2$, $\eta = 0.1$, $r = 0.06$, $\sigma_L = 0.2$, $\sigma_H = 0.3$, $\alpha = 0.25$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$.

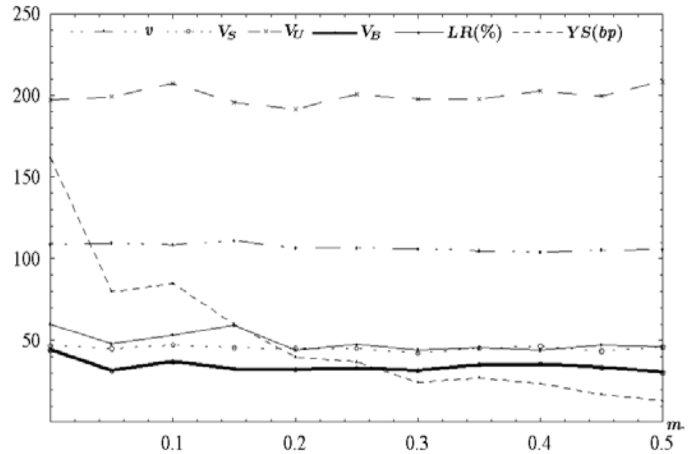


Figure 7 illustrates the effects of alternative debt retirement rates m. All things being equal, larger debt retirement rates can be associated with lower yield spreads YS. A higher retirement rate level, m, means that maturity debt, M, is weaker: what constitutes a "good signal" for the bondholders who will require, in return, a lower yield on risky debt. This is all the more true in the presence of shadow costs of incomplete information (Figure 8). As m increases, maximal firm value v decreases from 108.7 for $m = 0$ to 105.5 for $m = 0.5$. Asset value at which the debt is called V_U rises by 11.1 points in the same interval of debt retirement rate variation. The asset value at which default occurs V_B declines by 13.5 points. In like manner, optimal leverage ratio LR falls by 13.9 points. However, risk switching point V_S is relatively flat.

Figure 8 : Variation of optimal corporate financial structure with debt retirement rate in the presence of incomplete information.

The curves plot the firm value v, yield spreads YS, asset value at which the debt is called V_U, asset value at which default occurs V_B, risk switching point V_S, and optimal leverage LR for varying retirement rate levels m when an information cost regarding firm claims, $\lambda_F = 1\%$, is to taking into account and an information cost concerning firm assets, $\lambda_V = 1\%$, also. It is assumed that $\delta = 0.05$, $\tau = 0.2$, $\eta = 0.1$, $r = 0.06$, $\sigma_L = 0.2$, $\sigma_H = 0.3$, $\alpha = 0.25$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$.

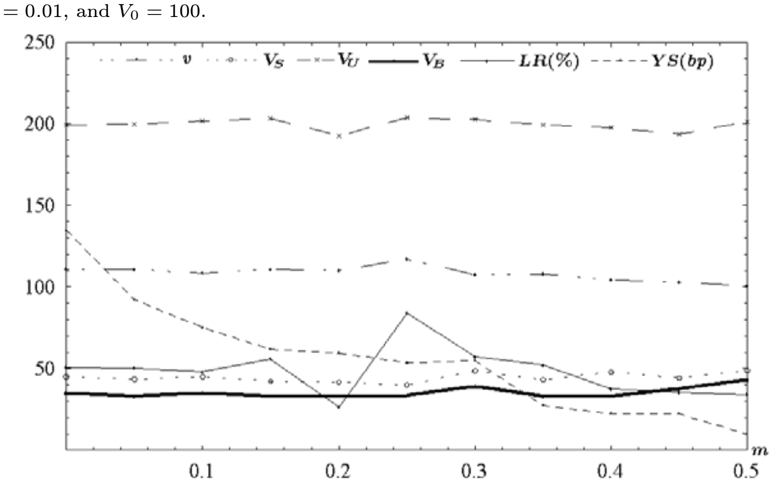


Figure 8 charts the effect of debt retirement rate changes by taking into account of information costs both regarding to corporate claims and asset value (that is, $\lambda_F = \lambda_V = 1\%$). Under this last assumption, the decrease in yield spread becomes more both consistent and spectacular throughout the interval. Relative to its level without information costs, maximal firm value decreases more importantly. Risk switching point, and asset value at which default occurs mark a light increase.

Figure 9 : Variation of optimal corporate financial structure with information costs regarding firm claims.

