Time-Varying Conditional Correlations and Volatilities of Stock Index Futures Returns

Yin-Feng Gau

Department of International Business Studies
National Chi Nan University

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Abstract

Utilizing the multivariate GARCH models, this paper analyzes the temporal dynamics of conditional volatilities and correlations across international index-futures markets. As the data of index futures provide a more efficiently informational measurement on the relationship among international stock indexes, the study using intraday returns to index futures not only directly investigates the comovement across index-futures markets, but also sheds light on the dynamic linkage among international stock indexes. The three index futures studied in this paper include the S&P500 index futures from the Chicago Merchantile Exchange (CME), Nikkei 225 index futures from the Osaka Securities Exchange (OSE), and FT-SE 100 index futures from the London International Financial Futures and Options Exchange (LIFFE). The empirical results imply that there exist time-varying conditional volatilities, as well as time-varying conditional correlations across index futures.

**Key words:** Multivariate GARCH; Volatility spillover; Stock index futures; Conditional correlation.
1 Introduction

The volatilities and correlations of asset returns are important in portfolio evaluation, derivative pricing, risk hedging, and calculating value at risk. The understanding on conditional correlations of asset returns could make portfolio diversification more effectively. Moreover, with the more growing integration of international financial markets, the study on the comovement among international asset markets becomes more important. As economic environments and market situations could vary over time, it is not necessary that conditional correlations across different international assets remain constant over time.¹ As a result, evaluating time-varying conditional covariances and volatilities of international assets would be major concerns of market participants in order to obtain an efficiently dynamic allocation of international assets and diversify risk effectively.

Nowadays, with the increasing liberalization and integration of global market activities, trading in international derivatives markets becomes feasible for investors and speculators. Since stock index futures started trading in 1982, hedging with stock index futures against the trading of the underlying index leads to very rapid growth in the volume and value of trading in index futures. The relatively low transaction cost, less restrictive rules on short sales, and high market liquidity in the index futures also make traders favor index futures.² By the returns to index futures across international markets, we can study the comovement of international asset markets in a more informational view-point. Moreover, the problem of stale prices or nonsynchronous trading occurred in stock index³ is also an important reason for us to utilize the index futures re-

¹For example, using monthly stock returns to seven international markets, Longin and Solink (1995) found that conditional covariances and conditional correlations vary over time. By the stock returns of the New York and Tokyo markets, Karolyi and Stulz (1995) also observed time-varying conditional covariances.

²Lower transaction costs and higher leverage from the trading of index futures allow traders to involve less financing of the transactions.

³As explained in Lo and MacKinlay (1990), statistical properties of market index returns are seriously
turns, instead of the index price changes, to investigate inter-reactions and comovements across international asset markets. Besides, a significant lead-lag relation between prices of the index futures and the underlying index has been documented in many empirical studies, such as Antoniou and Holmes (1995), Chan (1992), Chan, Chan, and Karolyi (1991), Stoll and Whaley (1990). Empirical evidences have concluded that index futures returns consistently lead returns to stock index. Therefore, the index futures is usually viewed as an informational signal in the price discovery for the cash price of the index.

Through an informative view of point, this paper employs the stock index futures to study the dynamic linkage among international index futures markets. The data analyzed in this paper include returns to the futures on the Standard & Poors 500 (S&P 500) Index, traded on the Chicago Merchantile Exchange (CME), the futures on the Nikkei 225 Stock Average (Nikkei 225), traded on the Osaka Securities Exchange (OSE), and the futures on the Financial Times - Stock Exchange 100 (FT-SE 100) Index, traded on the London International Financial Futures and Options Exchange (LIFFE).

 Apparently, relative to numerous studies of volatility transmission and comovement among international stock indexes, for example, including Hamao et al. (1990), King and Wadhwani (1990), Theodossiou and Lee (1993), Lin et al. (1994), Karloyi (1995), the investigation on the comovement of stock index futures markets, has rarely appeared in the literature. Kim, Szakmary, and Schwarz (1999) is the first one to analyze the relationships among stock index futures markets. Nevertheless, their VAR (vector autoregression) analysis ignored the problem of inefficient estimation resulted by heteroskedasticity in the data of futures returns. To consider the time-varying covariance matrix, we use the multivariate GARCH (generalized autoregressive conditional heteroskedastic) model in this paper. Using the multivariate GARCH framework, it allows affected by the existence of stale price. Due to infrequent trading or nonsynchronous trading across stocks included in the index, it results that index values often lag the actual market activities and new information in the market.

4The S&P 500 index futures contract, traded on the CME, is 250 times the index.
us to model the temporally varying volatilities and covariances, to analyze the multilateral feedback among index futures markets, and to examine the dynamic link between market information and volatility.

