# A Solvency Based Multi-period Corporate Short-term Credit Risk Model

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**ABSTRACT**

In current short-term credit risk literature, liquidity crisis prediction model is the main research area. Within this research field, two major models can be classified--- "classical statistical models" and "stochastic intensity models". However few of them can obtain probability of insolvency and expected ratio of liquidity gap at the same time. In addition, within the above two frameworks, few studies apply stochastic solvency ratio models to predict corporate liquidity crisis. Basing upon the two significant characteristics of solvency ratio --"mean-reversion" and "non-negative value" and the concept of varying coefficient model, the study develops a multi-period corporate short-term credit risk model by constructing an "time-dependent stochastic solvency ratio model". It considers the impacts of industrial economic state changes on the structure of a firm's solvency ratio process (i.e. the parameters of the solvency ratio model) through incorporating information generated from a stochastic model of industrial economic state. The solvency ratio model can simulate many solvency ratio paths and then the solvency ratio distributions of each future period. With the information of solvency ratio distribution and the criteria of insolvency (when solvency ratio is less than one), we can obtain both the probability of a company's short-term credit risk and the expected ratio of liquidity gap in future periods. To perform a multi-period firm's short-term credit risk analysis, this solvency ratio model needs only publicly available information of corporate finance and the industrial economic state (i.e. the industrial cyclicality information).

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# I. Introduction

Due to the fast development of corporate financing techniques and applications of derivative instruments, corporate credit risk evaluation becomes an important issue. Corporate credit risk can be roughly classified into two categories---short-term and long-term credit risk. A company's short-term credit depends upon its capability to meet its payment obligation. While a company's long-term credit relies upon the growing potential of its future net worth (asset value minus debt). In current short-term credit risk literature, liquidity crisis prediction model is the major research area. However, within this research field, neither "classical statistical models" nor "stochastic intensity models" can obtain probability of liquidity risk and ratio of insufficient liquidity at the same time. It is therefore that this line of research has its limitations in credit rating and in the valuation of related derivates.

Classical statistical models focus on searching for accounting-based measures as the predicting variables to forecast corporate failures through statistical techniques. These models can be divided into three generation[1]. The first-generation classical statistical models based on multivariate discriminant analysis originated with Beaver (1966), Beaver (1968a,1968b), and Altman (1968). The second- generation classical statistical models are represented as Ohlson (1980) and based on qualitative- response models, such as logit and probit. Among these models, Altman (1968) use the Z-Score and Ohlson (1980) employ the O-Score to be the composite measures of bankruptcy probability. And their failure prediction models are both one-period models. However, the latest-generation models can extend to do multi-period failure prediction by using duration analysis. Duration analysis is to add "default time-related" variables (e.g. age) to be a time-dependent covariate in original one-period model (Lee and Urrutia, 1996; Shumway, 2001). The related duration models include Donald & Van de Ducht (1999), Kavvathas (2001), Chava and Jarrow(2002), and Hillegeist, Keating, Cram, and Lundstedt (2003).

Stochastic intensity models emphasize on estimating the frequency of default occurs in a short time (default intensity). Different from classic statistical models, stochastic

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[1] Recently, many other new modeling techniques are mentioned. They include recursive partitioning analysis (or tree classification), neural networks and genetic algorithms. These methods are sometimes categorized into methods of "inductive learning". We therefore do not include them in the classic models.

intensity models rely on exogenous information such as credit rating (Litterman and Iben, 1991; Jarrow, Lando and Turnbull, 1997) or other default-related proxies to estimate a firm's bankruptcy probability through stochastic models of these proxies. Since stochastic intensity models employ market rating information rather than a firm's financial information, it better match market current pricing of credit risk. These models include Litterman and Iben (1991), Madan and Unal (1995), Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Lando (1998), Duffie and Singleton (1999), Duffie (1998) and Duffee (1999). The latter covers Wilson (1997a and 1997b), Guption, Finger and Bhatia (1997), McQuown (1997), Crosbie (1999). .

Within the frameworks of the above two failure prediction models, few of them use stochastic models to estimate future liquidity measure, let alone employ stochastic solvency ratio model to do corporate failure prediction. In this study, we define a measure for corporate solvency ability (later denoted as solvency ratio) based on a firm's capability to fulfill its payment obligation. The solvency ratio is defined as the ratio of a firm's disposable cash to its net payment obligations in a period. When a firm's solvency ratio is less than one, it will fail to fulfill its payment obligation and will enter into a situation of corporate failure. The solvency ratio is conceptually simpler than other liquidity measures in that it not only directly reflects a firm's exact periodic liquidity but also provides a straight indicator of a firm's failure. Through our observations of solvency ratio, we discover that the behavior of solvency ratio exhibits some stochastic characteristics, such as mean-reversion and non-negative values. Examining historical solvency ratios of sampled companies, we find that they are weakly stationary and comply with and lognormal distributions[2]. In addition, these behaviors are influenced by changes of industrial economic states.

In order to obtain a firm's solvency distributions, this study starts in building a stochastic solvency ratio model that can appropriately describe aforementioned characteristics of the solvency ratio. With the aim of allowing the solvency ratio model to reflect the changes of industrial economic states, the solvency ratio model is set to be time dependent. That is the parameters of the stochastic solvency ratio model are time varying

[2] The examinations for solvency ratio's historical distributions and its weakly stationary test (Dickey-Fuller test) please refer Appendix I.

and alter according to the changes of the states of industrial economy. Adopting the concept of varying coefficient model[3], we construct a stochastic model of industrial economic state[4], using industrial cyclical factors as proxies for the industrial economic states. The information forecasted by the industrial economic state model is used as the base for adjusting the parameters of the time-dependent solvency ratio model, which we call a ***"Time-dependent stochastic solvency ratio model".*** With the solvency ratio model, we can generate a firm's solvency ratio distributions in future periods. Knowing a firm's multi-period solvency ratio distributions and the criteria of insolvency (when solvency ratio is less than one), we are able to assess a firm's short-term credit risk and calculate its probability of insolvency and expected liquidity gap ratio (that is one minus expected solvency ratio conditioning on insolvency).

Comparing with the classical statistical models, our model is different in three aspects. First, we define a new liquidity measure that can direct reflect a firm's solvency. Second, our model incorporates the state of industrial economy to reflect its impact on a firm's solvency. Third, our solvency model is a mean-reverting and non-negative stochastic model that matches the common firm management principles of maintaining an optimal (or appropriate) firm liquidity, neither too high nor too low.

Comparing with stochastic intensity models, our model is different in two features. First, we use the firm-liquidity related information to be a stochastic variable instead of exogenous information such as credit rating. Second, comparing to those structural-form related corporate failure prediction models, our solvency ratio model can direct measure the probability of a firm's insolvency rather than indirectly relies on the relationship between debt and asset value. Therefore our model can consider the probability of corporate failures due to liquidity crunch when the firm's asset value is still higher than liabilities[5].