The curves plot the firm value v, yield spreads YS, asset value at which the debt is called V_U, asset value at which default occurs V_B, risk switching point V_S, and optimal leverage LR for varying information costs regarding firm claims levels λ_F. It is assumed that $m = 0.1$, $\delta = 0.05$, $\tau = 0.2$, $\eta = 0.1$, $r = 0.06$, $\sigma_L = 0.2$, $\sigma_H = 0.3$, $\alpha = 0.25$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$.

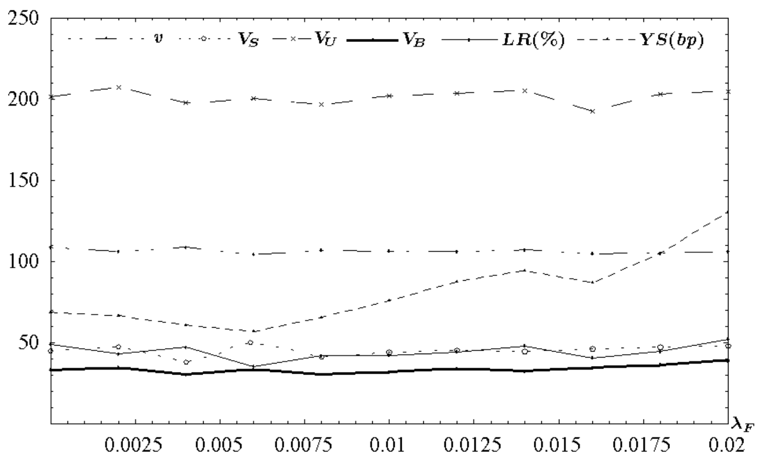
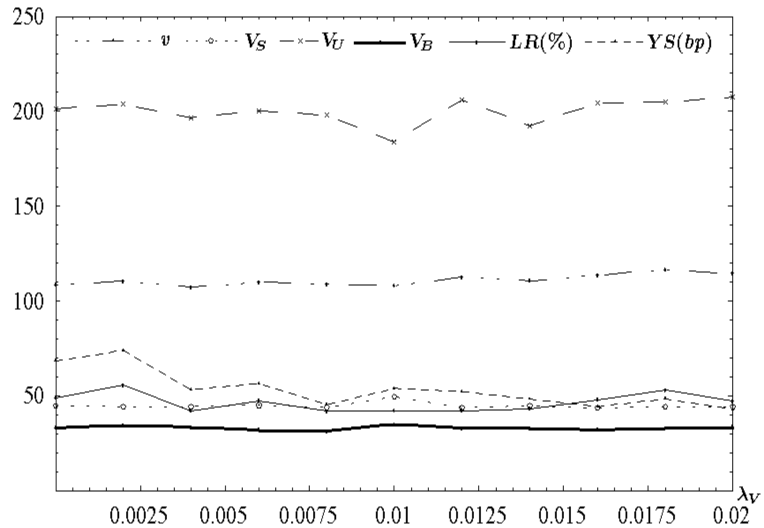


Figure 9 illustrates ex ante firm value v, yield spreads YS, asset value at which the debt is called V_U, asset value at which default occurs V_B, risk switching point V_S, and optimal leverage LR as functions of the information costs regarding firm claims levels λ_F. All things being equal, larger λ_F can be associated with a greater yield spread: YS increases from 68.6 for $\lambda_F = 0$ to 130.3 for $\lambda_F = 2\%$. Optimal leverage ratio, LR, increases by 3.2 points. Asset value at which default occurs, V_B, increases by 5.6 points in the same interval of information costs regarding firm claims levels variation. Whereas maximal firm value, asset value at which the debt is called, and risk switching point do not change significantly.

Figure 10 shows the relationship between the financial variables in question and the information cost regarding firm assets, λ_V. As expected, yield spread falls with increasing information costs λ_V: 68.6 for $\lambda_V = 0$ to 42.7 for $\lambda_V = 2\%$. Less expected is that maximal firm value and asset value at which the debt is called go up. Risk switching point, asset value at which default occurs, and optimal leverage remain relatively flat.

Figure 10 : Variation of optimal corporate financial structure with information costs regarding firm assets.