Various versions of multivariate GARCH models have been proposed, with different assumptions on parameters to ease the estimation complexity and ensure estimated conditional covariance to be positive definite. The simplest version of multivariate GARCH model that guarantees a positive definite conditional covariance matrix estimate is the Constant Conditional Correlation (CCC) model of Bollerslev (1990). Recently, Engle and Kroner (1995) proposed the BEKK (named after Baba, Engle, Kraft, and Kroner) model\(^5\) which ensures the positive definiteness of the covariance matrix as well. Another simplification of multivariate GARCH model is the factor model, including Diebold and Nerlove (1989), Engel and Rodrigues (1989), and Engle, Ng, and Rothschild (1990). The weakness of the BEKK and factor model is that the parameters can not readily reflect the intertemporal dynamics of variances and covariances, as the way readily seen in the univariate GARCH model.

With the advantage of computational simplicity, the CCC model is most popular among various multivariate GARCH models in applications.\(^6\) However, the crucial assumption of the CCC model is not necessarily satisfied by the empirical data. Longin and Solink (1995), and Karolyi and Stulz (1996) found evidences of time-varying conditional correlations between international equity markets. Therefore, some diagnostic tests have to been performed before estimating the CCC model. Bollerslev (1990) suggested the Ljung-Box portmanetau statistic on the cross products of standardized residuals across different univariate GARCH equations, judging by the critical values based on the \(\chi^2\) distribution. But as shown in Li and Mak (1994), the portmanteau statistic calculated

\(^5\)The model was initially proposed by Baba, Engle, Kraft, and Kroner (1991).

\(^6\)Empirical research that utilizes the CCC model includes, for example, Bollerslev (1990), Kroner and Claessens (1991), and Kroner and Sultan (1993), Longin and Solink (1995), Theodossiou and Lee (1993), Karolyi (1995), and Darbar and Deb (1997).
in such way is not asymptotically $\chi^2$ distributed, and the use of $\chi^2$ approximation is not appropriate in such situation. Instead of the above residual-based diagnostics, Tse (2000) suggested an Lagrange Multiplier (LM) statistic to test for the assumption of constant conditional correlation. In this paper we apply the LM statistic of Tse (2000) to test whether the data analyzed exhibit the constant conditional correlation. The results indicate that the conditional correlations between international index futures markets are not constant over time. Due to the setting of common factor needs further theoretical implication, the factor model is not considered in this paper, and it induces us to estimate the BEKK model to investigate temporal dynamics of the conditional covariance matrix of international index futures returns.

On the other hand, the possibility of asymmetric volatility effect has been discussed in many studies on univariate time-varying volatility. The asymmetric volatility effect characterizes a phenomenon that a negative returns shock (or unexpected price drop) at time $t - 1$ leads to a higher volatility, at time $t$, than a positive return shock of the same magnitude. Evidences of asymmetric volatility effects are usually found from applications of stocks, and it can be explained by the leverage effect – an increase in the riskiness of the stock due to an increase in the debt/equity ratio of the firm following a price drop. Estimating the data analyzed in this paper by the threshold GARCH model, proposed by Glosten, Jagannathan, and Runkle (1993), the estimation results, however, do not imply a significant asymmetry effect in the univariate process of conditional variance. Hence, we don’t extend the multivariate GARCH model considered in this paper for the effects of asymmetric volatilities and covariances.

This paper proceeds as follows. Section 2 documents the data set used in this study, and gives the statistical description of the data as well. Section 3 introduces the framework of multivariate GARCH models used in this paper. Empirical results are presented and interpreted in Section 4. Section 5 concludes.
2 The Data

This paper utilizes intraday returns to the S&P 500 index futures traded on the CME, Nikkei 225 index futures traded on the OSE, and FT-SE 100 index futures traded on the LIFFE, to explore the dynamics of conditional volatilities and correlations across international index-futures markets.

2.1 Features of Index Futures Markets

Futures contracts are exchange-traded instruments where one party agrees to purchase an asset at a future time for a certain price and the other party agrees to sell the asset at the same time for the same price. Trading of stock index futures started in the States in 1982, in the UK in 1984, and in Japan in 1988. The growth of trading in index futures in the States and Japan has been more rapid than in UK.

A stock index, the underlying asset of a stock index futures, is used to measure market-wide changes in the value of a stock market. According to different hypothetical portfolio of stocks, various stock indices have been constructed. For example, the Dow Jones Industrial Average is based on a portfolio consisting of 30 blue ship stocks in the States, with weight proportional to their prices. The S&P 500 Index is based on a portfolio of 500 different stocks: 400 industrial, 40 utilities, 20 transportation companies, and 40 financial institutions, with weights proportional to corresponding market capitalizations. The Nikkei 225 is based on a portfolio of 225 of the largest stocks trading on the Tokyo Stock Exchange (TSE), with weights according to their prices. The FT-SE 100 Index is based on a portfolio of 100 major UK shares listed on the London Stock Exchange.