In sum, solvency ratio has the following features: First, it is firm-liquidity related; second, it owns stochastic characteristics of mean-reversion and non-negative value; third,

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[3] It is usually applied in time-series sample data. Its characteristic is that it takes the changes of the model's coefficients as one or one more explainable variables in another regression model. And it makes the expected value of the coefficient be decided by a series of explaining variables.

[4] The characteristics of economic state (business cyclical factor) can be referred in appendix Ⅱ. We discover that its fluctuation obviously has the nature of mean-reversion.

[5] Structure form models employ asset value distribution or proxy such as distance-to-default to calculate probability of corporate failures (Merton 1976; KMV, 1998). The most recent and representative study is Duffie and Wang (2004)

it can extend to multi-period with future information of economic state forecasted by a stochastic model of industrial economic state; fourth, it direct provides a criterion for a firm's insolvency (when solvency ratio is less than one). The above four virtues are rarely simultaneously provided by other failure prediction models. Moreover, to perform our model needs only publicly available information (e.g. corporate financial data and industrial economic data). Our empirical analysis shows that the stochastic solvency ratio model is preliminarily supported.

The rest of the paper is divided into four sections：First, we construct a time-dependent stochastic solvency ratio model, including a discussion on the stochastic characteristics of solvency ratio, the time-dependent stochastic solvency ratio model, and a stochastic industrial economic state model; Second, we present a "Solvency Based Multi-period Corporate Short-term credit model"; Third, we empirically examine effectiveness of the model and apply our model results in pricing short-term credit securities. In the last section we conclude this study.

## II. The Time-Dependent Solvency Ratio Model

In this section, firstly, define solvency and solvency ratio. Second, we explore the characteristics of a firm's solvency ratio. Third, we construct our solvency ratio models that can appropriately describe solvency ratio's characteristics. Fourth, to consider future industrial economic state changes' impacts on a firm's solvency ratio, we introduce a stochastic model of industrial economic state. Finally, we introduce parameter estimation of the above two stochastic models.

### 1. Solvency and solvency Ratio

As introduced in previous section, the concept of a firm's solvency ability indicates a firm's capability to fulfill its payment obligation. The solvency ratio is therefore defined as the ratio of a firm's disposable cash to its net payment obligations in a period. To be more specific, solvency ratio can be defined as follows:

$$SR_t = \frac{Available\ Cash\ Bal._t}{Obligation\ Payments_t} = \frac{OCIF_t^{MA} + INCF_t + FNCF_t + C_{t-1} + SI_{t-1}}{OCOF_t^{MA} + Int_t + Tax_t^{MA} + DA_t} \quad (1)$$

The basic idea of solvency ratio defined in equation (1) is to distinguish a firm's operating, investing, and financing cash flows into a firm's "available cash balance" and "obligatory payments" individually. Regarding operating cash flow, there are three components belonging to obligatory payments in a short-term period: interest payment, income tax, and net decrease on account payables[6]. Concerning investing cash flow, it doesn't belong to an obligatory payment since investment activities in most cases are under management's discretion and do not have the compulsory payment nature. On the contrary, we can take net increase in investing cash flow as a component of solvency capacity. With respect to financing cash flow, it is the main source of obligatory payments including short-term and long-term debts payments. Considering both financing and refinancing activities, we view net decreases of total debts as obligatory payments and net increases of total debt as source of solvency capacity. In the following, we make a more detailed discussion. In equation (1),

- $OCIF_t^{MA}$: Indicating the "*adjusted* operating cash inflow" after a four-quarter moving average. As mentioned above, it is obviously that the items---interest payment, income tax, and net decrease on account payable are obligatory payments. Therefore they have to be added back to the original net operating cash flow. In addition, since we use quarterly data of a firm's cash flow in empirical analysis, we take the moving-average method to eliminate the effects caused by seasonality and management manipulation in credit policy. Based on the above, $OCIF_t^{MA}$ can be showed as equation (2):

$$OCIF_t^{MA} = MA(OCF_t + Int_t + Tax_t + NIAP_t) \tag{2}$$

In equation (2), $OCF_t$, $Int_t$ and $Tax_t$ stands for operating cash flow, interest expense and income tax paid during t period respectively. $NIAP_t$ indicates the net increases on account payables during t period.

- $OCOF_t^{MA}$: Standing for operating cash outflow after a four-quarter moving average. It primarily includes net decreases on account payables. With the same reasons for $OCIF_t^{MA}$, the four-quarter moving average is employed. As for other accrual

[6] We treat changes of accounting payables differently. A net decrease is classified into obligatory payments and a net increase is deemed as an added item to a firm's cash balances..

expenses, they have to be viewed as the adjustment for OCF because there is not any cash out flows.

- $INCF_t$: Representing investing net cash flow during time t. It is a component of solvency capacity for a firm[7].
- $FNCF_t$: Indicating financing net cash flow excluding cash dividend paid and debts' changes during time t. It is the net increase after equity and debt financing. It is also a component of solvency capacity for a firm.
- $C_{t-1}, SI_{t-1}$: Denoting the beginning balance of cash and short-term investments.
- $DA_t$: Indicating the amortization of debt principals. It is calculated from the net decreases of both total short-term debts and long-term debts in the period t.
- $Int_t, Tax_t$: Indicating the interest payments and tax expenses in the period t. It has to be noticed that income taxes is based on quarterly net income so that it is subject to the influence of seasonal effects. We therefore take moving average method for it.

From equation (1), we know that solvency ratio is mainly influenced by operating cash inflow and outflow, beginning solvency capacity (cash and short-term investments), and debt payment obligations. Because both a firm's operating performance and its solvency ability are primarily influenced by industrial economic state, there must exist a close relationship between solvency ratio and economic state.

## 2. The characteristics of firm's solvency ratio

Through our observations of solvency ratio, we discover that the behaviors of solvency ratio exhibit stochastic characteristics such as mean-reversion and non-negative values. Figure 1-4 display these characteristics. It is understandable that a normally managed firm tends to maintain its solvency ratio stable or stable with an upward trend due to the reasons of efficiently or time-optimally utilizing corporate funds and reducing agency problems.[8] In sum, based on the solvency ratio natures we found above, a "mean-reversion stochastic process" seems appropriate to depict solvency ratio's characteristics.

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[7] Regarding investing cash inflow, it primarily contains disposals of long-term investments and assets. It is not liquid and does not frequently happen. On the contrary, investing cash outflow is mainly capital expenditures. Since capital expenditures are necessary for maintaining stable growth, investing net cash flow is mostly negative. As a result, it has little influence on the cash inflow items.

[8] The case of time-optimally employing corporate funds can be included in financial slack phenomenon. The agency problem argument is discussed by cash flow hypothesis.