The curves plot the firm value v, yield spreads YS, asset value at which the debt is called V_U, asset value at which default occurs V_B, risk switching point V_S, and optimal leverage LR for varying information cost regarding firm claims levels λ_V. It is assumed that $m = 0.1$, $\delta = 0.05$, $\tau = 0.2$, $\eta = 0.1$, $r = 0.06$, $\sigma_L = 0.2$, $\sigma_H = 0.3$, $\alpha = 0.25$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$.



In contrast to Leland (1998), which treats the importance of the agency costs in the determination of optimal capital structure, the analysis above discusses the information costs effect on the financial variables allowing to determine the optimal capital structure. According to Leland (1998), agency costs increase with the hight risk level, are falling as debt retirement rate increases, and are impervious to default costs and payout rate variations. Our analysis concludes that shadow costs of incomplete information encourage the yield spread to increase more with the increases in hight risk level, bankruptcy costs, and payout rate. They encourage it to drop more when retirement rate increases. Leverage ratio increases more quickly with high risk level and bankruptcy costs. However, the firm value drops in the presence of information costs both regarding to corporate claims and asset value. Subsequent section applies preceding analysis to risk management.

IV. Risk Management with Shadow Costs of Incomplete Information

The benefit of hedging is measured by the percentage difference in firm value from using optimal hedging strategy compared with the no hedging case (ignoring costs of hedging). Consider a firm that has an exogenously given normal asset risk, σ_H. At any time, it is supposed that it can choose to reduce its risk costlessly to a given level σ_L. Furthermore, it can abandon its hedge and operates with normal risk σ_H at any time. A lower σ_L indicates a more effective available hedging strategy. In the same way as in section III, when $V \succeq V_S$, the firm chooses to hedge, with resultant risk σ_L. When $V \prec V_S$, the firm ceases hedging and operates with normal risk σ_H.

The following environments can be distinguish: (i) The firm can do no hedging whatever. (ii) The firm can precommit to hedge under all circumstances. (iii) The firm can precontract its hedging strategy. In this case, it will choose both its capital structure and hedging strategy ex ante to maximize market value. (iiii) The firm cannot precommit to any hedging strategy. In this case, it will choose its capital structure ex ante to maximize its market value, subject to constraint that the choice of hedging strategy, V_S, maximizes the value of equity ex post, given the debt in place.

In the subsection below, optimal hedging strategies are developed and numerical examinations of examples are suggested in order to show the influence of information costs both on optimal hedging strategies and on optimal capital structure. Subsequent subsection analyzes the behavior of financial variables ex ante in the presence of information costs.

IV.1. Optimal Hedging Strategies

This subsection studies several examples of optimal hedging strategies. Exogenous parameters are the same as in section III, except that volatility of the unhedged firm $\sigma_H = 0.20$. Table 2 examines optimal risk strategy and optimal capital structure for the ex ante and ex post hedging cases, and for both complete and incomplete information cases. For comparison, the case where the firm has no risk flexibility ($\sigma_H = \sigma_L = 0.20$) is also included. Panel A of Table 2 presents the base case when risk can be reduced to $\sigma_L = 15\%$. Panel B contains similar comparisons when risk can be reduced to $\sigma_L = 10\%$. Panel C presents the case where risk management might be used for speculative as well as hedging purposes. Two values of information costs regarding to firm claims value are assumed: $\lambda_F = 0\%$ and $\lambda_F = 1\%$. Symmetrically, two values of information costs concerning firm asset value are supposed: $\lambda_V = 0\%$ and $\lambda_V = 1\%$. In addition, the case where there are the two kinds of information costs, $\lambda_F = \lambda_V = 1\%$, is also considered. Agency costs, AC, are measured by the percentage difference between ex ante and ex post maximal firm values. The hedging benefits, HB, are measured by the percentage difference in firm value in comparison with no hedging.

We compare the ex ante optimal strategy, the ex post optimal strategy, and the always hedge strategy both in presence and in absence of shadow costs of incomplete information. When firm operates with no hedging strategy, there is no risk switching point, V_S, and the shadow costs of incomplete information regarding the firm and its cash flows can decrease optimal firm value. When the firm operates with one of the three hedging strategies considered, Panels A, B, and C show that the presence of the shadow costs of incomplete information regarding corporate claims decreases maximal firm value. Instead of decrease the maximal firm value, shadow costs of incomplete information concerning firm asset value raise it. The taking into account of information costs both regarding to corporate claims and asset value increases the maximal firm value. Another general result arising from Table 2 concerns yield spread, YS. While shadow costs of incomplete information on corporate claims increase sensitively yield spread, and information costs on firm's asset value lower it in the same way, the presence of information costs both on corporate claims and firm's asset value increases yield spread remarkably. This last result holds for the three possible levels of hedging considered (Panel A, B, and C). The principal intuition behind this result is that the investors require an additional yield to compensate for the expenditure concerning information acquisition. This last finding confirms the Merton's (1987) predictions: the effect of information costs is similar to an additional discounting rate required by the securityholders.