Futures contracts on stock indices are settled in cash, since it is impossible to deliver
the underlying asset. All contracts are marked to market on the last trading day,\(^7\) and the positions are then treated to be closed. Setting the spot price of index as \(S_0\), it yields, theoretically, the futures price \(F_0\),\(^8\) as

\[
F_0 = S_0 e^{(r-q)T}
\]

where \(T\) is the time to maturity of the contract, \(r\) is the risk-free rate for a maturity \(T\), and \(q\) is the dividend yield rate.

Over ten index futures are currently actively traded on four different US exchanges, and the value of transactions in S&P 500 index futures is almost equal to the values of shares traded on the New York Stock Exchange. The largest exchanges on which futures contracts are traded are the Chicago Board of Trade (CBOE) and the CME.

In Japan, there are three index futures traded on two exchanges. In 1991 the trading volume of the Nikkei index futures was the largest among all index futures in the world, and the value of index futures traded in Japan was more than five times larger than the value of shares traded on the Tokyo Stock Exchange.

The index futures analyzed in this paper include the S&P 500 index futures traded on the CME, FT-SE 100 index futures traded on the LIFFE, and Nikkei 225 index futures traded on the OSE. The S&P 500 index futures contracts on the CME expire in March, June, September, and December, with expiration occurring on the 3rd Thursday of the month. The CME opens at 8:30 a.m. and closes at 3:15 p.m. Chicago time. The FT-SE 100 index futures contracts on the LIFFE expire at the last trading day of the month in March, June September, and December. The LIFFE opens at 8:35 a.m. and closed

\(^7\)For most index futures contracts, the settlement price on the last trading day is set equal to the closing price of the index on that day, whereas for the S&P 500 it is set at the opening spot price the next day.

\(^8\)More clearly, \(S_0\) can be viewed as the price of the underlying asset today, and \(F_0\) is the futures price today.
at 4:10 p.m. London time. The Nikkei 225 index futures contracts on the OSE expire at the 2nd Friday of March, June, September, and December. The OSE opens at 9:00 a.m. and closes at 3:10 p.m. Tokyo time, with a lunch break from 11:10 a.m. to 12:10 p.m.

2.2 Statistical Description of Index Futures Returns

The data cover the period from September 1, 1988 through May 18, 1999.\textsuperscript{9} Prices of index futures studied in this paper are obtained from the Reuters Commodity, Energy, and Financial Futures database. Daily prices of index futures are based on the futures contracts with nearest expiration date. The problems with analyzing data from The States, UK, and Japan, is the nonsynchronous trading among these three markets and Saturday trading on the OSE.\textsuperscript{10} Following Hamao et al. (1990), the data are processed and yields 2522 observations for each series.

Table 1 summarizes the descriptive statistics of intraday returns to these index futures. By the sample skewness, these three series are less skewed. The high values of sample kurtosis, however, implies the empirical distributions of intraday returns to S&P 500 index futures, FT-SE 100 index futures, and Nikkei 225 index futures are leptokurtic or fat-tailed. The results of the Jarque-Bera statistic fail to accept the null hypothesis of normal distribution for returns. We also calculate augmented Dickey-Fuller (ADF) statistic and Phillips-Perron statistic to test for the null of unit root against the alternative of AR(1) process, and the results show that the three series are all stationary. The first-order sample autocorrelation of these three returns are not significant. \(Q(20)\) and \(Q^2(20)\) indicate the Ljung-Box statistic for up to 20th-order serial correlation for

\textsuperscript{9}Since the Nikkei 225 contracts on the OSE began to list on September 1, 1988, we choose this date as the beginning of data period.

\textsuperscript{10}Before January 1989, excepting weekday trading from Mondays till Fridays, the OSE also trades on the 1st and 3rd Saturdays each month.
returns and squared returns, respectively. The larger value of $Q^2(20)$, relative to the value of $Q(20)$, implies that squared returns exhibit significant serial correlation, though the returns are not so. Significant serial correlations in squared returns typically implies volatility clustering and time-varying variances or heteroskedasticity. Comparing with the stylized facts on the empirical distribution of stock returns, concluded by Fama (1965) and others, the empirical distribution of index-futures returns almost shares the same properties: fat-tail, weak serial correlation in returns, but strong serial correlation in squared returns.

3 Multivariate GARCH Models

Among multivariate extensions of the GARCH model, there are four popular multivariate GARCH models, namely the VECH model of Bollerslev, Engle, and Wooldridge (1988), the CCC model of Bollerslev (1990), the factor GARCH model of Engle, Ng, and Rothschild (1990), and the BEKK model of Engle and Kroner (1995). A brief discussion on these models is given below.

By assuming a constant correlation, Bollerslev (1990) estimated a multivariate GARCH model to explore spillover effect in conditional mean and variance of exchange rate returns. Numerous works, including, for example, Theodossiou and Lee (1993), Karolyi (1995), Longin and Solink (1995), and Darbar and Deb (1997), applied the CCC model to evaluate the mean and volatility spillover among international equity markets. However, the assumption under constant conditional correlation needs further diagnostic tests in order to impose reasonable estimation for the data investigated. On the other hand, Engle and Kroner (1995) proposed a version of multivariate GARCH model, namely the BEKK model, to ensure the positive definiteness of the conditional covariance matrix.