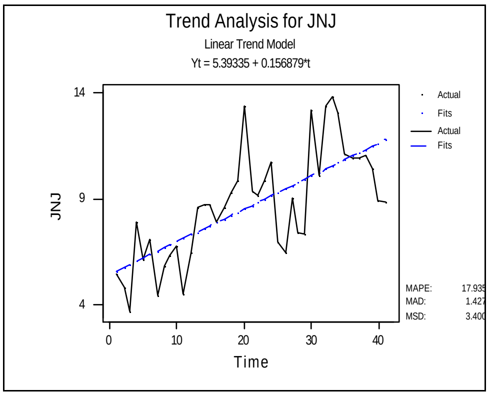


Figure1. JNJ's solvency ratio

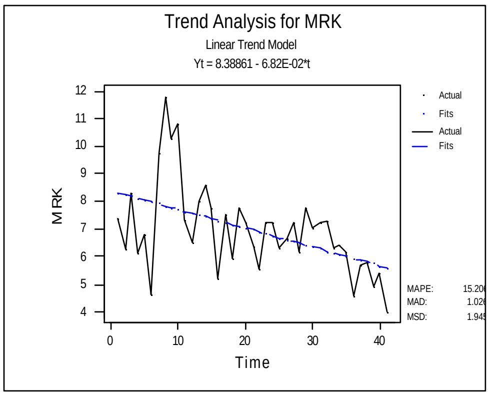


Figure2. MRK's solvency ratio

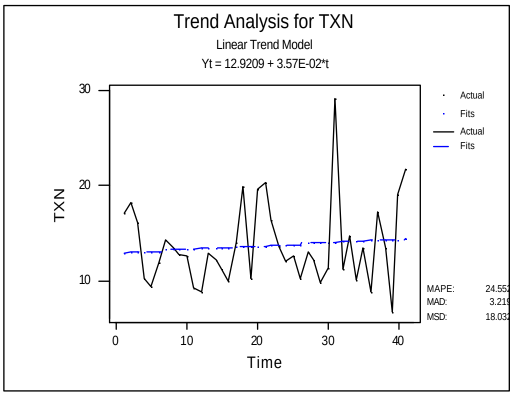


Figure3. TXN's solvency ratio

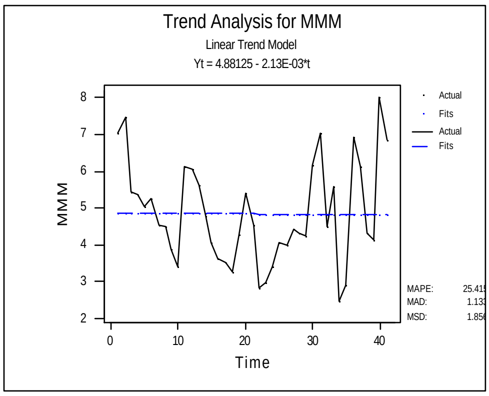


Figure4. MMM's solvency ratio

As mentioned in previous section, from historical solvency ratios of sampled companies, we find that they comply with lognormal distribution from goodness-of-fit tests. To simplify the model, we take natural log on solvency ratio (later denoted as lnSR) and then the lnSR will comply with normal distribution. Since taking natural log on solvency ratio is a monotonic transformation, it will not change solvency ratio's fundamental characteristics (e.g. its mean-reverting characteristic and trend characteristics). Comparing figures 1~4 with figures 5-8 we can find that the essential characteristics are still maintained.

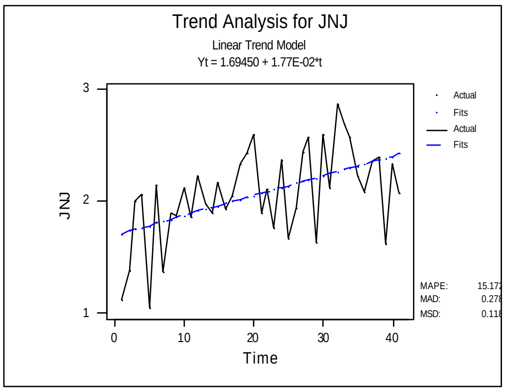


Figure 5. JNJ's lnSR trend analysis

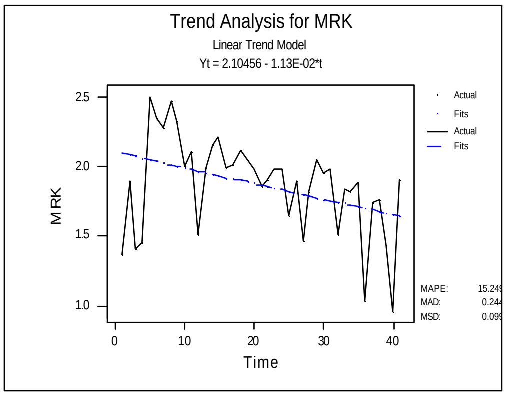


Figure 6. MRK's lnSR trend analysis

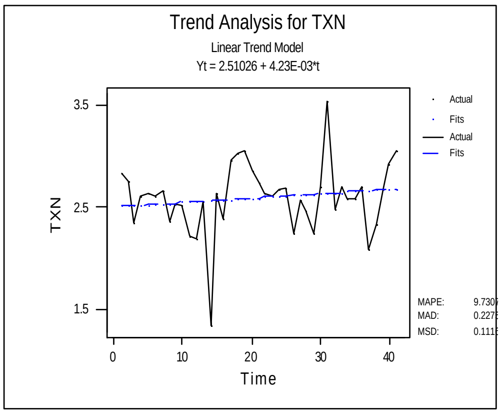


Figure 7. TXN's lnSR trend analysis

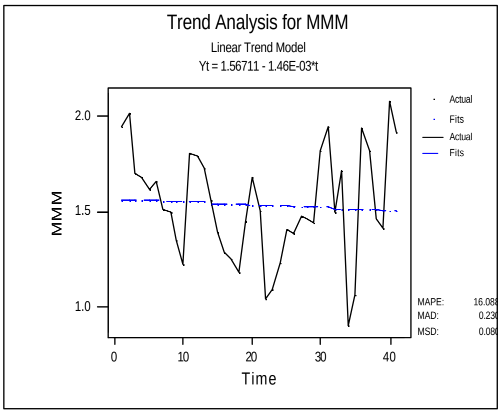


Figure 8. MMM's lnSR trend analysis

## 3. The setting of solvency ratio model

We adopt a time-dependent version of mean-reverting stochastic model because a firm's solvency ratio is severely influenced by changes of industrial economic states. Applying the concept of varying coefficient model, the parameters of the solvency ratio model are time varying to reflect the changes of future economic states. The expected future economic state changes are obtained from a stochastic industrial economic state model that will be discussed in the following section.