As for effect on optimal leverage ratio LR, information costs regarding to corporate claims bring it down in all cases. When information costs affect both the corporate claims and firm asset value, leverage ratio increases. The information costs only on firm asset value affects differently the leverage ratio. Except for the cases where firm is no hedging and that where it is hedging to $\sigma_L = 15\%$, λ_V increases the optimal leverage ratio.

Table 2 : Optimal Hedging Strategies and Capital Structure

This table presents the optimal capital structure and the optimal hedging strategies in with and without information costs cases. σ_L and σ_H are low and high risk levels. v denotes firm value. V_S is the risk switching point. V_B stands for the asset value at which default occurs and V_U is the asset value at which the debt is called. EM stands for expected debt maturity. LR, YS, and AC denote respectively the optimal leverage, yield spread, and agency costs. λ_F stands for shadow costs of incomplete information regarding to firm claims value and λ_V represents shadow costs of incomplete information concerning firm asset value. The values of base case parameters are defined in the text.

	v	V_S	V_U	EM_{max} (yrs)	EM_{min}	V_B	LR (%)	YS (bp)	HB (%)
No hedging : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	107.4	-	195	5.50	5.50	30.7	42.6	44	-
No hedging : $\lambda_F = 1\%$ and $\lambda_V = 0\%$	103.0	-	192	5.41	5.41	34.3	28.9	41	-
No hedging : $\lambda_F = 0\%$ and $\lambda_V = 1\%$	104.7	-	192	5.40	5.40	33.8	28.4	13	-
No hedging : $\lambda_F = 1\%$ and $\lambda_V = 1\%$	106.5	-	195	5.49	5.49	33.6	38.2	51	-
Panel A: Base Case, Hedging to $\sigma_L = 15\%$									
Ex ante optimal : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	109.2	52.5	176	4.97	4.91	41.0	51.7	33	1.80
Ex ante optimal : $\lambda_F = 1\%$ and $\lambda_V = 0\%$	104.6	48.3	176	4.97	4.91	39.8	33.5	34	1.62
Ex ante optimal : $\lambda_F = 0\%$ and $\lambda_V = 1\%$	111.6	48.6	177	5.01	4.95	40.5	48.3	21	6.92
Ex ante optimal : $\lambda_F = 1\%$ and $\lambda_V = 1\%$	113.2	50.0	176	4.95	4.89	40.5	61.7	47	6.70
Ex post optimal : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	108.9	69.2	171	4.79	4.73	38.1	50.0	41	1.44
Always hedge : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	107.9	-	173	4.87	4.87	43.2	48.3	29	0.45
Always hedge : $\lambda_F = 1\%$ and $\lambda_V = 0\%$	105.8	-	171	4.78	4.78	39.3	38.0	34	2.77
Always hedge : $\lambda_F = 0\%$ and $\lambda_V = 1\%$	122.3	-	179	5.06	5.06	40.5	77.4	28	17.58
Always hedge : $\lambda_F = 1\%$ and $\lambda_V = 1\%$	109.8	-	170	4.75	4.75	41.7	49.9	38	3.30
Panel B: Base Case, Hedging to $\sigma_L = 10\%$									
Ex ante optimal : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	110.6	61.0	156	4.22	4.12	52.6	55.8	15	3.17
Ex ante optimal : $\lambda_F = 1\%$ and $\lambda_V = 0\%$	106.0	61.8	153	4.10	4.00	50.4	38.7	28	3.00
Ex ante optimal : $\lambda_F = 0\%$ and $\lambda_V = 1\%$	123.2	61.5	154	4.26	4.16	49.6	64.2	16	18.47
Ex ante optimal : $\lambda_F = 1\%$ and $\lambda_V = 1\%$	111.5	62.8	157	4.26	4.16	57.5	58.7	33	5.00
Ex post optimal : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	111.3	80.1	146	3.73	3.63	46.6	60.6	36	3.60
Always hedge : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	110.6	-	153	4.09	4.09	54.3	55.6	14	3.15
Always hedge : $\lambda_F = 1\%$ and $\lambda_V = 0\%$	108.4	-	152	4.02	4.02	52.0	49.1	33	5.36
Always hedge : $\lambda_F = 0\%$ and $\lambda_V = 1\%$	122.8	-	150	3.92	3.92	53.9	78.8	13	18.09
Always hedge : $\lambda_F = 1\%$ and $\lambda_V = 1\%$	118.5	-	156	4.20	4.20	53.7	80.1	32	12.00
Panel C: Base Case, Hedging to $\sigma_L = 15\%$ and Speculation to $\sigma_H = 30\%$									
Ex ante optimal : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	113.4	67.4	184	5.23	5.05	48.7	59.2	84	5.96
Ex ante optimal : $\lambda_F = 1\%$ and $\lambda_V = 0\%$	106.4	67.4	183	5.19	5.01	46.0	55.0	93	3.40
Ex ante optimal : $\lambda_F = 0\%$ and $\lambda_V = 1\%$	114.1	70.5	186	5.29	5.10	49.7	72.1	82	9.40
Ex ante optimal : $\lambda_F = 1\%$ and $\lambda_V = 1\%$	109.4	65.0	183	5.21	5.02	47.9	59.2	85	2.90
Ex post optimal : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	108.5	84.9	162	4.48	4.26	35.4	53.8	105	1.02
Always hedge : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	109.0	-	173	4.87	4.87	43.2	48.3	29	1.56
Always hedge : $\lambda_F = 1\%$ and $\lambda_V = 0\%$	105.8	-	171	4.78	4.78	39.3	38.0	34	2.77
Always hedge : $\lambda_F = 0\%$ and $\lambda_V = 1\%$	122.3	-	179	5.06	5.06	40.5	77.4	28	17.58
Always hedge : $\lambda_F = 1\%$ and $\lambda_V = 1\%$	116.7	-	174	4.90	4.90	41.7	74.2	49	10.20