Let $y_t$ be the vector of returns of $N$ assets during period $t$, and let $\mu_t$ and $H_t$ be
the conditional mean vector and conditional covariance matrix of \( y_t \) given information available at time \( t-1, \Omega_{t-1} \). The most general version of multivariate GARCH model is the VECH model of Bollerslev, Engle, and Wooldridge (1990). With an \( L \times 1 \) vector of exogenous variables in mean equation (including constant and ARMA terms), \( X_t \), a GARCH(1,1) representation is written as

\[
\mu_t = E(y_t | \Omega_{t-1}) = \Gamma X_t + \epsilon_t \\
\epsilon_t | \Omega_{t-1} \sim N(0, H_t) \\
\text{vech}(H_t) = W + A \text{vech}(\epsilon_{t-1} \epsilon_{t-1}') + B \text{vech}(H_{t-1})
\]

where \( \text{vech}(\cdot) \) denotes the column stacking operator of the lower portion of a symmetric matrix, \( \Gamma \) is an \( N \times L \) parameter matrix, \( W \) is a \( \frac{1}{2}N(N+1) \times 1 \) vector. The VECH model suffers two disadvantages in estimation. First of all, number of parameters to be estimated in this model grows quickly as \( N \) increases. For an \( N \times N \) covariance matrix \( H_t \), there exist \( 2 \cdot [\frac{1}{2}N(N+1)]^2 + \frac{1}{2}N(N+1) \) parameters to be estimated for the conditional covariance equation. The “curse of dimension” is a serious problem in estimating the VECH model. Secondly, positive definiteness of estimated \( H_t \) is not ensured from this model.

To remedy for the problem of huge size of parameters and guarantee a positive definite \( H_t \), Bollerslev (1990) proposed the constant conditional correlation (CCC) model as follows:

\[
h_{ii}t = w_i + \alpha_{ii} \epsilon_{it-1} + \beta_{ii} h_{ii}t-1, \quad i = 1, \cdots, N \\
h_{ij}t = \rho_{ij} \sqrt{h_{ii}t} \sqrt{h_{jj}t}, \quad i = 1, \cdots, N, j = 1, \cdots, N, 1 \leq i < j \leq N
\]

where \( h_{ij}t \) refers to the \( ij \)th element of \( H_t \), \( \alpha_{ii} \) and \( \beta_{ii} \) correspond to the \( ij \)th element of \( A \) and \( B \), respectively. \( R = [\rho_{ij}] \) is the conditional correlation matrix. The number
of parameters is $\frac{1}{2}N^2 + \frac{7}{2}N$. The crucial assumption of the CCC model is the constant conditional correlation, that is, $\rho_{ijt} = \rho_{ij}$ for $t = 1, \ldots, T$.

To guarantee positive definiteness of estimated covariance matrix, Engle and Kroner (1995) use quadratic forms of parameterizations to model the covariance. The BEKK model is

$$H_t = CC' + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BH_{t-1}B'$$

where $C$, $A$, and $B$ are all $N \times N$ parameter matrices. The BEKK model allows the conditional covariance matrix of asset returns to be determined by the outer product matrices of the vector of past return shocks. However, the number of parameters in the BEKK model is $\frac{5}{2}N^2 + \frac{N}{2}$, which is large even for moderate size of $N$.

An alternative model with positive definite $H_t$ but fewer parameters is the Factor ARCH model, suggested by Engle, Ng, and Rothschild (1990). Assuming there exists a single portfolio whose variance is driving all the conditional variances and covariances of asset returns, a one factor ARCH model is

$$H_t = CC' + \lambda\lambda'[\phi \cdot w'H_{t-1}w + \eta \cdot (w'\varepsilon^2_{t-1})]$$

where $\lambda$ and $w$ are $N \times 1$ vectors, and $\eta$ and $\phi$ are scalars. The number of parameters in this model is reduced to $\frac{1}{2}N^2 + \frac{3}{2}N + 2$.

### 3.1 Tests for Constant Conditional Correlations in a Multivariate GARCH Model

Of these multivariate GARCH models reviewed in this paper, the CCC model of Bollerslev (1990) is the most popular one in empirical studies. However, in most applications, the crucial assumption for constant conditional correlation is usually taken as given
without test. This will lead to model misspecification and invalid inference from the estimation results. One adequate approach is to test for the assumption of constant conditional correlation before estimating the CCC model. One diagnostic test suggested by Bollerslev (1990) is the Ljung-Box statistic based on the product of standardized residuals. However, Li and Mak (1994) pointed out that the Ljung-Box statistic of the product of standardized residuals is not asymptotically $\chi^2$ distributed under the null hypothesis of constant conditional correlation. On the other hand, Tse (2000) suggested an alternate statistic to test for constant conditional correlation.