Basic Model Setting

From above discussion, we set our solvency ratio model as a "Time-dependent stochastic model". As previously stated the natural log value of solvency ratio (lnSR) is normal-distributed. Therefore, we can utilize a Gaussian process to describe the stochastic fluctuation of lnSR. We establish the "Time-dependent stochastic solvency ratio model" as equation (3):

$$d(\ln SR_t) = a(t)\cdot[b(t) - \ln SR_{t-1}]\cdot dt + s(t)\cdot dz\,,\quad dz = e\sqrt{dt}\,,\quad e \sim N(0,1) \qquad (3)$$

where,

$d(\ln SR_t)$: lnSR's term variation (or instantaneous changes in continuous time)

$a(t)$: lnSR's mean-reversion speed.

$b(t)$: lnSR's long-term average level

$s(t)$: standard deviation of lnSR's term variation, namely $\sqrt{Var(d(\ln SR_t))}$.

In order to simplify our model and without loss of generalization, we assume that a(t) in equation (3) is equal to a constant[9]. The a(t) stands for long-run mean-reversion speed of a firm's lnSR. While, b(t) and $s(t)$ represent for long-term average lnSR and standard deviation of the term changes of lnSR's respectively. They both vary over time according the changes of state of industrial economy. The basic concept of this idea is that industrial states influence a firm's operating performance and therefore its periodic liquidity. When the state of industrial economy changes, the structure (parameters) of the solvency model change accordingly. These three parameters can be estimated by the MLE (Maximum Likelihood Estimation) method and optimization technique. In the following, we introduce the stochastic economic state model and the parameter adjustment of solvency model based on the state model.

## 4. Stochastic economic state model and parameters' adjustments of the stochastic solvency ratio model

In this study, we use coincident indictors of the industry to which the firm belongs to proxy economic state and build a stochastic industrial economic state model as equation (4). With this state model, the economic states in the future periods can be estimated.

$$d(h_t) = a_h \cdot [b_h - h_{t-1}] \cdot dt + s_h \cdot dz \qquad (4)$$

where,

$h_t$ : the growth rate of industrial coincident indictor in time t.

$a_h$ : the mean-reverting speed of industrial coincident indictor's growth rate

$b_h$ : the long-term average of industrial coincident indictor's growth rate

$s_h$ : the standard deviation of the changes of industrial coincident factor's growth rate

The adjustment of the parameters b(t) and s(t) of the stochastic solvency ratio model in equation (3) are shown as bellow (see appendix III for more detailed discussion):

$$b(t) = b \cdot (1 + y_t^b) \qquad (5)$$

$$s(t) = s \cdot (1 + y_t^s) \qquad (6)$$

In equations (5) and (6),

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[9] The a(t) indicates the growth trend of individual enterprise. Since later in our model, the growth trend of individual firm can be reflected in the changes of long-term average level, setting a(t) to be constant is appropriate and without loss of generalization.

$b$ : the long-term average of lnSR estimated by MLE method.

$s$ :the standard deviation of lnSR's term changes estimated by MLE method.

Where[10]:

$$y_t^b = \left(\frac{a_0 + a_1 \cdot w(t)}{b}\right)^{\frac{1}{t}} - 1 \tag{7}$$

$$y_t^s = \left(\frac{a_0 + a_1 \cdot w(t)}{b}\right)^{\frac{1}{t}} - \left(\frac{a_0 + a_1 \cdot w(t-1)}{b}\right)^{\frac{1}{t-1}} \tag{8}$$

In equation (7) and (8),

$w(t)$ : the estimated state of industrial economy in future period t from stochastic industrial economic state model.

$a_0, a_1$ : the intercept and sensitivity of lnSR relative to industrial economic state (namely the regression coefficient of $\ln SR_t = a_0 + a_1 \cdot w(t) + e$ ).

In the above parameter adjustment methods, $a_1$ reflects the sensitivity of lnSR to state of industrial economy. The long-term average growth rate of industrial economic state ($b_h$) and the standard deviation of changes of industrial economic state's growth rate ($s_h$) are both constants.

## 5. Parameters estimation of stochastic economic state model

In equation (4), all the parameters, $a_h$, $b_h$ and $s_h$, are estimated by MLE method. We use the estimates from AR(1) method (Chen, 1996) as initial values for MLE optimization. Because the model of the state of the industrial economy is an O-U process, the conditional density of a specific future industrial economic state is a normal distribution with the mean and variance as follows:

$$E(h_s \mid h_t) = h_t \cdot e^{-a_h(s-t)} + b_h \cdot (1 - e^{-a_h(s-t)}) \tag{9}$$

[10] When employ leading indictor as proxy for industrial economic state, then

$$y_t^b = \left(\frac{a_0 + a_1 \cdot w(t-1)}{b}\right)^{\frac{1}{t}} - 1 \qquad y_t^s = \left(\frac{a_0 + a_1 \cdot w(t-1)}{b}\right)^{\frac{1}{t-1}} - \left(\frac{a_0 + a_1 \cdot w(t-2)}{b}\right)^{\frac{1}{t-2}}$$

$$Var(h_s | h_t) = \frac{s_h^{\ 2}[1 - e^{-2a_h(s-t)}]}{2a_h} \tag{10}$$

In equation (9) and (10), *s* indicates the observed time point in the future.

Moreover, the unconditional distribution of industrial economic state complies with $N(b_h, \frac{s_h^{\ 2}}{a_h})$.

We therefore introduce a likelihood function of the state variable of the industrial economy as follows:

$$L(a_h, b_h, s_h^{\ 2}; h_0, h_{t-1}) = f(h_{t=0}) \cdot \prod_{t=1}^{T} f(h_t | h_{t-1}) \tag{11}$$

$$\underset{a_h, b_h, s_h^{\ 2}}{Max} \ln L(a_h, b_h, s_h^{\ 2}; h_0, h_{t-1}) = \ln f(h_{t=0}) + \sum_{t=1}^{T} \ln f(h_t | h_{t-1}) \tag{12}$$

According to equation (12), we can estimate model's parameters by optimization technique and the initial value is estimated by AR(1) method.

Chen's estimate method (AR(1)) is to rewrite the equation (9) as a discrete autoregressive process for order as follows:

$$h(s) = h(t) \cdot e^{-a_h(s-t)} + b_h \cdot (1 - e^{-a_h(s-t)}) + x(s) \tag{13}$$

$$h_{t+\Delta t} = h_t \cdot e^{-a_h \Delta t} + b_h \cdot (1 - e^{-a_h \Delta t}) + x_{t+\Delta t} \tag{14}$$

Where the error term $x$ is normal distributed with mean 0 and variance as described in equation (10). And $\Delta t$ is a length of time interval. The AR(1) process allows $h_t$ to satisfy all three properties of the OU process, i.e., mean, variance, and white noise with normal density. Obtaining this exact form from discretization is essential for simplifying the estimation process of the parameters. Equation (14) can be written as the following regression model:

$$h_t = a + b \cdot h_{t-\Delta t} + e_t \tag{15}$$

where $a = b_h(1 - b)$, $b = e^{-a_h \Delta t}$, so all the three parameters can be solved from equation (15).

$$a_h = \frac{-Lnb}{\Delta t} \qquad b_h = \frac{a}{1-b_1} \qquad s_h^2 = \frac{2a_h \cdot MSE}{1-e^{-2a_h \Delta t}} \tag{16}$$

According to equation (16), we therefore obtain the initial values for the three parameters, $a_h$, $b_h$ and $s_h$, of the stochastic industrial economic state model.