Hedging to $\sigma_L = 15\%$ provides modest benefits (1.80). Hedging to $\sigma_L = 10\%$ broughts more benefits (3.17). Panel C shows that the most important benefit results from hedging to $\sigma_L = 15\%$ and speculation to $\sigma_H = 30\%$ (5.96). It should be noted that when there are information costs on both corporate claims and asset firm value, hedging benefits go up. This result is robust for the three risk strategies considered. Whereas a shadow costs of incomplete information on firm's claims brings down the hedging benefits, the information costs concerning asset value increase it remarkably: HB reaches 18.47% when firm chooses to hedge to $\sigma_L = 10\%$ and a information cost, $\lambda_V = 1\%$, is to taking into account. Ex post optimal base cases in Panels A and B shows the relationship between agency costs and hedging benefits. Note that at lower average volatility, higher optimal leverage ratio is associated that, in return, generates greater tax benefits and then greater hedging benefits are realized. We also notice a fall of yield spread which cuts the default likelihood.

Overall, the results displayed in Table 2 suggest that, in comparison with the without information costs case, both shadow costs of incomplete information regarding to corporate claims and those concerning firm's asset value insrease optimal firm value, yield spread, and optimal leverage ratio. Furthermore, they increase hedging benefits above all when they concern the firm's asset value. Subsequent subsection analyzes the behavior of financial variables ex ante in the presence of information costs.

IV.2. Comparative Statics

Table 3 investigates the comparative statics of firm value v, the risk switching point V_S, asset value at which the debt is called V_U, expected debt maturity EM, asset value at which default occurs V_B, optimal leverage ratio LR, yield spread YS, and hedging benefits HB for the optimal ex ante risk strategies and optimal capital structure when risk can be reduced to $\sigma_L = 15\%$. All exogenous parameters are as in the base case above, exept for the parameter heading each row. Recall that hedging benefits are the percentage increase in firm value compared to an otherwise identical firm that operates without any hedging strategy and that has the same value of the parameter heading the row in question.