To test for the assumption of constant conditional correlation, Tse (2000) favored an LM statistic which only requires estimates under the CCC model. Rewrite the conditional correlations as:

$$\rho_{ijt} = \rho_{ij} + \delta_{ij} y_{it-1} y_{jt-1}$$

where $\delta_{ij}$ for $1 \leq i < j \leq N$ are additional parameters in the extended equation of conditional correlation. Therefore, the constant-correlation hypothesis can be tested by examining the hypothesis $H_0 : \delta_{ij} = 0$, for $1 \leq i < j \leq N$. Applying the LM statistic to the above restrictions, one only needs to estimate the CCC model before diagnostic test, without estimating the complex multivariate model with time-varying conditional correlations.

As derived from Tse (2000), the LM statistic for testing the above null hypothesis can be calculated by the following formula:

$$\text{LMC} = \mathbf{\iota}' \hat{\mathbf{S}} (\hat{\mathbf{S}}' \hat{\mathbf{S}})^{-1} \hat{\mathbf{S}}' \mathbf{\iota}$$

where $\mathbf{\iota}$ is the $T \times 1$ column vector of ones and $\hat{\mathbf{S}}$ is $\mathbf{S}$ evaluated at $\hat{\theta}, \theta = (w_1, \alpha_1, \beta_1, w_2, \alpha_2, \beta_2, \cdots, \beta_N, \rho_{12}, \rho_{13}, \cdots, \rho_{N-1,N}, \delta_{12}, \cdots, \delta_{N-1,N})'$, $\hat{\theta}$ is the MLE (maximum likelihood estimator) of $\theta$ under $H_0$, $\mathbf{S}$ is the $T \times K$ matrix and its rows are the partial derivative
\[ \frac{\partial l_t}{\partial \theta'} \text{ for } t = 1, \cdots, T, \text{ and } K \text{ is the rank of the score vector. Under the usual regularity conditions this LMC statistic is asymptotically distributed as a } \chi^2_M, \quad M = N \times (N - 1)/2. \] By the above equation, the LMC statistic can also be calculated as \( T \cdot R^2 \), where \( R^2 \) is the uncentered coefficient of determination of the regression of \( \mathbf{l} \) on \( \hat{S} \).

### 4 Empirical Results

By assuming \( \varepsilon_t \) is conditionally multivariate-normally distributed, we apply QMLE (quasi maximum likelihood estimation) method to estimate the CCC model and BEKK model. The conditional log likelihood function under the BEKK model is

\[
L = \sum_{t=1}^{T} l_t(\theta), \quad l_t(\theta) = -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln |H_t(\theta)| - \frac{1}{2} \varepsilon_t(\theta)'H_t^{-1}\varepsilon_t(\theta)
\]

where all parameters have been combined into \( \theta' = (\text{vech}(\Gamma)', \text{vech}(C)', \text{vech}(A)', \text{vech}(B)') \), an \( m \times 1 \) parameter vector, and \( T \) is the sample size.

#### 4.1 Estimation Results of the CCC Model

For intraday returns of S&P 500 index futures, FT-SE 100 index futures, and Nikkei 225 index futures, a trivariate CCC model is fitted. The estimation results of CCC model as well as the computed LMC statistic are presented in Table 2.

The estimated correlations for pair-returns of S&P 500 index futures and FT-SE 100 index futures, S&P 500 index futures and Nikkei 225 index futures, as well as FT-SE 100 index futures and Nikkei 225 index futures, are also reported in Table 2. Significantly positive estimated correlations of each pair-returns reflect a significant, though low, degree of co-movement between every two of the three index futures returns. By the LMC statistic, it implies time-varying correlations among intraday returns to S&P 500
futures, FT-SE 100 futures, and Nikkei 225 futures. It shows that the correlations of index futures returns are time-varying.

4.2 Estimation Results of the BEKK Model

Based on the Lagrange multiplier statistic suggested by Tse (2000), we have found evidence against the assumption of constant conditional correlations. To ensure the estimated conditional-variance matrix to be always positive definite, we estimate the BEKK model to allow conditional correlation to be time-varying.

A trivariate BEKK model for intraday returns to S&P 500 futures, FT-SE 100 futures, and Nikkei 225 futures is estimated by QMLE. The estimation results are displayed in Table 3. To interpret the estimation results from a BEKK model, we write the full representation of the trivariate BEKK model below. Denote returns to S&P500 index futures, FT-SE 100 index futures, and Nikkei 225 index futures, as SP500, FTSE100, and NIKKEI225, respectively. Specify SP500, FTSE100, and NIKKEI225, as \( y_1, y_2, \) and \( y_3 \), respectively. For \( y_t = (y_{t1}, y_{t2}, y_{t3})' \), \( H_t \) denotes the conditional variance matrix of \( y_t \), \( H_t = [h_{ij}]_t, i, j = 1, 2, 3 \). The trivariate BEKK model is characterized as