### 6. Parameters estimation of stochastic solvency ratio model

For stochastic solvency ratio model, we also employ MLE optimization to estimate its parameters $a$, $b$ and $s$. Under the assumption that the parameters of industrial state model are fixed and the parameter adjustment method stated previously, equation (3) can be rewritten as equation (17):

$$d(\ln SR_t) = a \cdot [b \cdot (1+y_t^b) - \ln SR_{t-1}] \cdot dt + s \cdot (1+y_t^s) \cdot dz \tag{17}$$

## III. The Time Dependent Solvency Ratio Model and Multi-Period Short-term Credit Risk

A multi-period short-term credit risk model should be able to estimate a firm's multi-period probability of insolvency and expected liquidity gap ratio (i.e. the ratio liquidity gap to total obligatory payment). To achieve the goal, we need to know the firm's solvency ratio distributions in the future periods. With our "time-dependent stochastic solvency ratio model", we first simulate appropriate number of lnSR paths. And then we switch these paths back to "solvency ratio paths" by taking exponential transformation to obtain the firm's solvency ratio distributions in the future periods. With the solvency ratio distribution, the firm's multi-period short-term credit risk assessments can be done. A more detailed description of the above process is as follows.

Employing the solvency model to perform a multi-period short-term credit analysis of a firm, first one should choose an appropriate industrial indicator that can reflect the state of the specific industrial economy to which the firm belongs. For this indicator, we build a stochastic model of the state of the industrial economy. Using historical data and maximum likelihood estimation methodology, we can obtain the parameters of the state model. With the state model, we can get expected state value[11] of future periods. With the information of future state of the industrial economy, and employ equation (5)-(8), we can adjust the parameters of the solvency ratio model in future periods. Using the time-dependent solvency model, we can get one future solvency ratio path of a firm by simulating once according to equation (17). Repeating above process for N times, we can have a firm's N solvency ratio paths. Through a cross-sectional analysis in each period, we can obtain the firm's multi-period solvency ratio distributions. A firm is deemed as insolvency when solvency ratio is less than one. It can be illustrated as figure 9. In figure 9, the solvency ratio distribution complies with lognormal distribution as previously mentioned.

Since liquidity crisis occurs when the solvency ratio is less than one, we can calculate the probability of insolvency (latter denoted as PIS), expected liquidity ratio given insolvency (latter denoted as ELRGI), and the expected liquidity gap ratio (latter denoted as ELGR) from the future solvency ratio's distributions. PIS and ELGR can be showed as below:

$$\textit{Probability of Insolvency}(t) = \int_0^1 f(SR_t) \cdot d(SR_t) \tag{18}$$

$$\textit{Expected Liquidity Ratio Given Insolvency}(t) = \int_0^1 SR_t \cdot f(SR_t) \cdot d(SR_t) \tag{19}$$

$$\textit{Expected Liquidity}\ \text{Gap}\ \textit{Ratio}(t) = \int_0^1 (1 - SR_t) \cdot f(SR_t) \cdot d(SR_t) \tag{20}$$

In equation (18), probability of insolvency (PIS) is the area of solvency ratio's distribution when it is smaller than one. In equation (19), expected liquidity ratio given insolvency (ELRGI) is the expected value when solvency ratio is less than one in its future distributions. It means that how much can be recovered per dollar obligatory payments when insolvency. The expected liquidity gap ratio (ELGR) indicates the liquidity gap per dollar obligatory payments given insolvency. It is shows in equation (20).

Based upon these equations above, we can perform a short-term credit risk assessments for a firm in the future periods. In summary, the process of the solvency based multi-period short-term credit risk model can be illustrated as figure 10.

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[11] That is the $w(t)$ in equation (7) and (8).