Table 3 : Comparative Statics of Financial Variables : Hedging to $\sigma_L = 15\%$ ex ante Case

This table examines the comparative statics of firm value v, risk switching point V_S, asset value at which the debt is called V_U, expected debt maturity EM, asset value at which default occurs V_B, optimal leverage LR, yield spread YS, and hedging benefits HB when the firm hedges to $\sigma_L = 15\%$ ex ante. σ_L and σ_H are low and high risk levels. λ_F stands for shadow costs of incomplete information regarding to firm claims value and λ_V represents shadow costs of incomplete information concerning firm asset value. The values of base case parameters are defined in the text.

	v	V_S	V_U	EM_{max} (yrs)	EM_{min} (yrs)	V_B	LR (%)	YS (bp)	HB (%)
Base Case : $\lambda_F = 0\%$ and $\lambda_V = 0\%$	109.2	52.5	176	4.97	4.91	41.0	51.7	33	1.80
Base Case : $\lambda_F = 1\%$ and $\lambda_V = 1\%$	113.2	50.0	176	4.95	4.89	40.5	61.7	47	6.70
$\alpha = 0.10$: $\lambda_F = 0\%$ and $\lambda_V = 0\%$	112.1	49.7	180	5.09	5.03	39.6	55.0	28	3.10
$\alpha = 0.10$: $\lambda_F = 1\%$ and $\lambda_V = 1\%$	111.6	51.9	172	4.84	4.78	39.7	52.1	39	0.41
$\alpha = 0.50$: $\lambda_F = 0\%$ and $\lambda_V = 0\%$	112.2	49.7	183	5.19	5.13	42.1	71.8	59	11.92
$\alpha = 0.50$: $\lambda_F = 1\%$ and $\lambda_V = 1\%$	108.7	48.8	180	5.11	5.05	41.0	51.2	52	2.07
$\delta = 0.04$: $\lambda_F = 0\%$ and $\lambda_V = 0\%$	117.7	50.3	176	4.62	4.60	41.2	66.1	32	19.23
$\delta = 0.04$: $\lambda_F = 1\%$ and $\lambda_V = 1\%$	119.2	50.3	184	4.88	4.85	40.8	68.5	39	1.89
$\delta = 0.06$: $\lambda_F = 0\%$ and $\lambda_V = 0\%$	113.3	48.6	180	5.48	5.37	39.6	74.9	60	4.83
$\delta = 0.06$: $\lambda_F = 1\%$ and $\lambda_V = 1\%$	111.5	48.6	179	5.46	5.36	40.9	65.2	65	7.55
$m = 0.05$: $\lambda_F = 0\%$ and $\lambda_V = 0\%$	109.1	50.1	176	6.38	6.20	38.5	46.4	35	1.22
$m = 0.05$: $\lambda_F = 1\%$ and $\lambda_V = 1\%$	107.2	55.5	179	6.53	6.35	46.1	43.5	54	2.88
$m = 0.25$: $\lambda_F = 0\%$ and $\lambda_V = 0\%$	106.7	47.0	175	3.05	2.93	39.0	42.5	10	4.15
$m = 0.25$: $\lambda_F = 1\%$ and $\lambda_V = 1\%$	108.9	46.9	178	3.08	2.96	38.9	49.1	21	6.88
$\eta = 0.05$: $\lambda_F = 0\%$ and $\lambda_V = 0\%$	108.3	49.7	174	4.89	4.83	39.1	46.1	26	11.75
$\eta = 0.05$: $\lambda_F = 1\%$ and $\lambda_V = 1\%$	107.5	50.0	176	4.98	4.92	41.8	42.1	36	3.50

In the complete information case, all things being equal and as expected, the extent of hedging benefits increases with default costs α. Leverage ratio, yield spread and asset value at which default occurs raise substantially. However, the optimal firm value and the risk switching point remain almost unchanged. In fact, when the risk switching point, capital structure, and default value can be committed ex ante, a higher bankruptcy costs lead securityholders to require a higher yield spread. On the other hand, optimal leverage ratio increases and generates greater tax benefits that offset the higher yield spread. Overall, the maximal firm value remains unchanged.

At higher payout rates δ, the hedging benefits and firm value drop. But yield spread and leverage ratio increase. These results can be interpreted in the same way as those concerning increase in bankruptcy costs except that here yield spread double while leverage ratio increases only by 13 percent. Outcome is that maximal firm value falls. Lines 11 and 13 of Table 3 show that hedging benefits vary increasingly when debt retirement rate rises. This reflects the fact that short term debt is more incentive-compatible with hedging than long term debt. Obviously, the increase in retirement rate (that is use for short term debt) leads to a fall in yield spread. Lowering net cash flow η from 10 to 5 percent of asset value increases hedging banefits from 1.80 to 11.75. Yield spread, leverage ratio, and optimal firm value are lower when cach flow rate rises.