\[
y_t = E(y_t | \Omega_{t-1}) + \varepsilon_t
\]

\[
E(y_t | \Omega_{t-1}) = K_0 + K_1 y_{t-1}
\]

\[
\text{Var}(\varepsilon_t | \Omega_{t-1}) = H_t
\]

\[
H_t = C'C + A' \varepsilon_{t-1} \varepsilon_{t-1}' A + B' H_{t-1} B
\]

\[
= \begin{pmatrix}
c_{11} & c_{12} & c_{13} \\
0 & c_{22} & c_{23} \\
0 & 0 & c_{33}
\end{pmatrix}
\begin{pmatrix}
c_{11} & c_{12} & c_{13} \\
0 & c_{22} & c_{23} \\
0 & 0 & c_{33}
\end{pmatrix}
\]
Manipulating the products of matrices, the $i$th conditional variance, $h_{ii}, t = 1, 2, 3$, can be written as

$$h_{11t} = c_{11}^2 + a_{11}^2 \varepsilon_{1,t-1}^2 + a_{21}^2 \varepsilon_{2,t-1}^2 + a_{31}^2 \varepsilon_{3,t-1}^2$$

$$+ 2a_{11}a_{21} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + 2a_{11}a_{31} \varepsilon_{1,t-1} \varepsilon_{3,t-1} + 2a_{21}a_{31} \varepsilon_{2,t-1} \varepsilon_{3,t-1}$$

$$+ b_{11}^2 h_{11,t-1} + b_{21}^2 h_{22,t-1} + b_{31}^2 h_{33,t-1} + 2b_{11}b_{21} h_{12,t-1} + 2b_{11}b_{31} h_{13,t-1} + 2b_{21}b_{31} h_{23,t-1}$$

$$h_{22t} = c_{12}^2 + c_{22}^2 + a_{12}^2 \varepsilon_{1,t-1}^2 + a_{22}^2 \varepsilon_{2,t-1}^2 + a_{32}^2 \varepsilon_{3,t-1}^2$$

$$+ 2a_{12}a_{22} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + 2a_{12}a_{32} \varepsilon_{1,t-1} \varepsilon_{3,t-1} + 2a_{22}a_{32} \varepsilon_{2,t-1} \varepsilon_{3,t-1}$$

$$+ b_{12}^2 h_{11,t-1} + b_{22}^2 h_{22,t-1} + b_{32}^2 h_{33,t-1} + 2b_{12}b_{22} h_{12,t-1} + 2b_{12}b_{32} h_{13,t-1} + 2b_{22}b_{32} h_{23,t-1}$$

$$h_{33t} = c_{13}^2 + c_{23}^2 + c_{33}^2 + a_{13}^2 \varepsilon_{1,t-1}^2 + a_{23}^2 \varepsilon_{2,t-1}^2 + a_{33}^2 \varepsilon_{3,t-1}^2$$

$$+ 2a_{13}a_{23} \varepsilon_{1,t-1} \varepsilon_{2,t-1} + 2a_{13}a_{33} \varepsilon_{1,t-1} \varepsilon_{3,t-1} + 2a_{23}a_{33} \varepsilon_{2,t-1} \varepsilon_{3,t-1}$$

$$+ b_{13}^2 h_{11,t-1} + b_{23}^2 h_{22,t-1} + b_{33}^2 h_{33,t-1} + 2b_{13}b_{23} h_{12,t-1} + 2b_{13}b_{33} h_{13,t-1} + 2b_{23}b_{33} h_{23,t-1}$$

The $ij$th covariance, $h_{ij}$, can be written as

$$h_{ij} = w_{ij} + \varepsilon_{p,t-1} \varepsilon_{q,t-1} + \text{Cov}_{t-1}(\varepsilon_{rt}, \varepsilon_{st})$$

where $\varepsilon_p, \varepsilon_q, \varepsilon_r,$ and $\varepsilon_s$ are the unexpected shocks to portfolios $p, q, r,$ and $s,$ and $w_{ij}$ is the $ij$th element of $C'C$ matrix. Portfolios $p, q, r,$ and $s$ are portfolios composed of
$y_1$, $y_2$, and $y_3$, whereas the weights in portfolios $p$ and $q$ are decided by the $i$th and $j$th columns of the $A$ matrix, and the weights in portfolios $r$ and $s$ are determined by the $i$th and $j$th columns of the $B$ matrix.

To test for model misspecification, we further examine the serial correlation in standardized residuals estimated from the BEKK model. The Ljung Box statistics for up to 15th-order serial correlation in $\varepsilon_{it}^2/h_{iit}$, for $i =$ SP500, FTSE100, and NIKKEI225, as well as $\varepsilon_{it}\varepsilon_{jt}/h_{ijt}, i \neq j$, are given in the bottom panel of Table 3. These Ljung-Box statistics fall below 25, the critical value at 5% significance level. It indicates that the estimated BEKK model captures the dynamics of conditional volatilities and covariances well.