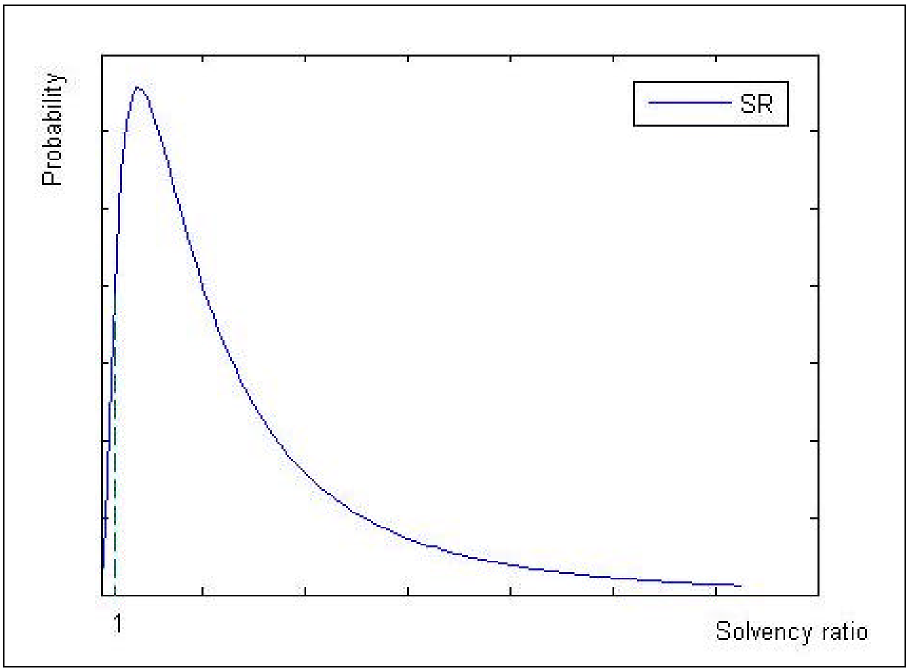


Figure 9. Illustration of Insolvency determination

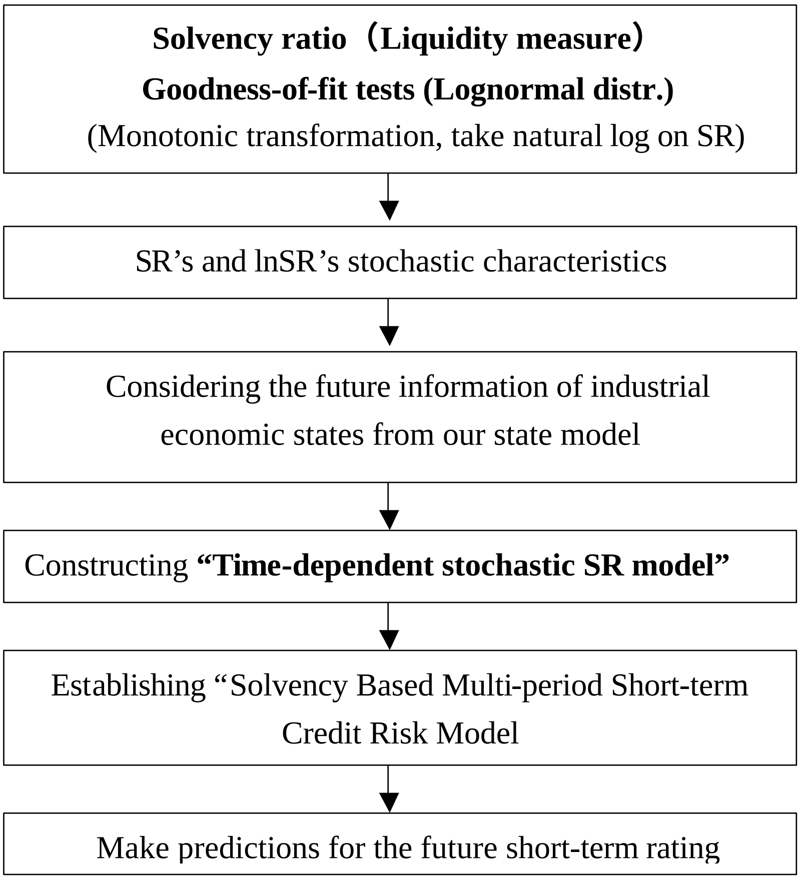


Figure 10. Flow Chart of the Solvency-Based Multi-period short-term credit Risk Model

## IV. Empirical Analysis

In this section, we use formerly developed models to empirically assess 54 firms with short-term credit rating from S&P 100 component stocks to examine the validity of the model. In the following, we introduce our data, the results of parameter estimation of industrial economic state model and stochastic solvency ratio model, the results of short-term credit risk analysis, and in the last the application of the models in pricing debt securities with short-term credit risk.

### 1. Data

The sample companies are from S&P 100 component stocks. The industry distribution of the sample companies is illustrated in table 1. All company related financial information is from COMPUSTAT database and credit rating information of sampled companies are from Standard and Poor's Corporation (later denoted as S&P). We employ three criteria in sample selection. First, we select sample companies with both S&P short- and long-term credit ratings. Second, since our model is not appropriate for analyzing financial firms, we exclude financial firm or firms operated like financial institutions[12]. Third, companies with missing data are also left out. The estimation period for parameters in our solvency ratio model is from 1994 to 2004 Q1, except for some three companies lacking data in the beginning or ending periods. The three companies are MDT, WY, CPB and their estimation period are 1996~2004Q1, 1994~2002Q1, and 1994~2001Q1 respectively. The sources and information of industrial state (industrial cyclical factors), including coincident and leading indicators, exhibit in table 3[13]. To sum up, all data sources are completely illustrated in table 2.

Table 1. The industrial categories' distribution of empirical sample

| **Industry** | Firms with Short-term Credit Rating |
|---|---|
| 1. Paper Product | 3 |
| 2. Oil and Gas | 3 |
| 3. Food & its Services | 7 |

[12] For some companies, though their SIC codes belong to manufacturing industries, their business nature are close to financial companies, such as General Motor, General Electronic, and Ford. Their ratios of debt to total asset are over 80%.

[13] The applicable proxies for industrial economic states in table 2 selected are according to the criteria of NBER.

| | |
|---|---|
| 4. Chemicals | 10 |
| 5. Metal | 3 |
| 6. Electronic (Computer) | 5 |
| 7. Semiconductors | 2 |
| 8. Aircrafts | 4 |
| 9. Surgical, Search, Detection | 4 |
| 10. Railroads &Transportation | 3 |
| 11. Communication | 2 |
| 12. Retails | 6 |
| 13. Services | 2 |
| **Total Sample Num.** | **54** |

Table 2. All related empirical data sources

| Items | Corporate financial data and ratings | Industrial economic state (business cyclical factors) |
|---|---|---|
| Sources | COMPUSTAT, S&P website | U.S. Department of Commerce, Datastream, SEMI |

Table 3: The applicable proxies for industrial economic state

| **Leading indictors** | sources | **Coincident indictors** | sources |
|---|---|---|---|
| 1. The ratio of unfilled orders-to-shipments of each industry | U.S. Department of Commerce | 1.each industrial sales revenues | U.S. Department of Commerce |
| 2. The change rate of each industrial new-orders | U.S. Department of Commerce | 2.the change rate of each industrial production | Datastream |
| 3. For specified industry (e.g. Semiconductor, DRAM) with compiled index or goods' price | SEMI, Bloomberg, Datastream | | |
| *. Data period : 1992-2002<br>**.The decisions of leading indictors or coincident indictors primarily depend on the business cyclical indictors selected and announced by The Conference Board | | | |

## 2. Parameters estimation of the stochastic model of solvency ratio and industrial economic state

For our stochastic solvency ratio model, we employ MLE optimization to estimate its parameters. Moreover, we also estimate the coefficients $(a_0, a_1)$ of the lnSR relative to industrial economic state by linear regression model. The above results are illustrated as table A4-1.

To adjust the parameters of our solvency ratio model, we have to know the industrial economic state model first. Here we select some business cyclical factors (industrial economic state) as shown in table 3 to be a proxy for industrial economic state. We also employ MLE optimization to estimate its parameters and the results are illustrated in table A5-1.

### 3. Empirical results of firm's short-term credit analysis

To examine the validity of our models, we estimate one year probability of insolvency (PIS) and expected liquidity gap ratio (ELGR) of each sample firm and convert them into corresponding one year long-term rating according to one-year ***average forward default rates*** provided by S&P (1981~2003). Since we are doing the short-term credit analysis, we use the correlation table of long- and short-term rating from S&P to translate a firm's one-year long-term rating to short-term rating. In the correlation table, a long-term rating may be converted to more than one short-term rating such as the corresponding short-term ratings of A+ long-term rating are A-1+ or A-1. To avoid subjective (or selective) bias in the transformation process from long-term rating to short-term rating, we exhibit two sets of empirical analysis results. When a sample firm's long-term rating has more than one corresponding short-term ratings, the first set exhibits the results we assign the firm the rating that are closest to the actual short-term rating of the firm (denoted as best choice situation). The other set contains the results we assign the firm the farthest rating to the current firm short-term rating (denoted as worst choice situation). Because the short-term ratings of our sample firms are all fall into the four ratings, A-1+, A-1, A-2, and A-3, we examine how well our models can correctly assign sample firms the four short-term rating groups. Since, the probability of insolvency short-term rating A-1 is around 0.05%, the PIS of A-1+ and A-1 are very close (see more details in appendix VI). So we combine these two ratings into one group and also examine the validity of our model again in a three rating group scenario. The summarized results are exhibited in table 4. A more detailed empirical credit analyses results are illustrated in table A7-1.