When we locate in a world with shadow costs of incomplete information, some changes in the comparative statics are to point out. As regards changes resulting from an increase in bankruptcy costs, let us note that the fall of leverage ratio causes that of firm value. Increasing payout rates lead now to a small cut in leverage ratio and a large increase in hedging benefits level. Higher debt retirement rate might be expected to increase the leverage ratio and then maximal firm value. We may also add that unlike the complete information case, lower net cash flow lead to decrease hedging benefits. This reflects the large decrease in leverage.

V. Concluding Remarks

This research outlines the role and implications of information in the interaction of financing decisions and investment risk strategies. Optimal firm decisions are studied in a context with information costs. We use an optional approach to derive a model that provides quantitative guidance on the amount and maturity of debt. Optimal capital structure, risk management, agency costs and shadow costs of incomplete information are examined in a unified framework. Our paper shows that information costs prompts the yield spread to increase more when average risk, bankruptcy costs, and payout rate increase. Information costs encourage also leverage ratio to rise with more quickly with average risk and bankruptcy costs. The numerical examination of examples shows that both shadow costs of incomplete information regarding to corporate claims and those concerning firm's asset value increase optimal firm value, yield spread, and optimal leverage ratio. The numerical study of optimal risk strategies reveals an increase in hedging benefits above all when information costs concern the firm's asset value.

Mathematical Appendix

In this appendix, the coefficients a, b, c, and d are determined by using the value-matching, smoothness, boundary, and the scaling conditions.

A.1. Debt Coefficients

Boundary conditions include the value-matching and smoothness condition (10) at $V = V_S$:

$$a_{1L}V_S^{y_{1L}} + a_{2L}V_S^{y_{2L}} - a_{1H}V_S^{y_{1H}} - a_{2H}V_S^{y_{2H}} = 0$$

$$y_{1L}a_{1L}V_S^{y_{1L}-1} + y_{2L}a_{2L}V_S^{y_{2L}-1} - y_{1H}a_{1H}V_S^{y_{1H}-1} - y_{2H}a_{2H}V_S^{y_{2H}-1} = 0$$

The boundary condition (8) at V_U, with $\sigma = \sigma_L$ is :

$$\frac{C + mP}{r + \lambda_F + m} + a_{1L}V_U^{y_{1L}} + a_{2L}V_U^{y_{2L}} = P$$

and boundary condition (9) at default with $\sigma = \sigma_H$ is :

$$\frac{C + mP}{r + \lambda_F + m} + a_{1H}V_B^{y_{1H}} + a_{2H}V_B^{y_{2H}} = (1 - \alpha)V_B$$

Solving for a gives

$$\begin{pmatrix} a_{1L} \\ a_{2L} \\ a_{1H} \\ a_{2H} \end{pmatrix} = \begin{pmatrix} V_S^{y_{1L}} & V_S^{y_{2L}} & -V_S^{y_{1H}} & -V_S^{y_{2H}} \\ y_{1L}V_S^{y_{1L}-1} & y_{2L}V_S^{y_{2L}-1} & -y_{1H}V_S^{y_{1H}-1} & -y_{2H}V_S^{y_{2H}-1} \\ V_U^{y_{1L}} & V_U^{y_{2L}} & 0 & 0 \\ 0 & 0 & V_B^{y_{1H}} & V_B^{y_{2H}} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ P - \frac{C+mP}{r+\lambda_F+m} \\ (1-\alpha)V_B - \frac{C+mP}{r+\lambda_F+m} \end{pmatrix}$$

A.2. Tax Benefit Coefficients

Boundary conditions include the scaling condition

$$TBL(V_U) = \frac{V_U}{V_0}TBL(V_0)$$

the default condition

$$TBT(V_B) = 0$$

and the value-matching and smoothness condition at V_S and at V_T :

$$TBL_V(V_S) = TBH_V(V_S)$$

$$TBL(V_S) = TBH(V_S)$$

$$TBH_V(V_T) = TBT_V(V_T)$$

$$TBH(V_T) = TBT(V_T)$$

Substituting the appropriate equations for TBL, TBH, and TBT from equation (16) into the boundary conditions and recalling $\rho = V_U/V_0$ leads to the following solutions for the coefficients b:

$$\begin{pmatrix} b_{1L} \\ b_{2L} \\ b_{1H} \\ b_{2H} \\ b_{1T} \\ b_{2T} \end{pmatrix} = \Omega^{-1}\Psi$$

where

$$\Omega = \begin{pmatrix} V_U^{x_{1L}} - \rho V_0^{x_{1L}} & V_U^{x_{2L}} - \rho V_0^{x_{2L}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & V_B^{x_{1H}} & V_B^{x_{2H}} \\ x_{1L} V_S^{x_{1L}-1} & x_{2L} V_S^{x_{2L}-1} & -x_{1H} V_S^{x_{1H}-1} & -x_{2H} V_S^{x_{2H}-1} & 0 & 0 \\ V_S^{x_{1L}} & V_S^{x_{2L}} & -V_S^{x_{1H}} & -V_S^{x_{2H}} & 0 & 0 \\ 0 & 0 & x_{1H} V_T^{x_{1H}-1} & x_{2H} V_T^{x_{2H}-1} & -x_{1H} V_T^{x_{1H}-1} & -x_{2H} V_T^{x_{2H}-1} \\ 0 & 0 & V_T^{x_{1H}} & V_T^{x_{2H}} & -V_T^{x_{1H}} & -V_T^{x_{2H}} \end{pmatrix}$$

$$\Psi = \begin{pmatrix} (\rho - 1)\frac{\tau C}{r+\lambda_F} \\ \frac{-\tau\eta V_B}{\delta+\lambda_F-\lambda_V} \\ 0 \\ 0 \\ \frac{\tau\eta}{\delta+\lambda_F-\lambda_V} \\ \frac{\tau C}{\delta+\lambda_F-\lambda_V} - \frac{\tau C}{r+\lambda_F} \end{pmatrix}$$

A.3. Default Cost Coefficients

Under the assumption that the risk switching value $V_S \prec V_0$, boundary conditions include the scaling property

$$BCL(V_U) = \rho BCL(V_0)$$

and default condition

$$BCH(V_B) = \alpha V_B$$

Substituting for BCL and BCH from equation (17) into the equations above, together with the smoothnes and value matching conditions at V_S, gives

$$\begin{pmatrix} c_{1L} \\ c_{2L} \\ c_{1H} \\ c_{2H} \end{pmatrix} = \begin{pmatrix} V_U^{x_{1L}} - \rho V_0^{x_{1L}} & V_U^{x_{2L}} - \rho V_0^{x_{2L}} & 0 & 0 \\ 0 & 0 & V_B^{x_{1H}} & V_B^{x_{2H}} \\ x_{1L} V_S^{x_{1L}-1} & x_{2L} V_S^{x_{2L}-1} & -x_{1H} V_S^{x_{1H}-1} & -x_{2H} V_S^{x_{2H}-1} \\ V_S^{x_{1L}} & V_S^{x_{2L}} & -V_S^{x_{1H}} & -V_S^{x_{2H}} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \alpha V_B \\ 0 \\ 0 \end{pmatrix}$$

A.4. Debt Reissuance Cost Coefficients

The scaling property at the restructure point implies

$$T\hat{C}L(V_U) = \rho(T\hat{C}L(V_0) + k_1 P)$$

and the default boundary condition is

$$T\hat{C}H(V_B) = 0$$

Substituting for the functions $T\hat{C}L$ and $T\hat{C}H$ from equation (18) into the equations above, together with the smoothness and value matching condition at V_S, gives

$$\begin{pmatrix} d_{1L} \\ d_{2L} \\ d_{1H} \\ d_{2H} \end{pmatrix} = \begin{pmatrix} V_U^{x_{1L}} - \rho V_0^{x_{1L}} & V_U^{x_{2L}} - \rho V_0^{x_{2L}} & 0 & 0 \\ 0 & 0 & V_B^{x_{1H}} & V_B^{x_{2H}} \\ x_{1L} V_S^{x_{1L}-1} & x_{2L} V_S^{x_{2L}-1} & -x_{1H} V_S^{x_{1H}-1} & -x_{2H} V_S^{x_{2H}-1} \\ V_S^{x_{1L}} & V_S^{x_{2L}} & -V_S^{x_{1H}} & -V_S^{x_{2H}} \end{pmatrix}^{-1} \begin{pmatrix} \frac{(\rho-1)k_2 m P}{r} + \rho k_1 P \\ -\frac{k_2 m P}{r} \\ 0 \\ 0 \end{pmatrix}$$

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