As shown in Table 3, we find that most estimated coefficients in matrix $A$ are significant, except for the estimates of $a_{13}$ and $a_{31}$. It indicates that, in terms of volatility of S&P 500 index futures returns, the news impact from innovations of Nikkei 225 index futures (that is, $\varepsilon_{3,t-1}$) is weaker than innovation from FT-SE 100 index futures returns. By the estimated coefficients in matrix $B$, we find that most estimates, except for estimated $b_{11}$, $b_{12}$, $b_{21}$, $b_{22}$, and $b_{33}$, are not significant. If we characterize the volatility spillover among the three index futures markets, by means of conditional variance, we can conclude that volatility spillover only exists between markets of FT-SE 100 index futures and S&P 500 index futures. For the part of conditional covariances, since not all coefficients of matrices $A$ and $B$ are jointly insignificant, we can conclude that there does exist time-varying conditional covariances and conditional correlations. =

5 Conclusion

Employing the multivariate GARCH models that allow the conditional variance matrix to be time varying, this paper analyzes the dynamics of conditional volatilities and
conditional correlations across international index-futures markets. Existing empirical studies on the comovement across international asset markets generally impose ad hoc restrictions on the dynamics of conditional correlation without further testing. On the other hand, we utilize a formal statistic to test for the crucial assumption of constant correlation before choosing the preferred multivariate model.

Moreover, by the LM statistic testing for constant conditional correlation, the results suggest that conditional correlations are time-varying and it is invalid to use the CCC model of Bollerslev (1990) to estimate the system of multivariate GARCH for futures returns analyzed in this paper. To ensure a positive definite conditional variance matrix, the BEKK model is used to estimate time-varying conditional volatility and correlation of the data.

Although numerous studies have investigated the volatility spillover among international stock markets, the relation across international index-futures markets has been rarely studied. Through a more informative aspect, this paper studies the cross-volatility spillover across different index futures markets. Using intraday returns to the S&P 500 index futures, FT-SE 100 index futures, and Nikkei 225 index futures, the empirical results show that volatility spillover is significant between the CME and LIFFE futures markets, as well as between the LIFFE and OSE futures markets, however, insignificant between the OSE and CME futures markets. This is somewhat different with the conclusion found from most evidences on the underlying indexes: a low, but significant comovement across the S&P 500 index, FT-SE 100 index, and Nikkei 225 index.
References


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Table 1: Intraday Returns of Index Futures
(August 1988 – May 1999)

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500 (CME)</th>
<th>FT-SE 100 (LIFFE)</th>
<th>Nikkei 225 (OSE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.023</td>
<td>-0.006</td>
<td>-0.024</td>
</tr>
<tr>
<td>Median</td>
<td>0.026</td>
<td>0.000</td>
<td>-0.011</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.889</td>
<td>2.394</td>
<td>3.315</td>
</tr>
<tr>
<td>Minimum</td>
<td>-3.689</td>
<td>-1.860</td>
<td>-2.683</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.387</td>
<td>0.379</td>
<td>0.558</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.410</td>
<td>-0.008</td>
<td>0.005</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>12.627</td>
<td>5.164</td>
<td>5.815</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>9808.852</td>
<td>492.078</td>
<td>832.681</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>0.000</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>2522.000</td>
<td>2522.000</td>
<td>2522.000</td>
</tr>
<tr>
<td>ADF</td>
<td>-24.436</td>
<td>-23.183</td>
<td>-21.303</td>
</tr>
<tr>
<td>Phillips-Perron</td>
<td>-55.438</td>
<td>-51.814</td>
<td>-52.503</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>-0.083</td>
<td>-0.029</td>
<td>-0.046</td>
</tr>
<tr>
<td>$Q(20)$</td>
<td>72.777</td>
<td>23.373</td>
<td>39.794</td>
</tr>
<tr>
<td>$Q^2(20)$</td>
<td>361.370</td>
<td>355.310</td>
<td>555.400</td>
</tr>
</tbody>
</table>

Notes: The Jarque-Bera statistic is used to test for the null hypothesis of normal distribution, and values in the below parentheses indicate the p-value. 1%, 5%, and 10% critical values for the ADF and Phillips-Perron tests are -3.436, -2.864, and -2.568, respectively. $\rho_1$ implies the 1st-order autocorrelation coefficient. $Q(20)$ and $Q^2(20)$ represent the ljung-Box statistic for up to 15-th order serial correlation in the returns and squared returns, respectively.
Table 2: Estimation Results of CCC Multivariate GARCH(1,1)