In table 4, the first column indicates the industry the sample firms belong to. The second and the third columns of table 4 show the results of the number of firms being correctly short-term ratings by our model under the best choice condition. Similarly, the fourth and the fifth columns of table 4 show the results under the worst choice condition. From table 4, we found that even in worst choice situation, our model still can classify

64.81% firms into correct rating group (in three group scenario). While the correct classification ratio is as high as 87.04% under best choice condition in the three-group scenario.

To investigate the robustness of previous empirical results, we employ multinomial logit model and the information generated from our model to classify sample firms into appropriate rating groups. We create a new liquidity proxy (latter called as PIS-ELGR factor) incorporating our model's one-year PIS and ELGR through factor analysis[14]. The classification results are shown in table 5. From last two columns of table 5, we find that correct classification is 59.3% for four short-term rating groups scenario and 77.8% for three groups scenario. Comparing the results with those of table 4, we find that they both fall into the intervals of correct classification rates between best choice and worst choice situations for four groups scenario and three groups scenario respectively[15]. Furthermore, we introduce a non-liquidity related variable, a firm's current D/E ratio, into multinomial logit model to do the short-term rating classification for sample firms. The current D/E is the ratio of debt's book value over equity's market value. The ratio includes long-term credit information of a firm. The classification results are 72.2% and 85.2% for four and three group scenarios respectively. These classification results are very close to the best choice situation of table 4. The robust investigations also show similar results to those of our models. According to empirical results illustrated in table 4 and 5, the models' effectiveness seems preliminarily supported by the empirical evidences. However, it is our models' limitation that they cannot consider all the uncertainty (e.g. suddenly changes). It could be improved by adding extra stochastic terms into our model, such as "jump diffusion model" to take care of more uncertainties.

[14] Due to PIS and ELGR are highly-correlated in our sample (the correlation coefficient is 0.997), we then create a new factor, called PIS-ELGR factor by factor analysis and it can explain 99.851% of the variances.

[15] That is for four group scenario, the result of multinomial logit model 59.3% fall between the interval of correction classification rates of previous best and worst choice situation results 51.85% and 74.07%.; while similarly 77.8% is between 64.81% and 87.04% for three group scenario.

Table 4. Short-term Rating Classification Results of the Solvency Ratio Model
(Basing upon S&P Average Forward Default Date :1981-2003)

| **Empirical Results of S&P 100 Components' Short-Term Credit Risk by Solvency Ratio Model (Sample=54)** | | | | |
|---|---|---|---|---|
| **Industry** | Best Choice Situation | | Worst Choice Situation | |
| | Four-Group | Three-Group | Four-Group | Three-Group |
| 1. Paper Product | 3 | 3 | 1 | 1 |
| 2. Oil and Gas | 3 | 3 | 3 | 3 |
| 3. Food & its Services | 4 | 5 | 3 | 4 |
| 4. Chemicals | 8 | 10 | 6 | 8 |
| 5. Metal | 1 | 1 | 0 | 0 |
| 6. Electronic(Computer) | 4 | 5 | 3 | 4 |
| 7. Semiconductors | 1 | 2 | 1 | 2 |
| 8. Aircrafts | 3 | 4 | 3 | 4 |
| 9. Surgical, Search, Detection | 2 | 2 | 2 | 2 |
| 10. Railroads &Transportation | 3 | 3 | 1 | 1 |
| 11. Communication | 1 | 1 | 1 | 1 |
| 12. Retails | 5 | 6 | 3 | 4 |
| 13. Services | 2 | 2 | 1 | 1 |
| Total | 40 | 47 | 28 | 35 |
| **correct classification rate** | **74.07%** | **87.04%** | **51.85%** | **64.81%** |

1. Four-group indicates the scenario classifying sample firms into short-term rating groups of A-1+, A-1, A-2, and A-3. Three-group denotes rating groups of A-1+ & A-1, A-2, A-3.
2. To obtain short-term rating of each sample firm, we first convert each sample firm' PIS and ELGR into corresponding one year long-term rating according to one-year ***average forward default rates*** provided by S&P (1981~2003). Then, we get a firm's short-term credit ratings through the "Correlation of Long- and Short-term Rating" table provided by S&P website.
3. When a sample firm's long-term rating has more than one corresponding short-term ratings, "best choice situation" indicates that we assign the firm the rating that are closest to the actual short-term rating of the firm and "worst choice situation" indicates that we assign the firm the farthest rating to the current firm short-term rating.

Table 5: Short-term Rating Classification of the Multinomial Logit Model

| **Empirical Results of S&P 100 Components' Short-Term Credit Risk by Solvency Ratio Model (Sample=54)** | | | | |
|---|---|---|---|---|
| **Item** | PIS-ELGR factor and D/E | | PIS-ELGR factor | |
| | Four Groups | Three Groups | Four Groups | Three Groups |
| Model fitting information (LR test: chi-square statistics) | 56.625*** | 45.457*** | 39.012*** | 34.895*** |
| Goodness of Fit (Deviance) (Ho: Multinomial dist.) | 1.000 | 1.000 | 0.996 | 0.955 |
| Pseudo R-square (Cox and Snell) | 0.650 | 0.569 | 0.514 | 0.476 |
| **Variables:(LR tests)** | | | | |
| ---PIS-ELGR | 21.626*** | 20.311*** | 39.012*** | 34.895*** |
| ---D/E | 17.613*** | 10.563*** | | |
| **Precise prediction rate** | **72.20%** | **85.20%** | **59.30%** | **77.80%** |
| ***: at a significant level of 1%. | | | | |

## 4. Application of the models in pricing debt securities with short-term credit risk

Our model employs solvency ratio model performs multi-period corporate short-term credit risk assessment. In this section we show the application of our model in the valuation of corporate issues such as commercial paper. Our models are especially useful in the valuation of revolving corporate issues since our models can provide forward probability of insolvency (PIS) and expected liquidity gap ratio (ELGR) of a firm. Knowing the forward PIS and ELGR of a firm, one can easily price the future issues of the firm. We use HPQ as an example and describe its multi-period distributions in figure 11 & 12. In the following, we show an more detailed example of applying the models to price a complicate debt issues collateralized by several firms' short-term credits. Regarding the relationship between firms, they are not cross-collateralized and do not have cross-default contracts.