\[ y_{it} = c_i + a_i y_{it-1} + b_i \varepsilon_{it-1} + \varepsilon_{it} \]
\[ h_{iit} = w_i + \alpha_{ii} \varepsilon_{it-1} + \beta_{ii} h_{iit-1}, \quad i = \text{SP500, FTSE100, NIKKEI225}. \]
\[ h_{ijt} = \rho_{ij} \sqrt{h_{iit}} \sqrt{h_{jjt}}, \quad i, j = \text{SP500, FTSE100, NIKKEI225}, i < j. \]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SP500</th>
<th>FTSE100</th>
<th>NIKKEI225</th>
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<td>(c_i)</td>
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<td>.003</td>
<td>-.005</td>
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<tr>
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<td>(4.278)</td>
<td>(.444)</td>
<td>(-.638)</td>
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<tr>
<td>(a_i)</td>
<td>.851</td>
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<td>-.432</td>
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<td></td>
<td>(13.696)</td>
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<td>(b_i)</td>
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<td>.375</td>
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<td></td>
<td>(-16.604)</td>
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<td>(w_i)</td>
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<td>.001</td>
<td>.003</td>
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<td></td>
<td>(2.426)</td>
<td>(2.370)</td>
<td>(1.908)</td>
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<tr>
<td>(\alpha_{ii})</td>
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<td>.039</td>
<td>.066</td>
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<tr>
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<td>(4.072)</td>
<td>(5.008)</td>
<td>(5.058)</td>
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<tr>
<td>(\beta_{ii})</td>
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<td>.954</td>
<td>.929</td>
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<tr>
<td></td>
<td>(97.210)</td>
<td>(106.974)</td>
<td>(72.759)</td>
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<tr>
<td>Q2(15)</td>
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<tr>
<td>(\rho_{12})</td>
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<tr>
<td>(\rho_{13})</td>
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<td>(\rho_{23})</td>
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<tr>
<td>LMC</td>
<td>23.83*</td>
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Note: Bollerslev-Wooldridge robust \(t\) statistics are in parentheses; \(Q2(15)\) represents Ljung-Box statistics for series correlation up to 15th order in the Ljung-Box statistic for serial correlation in \(\varepsilon_{it}^2/h_{iit}\) for up to the 15th lag. The LMC statistic represents the Lagrange multiplier statistic for constant correlation, as derived from Tse(2000). LMC \(\sim \chi^2_3\) and * for the LMC indicates statistical significance at the 5% level.
Table 3: Estimation Results of BEKK Multivariate GARCH(1,1)

\[ y_t = E(y_t | \Omega_{t-1}) + \epsilon_t \]
\[ E(y_t | \Omega_{t-1}) = K_0 + K_1 y_{t-1} \]
\[ \text{Var}(\epsilon_t | \Omega_{t-1}) = H_t \]

\[ H_t = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{pmatrix} \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{pmatrix} \]
\[ + \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \epsilon_{1,t-1}^2 & \epsilon_{1,t-1} \epsilon_{2,t-1} & \epsilon_{1,t-1} \epsilon_{3,t-1} \\ \epsilon_{2,t-1} \epsilon_{1,t-1} & \epsilon_{2,t-1}^2 & \epsilon_{2,t-1} \epsilon_{3,t-1} \\ \epsilon_{3,t-1} \epsilon_{1,t-1} & \epsilon_{3,t-1} \epsilon_{2,t-1} & \epsilon_{3,t-1}^2 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \]
\[ + \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} H_{t-1} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>Robust s.e.</th>
<th>Robust t</th>
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<tr>
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<tr>
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<td>.059</td>
<td>.508</td>
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<tr>
<td>( c_{23} )</td>
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<td>.137</td>
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<td>( c_{33} )</td>
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<td>( a_{33} )</td>
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<td>( b_{31} )</td>
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<td>.028</td>
<td>-.357</td>
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<tr>
<td>( b_{32} )</td>
<td>.181</td>
<td>.248</td>
<td>.730</td>
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<tr>
<td>( b_{33} )</td>
<td>.961</td>
<td>.008</td>
<td>120.125</td>
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Log likelihood = 1528.55

<table>
<thead>
<tr>
<th>Ljung-Box Q(15) of ( \epsilon_{it}^2/h_{it} ) and ( \epsilon_{it}\epsilon_{jt}/h_{jt} )</th>
<th>SP500</th>
<th>FTSE100</th>
<th>NIKKEI225</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP500</td>
<td>15.178</td>
<td>7.217</td>
<td>24.915</td>
</tr>
<tr>
<td>FTSE100</td>
<td>–</td>
<td>13.151</td>
<td>4.605</td>
</tr>
<tr>
<td>NIKKEI225</td>
<td>–</td>
<td>–</td>
<td>11.406</td>
</tr>
</tbody>
</table>

Note: Q(15) is the Ljung-Box statistic for up to 15-th order autocorrelation in \( \epsilon_{it}^2/h_{it} \) and \( \epsilon_{it}\epsilon_{jt}/h_{jt} \), for \( i = \text{SP500, FTSE100, and NIKKEI225, } i \neq j \). Robust s.e. and robust t denote Bollerslev-Wooldridge robust standard error and robust t statistic, respectively.