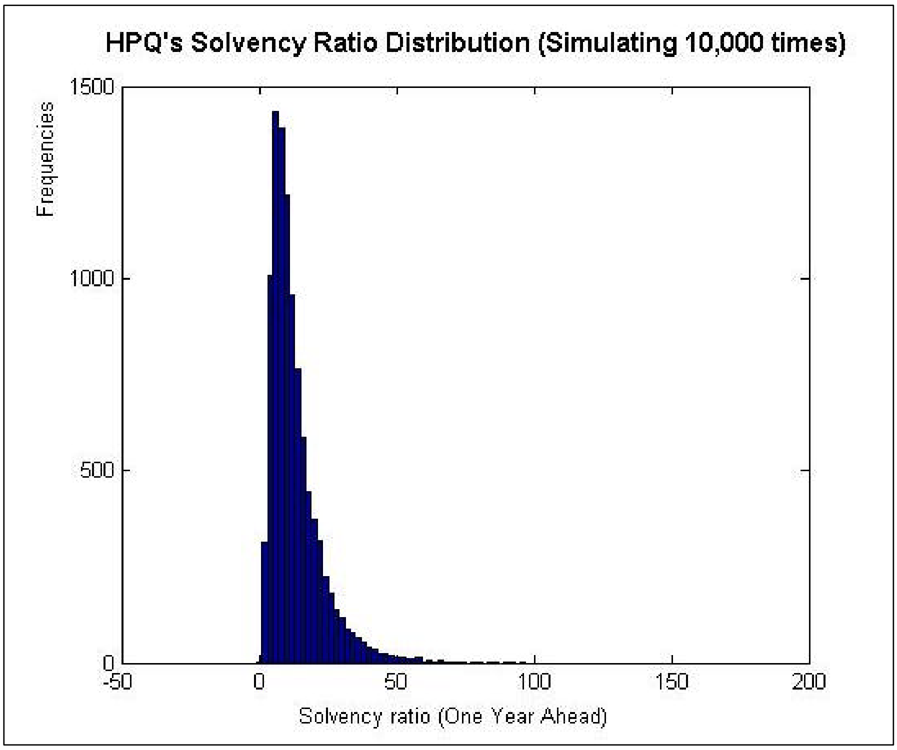


Figure 11. HPQ's One-Year Solvency Ratio Distribution

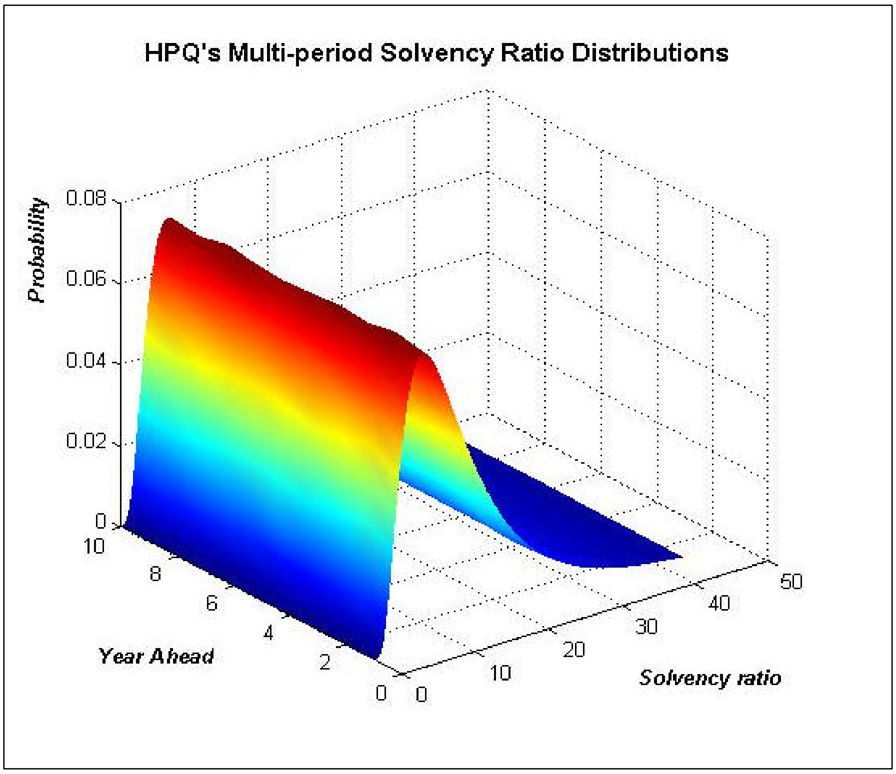


Figure 12. HPQ's Multi-period Solvency Ratio Distributions

Though most of sample firms' lnSR are normal-distributed by the normality test, the joint probability density function of several normal-distributed lnSRs is not guaranteed to comply a multivariate normal distribution. In order to simplify the example, we assume that multi-firm solvency ratio is multivariate normal-distributed. Under the above assumption a portfolio's probability of insolvency (PIS) can be estimated by considering correlation matrix. A detailed discussion is as follows.

In equation (21), we set the lnSR of the multi-firm credit portfolio follow *n*-dimensional (*n*-firms) multivariate normal distribution with mean vector $m$ and covariance matrix $\Sigma$ as $N_n(m, \Sigma)$. If $\Sigma$ is positive definite, the probability density function for lnSR is:

$$f_n(\ln SR) = \frac{\exp\left(-\frac{1}{2}(\ln SR - m)'\Sigma^{-1}(\ln SR - m)\right)}{\sqrt{(2p)^n \cdot |\Sigma|}} \quad (21)$$

According to equation (21), the PIS and ELGR of the multi-firm credit portfolio can be written as equation (22) and (23).

$$PIS_t^n = 1 - \int_0^\infty \int_0^\infty \ldots \int_0^\infty \int_0^\infty f_n(\ln SR) \cdot d(\ln SR_1) \ldots d(\ln SR_n) \quad (22)$$

In equation (22), the portfolio's PIS covers these situations for one firm's insolvency, two firms' insolvency,…..., N firms' insolvency; namely one minus the probability of no firm insolvency at the same time.

$$ELGR_t^n = \sum_{k=1}^{n} \sum_{t=1}^{C_t^n} \left( \prod_{j=n-i}^{0} \int_0^\infty \left( \prod_{i=1}^{n} \int_{-\infty}^{0} \left( \prod_{i=1}^{k} (1 - SR_{it}) \cdot f_n(\ln SR) \right) \prod_{i=1}^{n} d(\ln SR_i) \right) \prod_{j=n-i}^{0} d(\ln SR_j) \right) \quad (23)$$

In equation (23), the portfolio's ELGR can be calculated by considering all the above situations (one firm's insolvency,……, N firms' insolvency). k refers to the k-th situation and t stands for the t-th firm in k-th situation.

Based upon the above discussion, we can further evaluate the derivative with short-term credit risk such as Asset backed commercial paper (ABCP). For example, a firm issues a 3-month ABCP and its asset portfolio includes commercial papers of three obligors (data shown in table 6). Therefore to price the ABCP, we have to evaluate the three obligors' short-term credit risk (solvency ability) first. And then we can use J-T model to pricing the ABCP for considering both interest risk and obligors' short-term credit risk.

Table 6 introduces the ABCP components (three commercial papers). Table 7 shows the three-CP portfolio's "spot" and "forward" PIS & ELGR. And table 8 is the ABCP's pricing results under different portfolio compositions. We also evaluate the revolving ABCP based upon the forward PIS and ELGR of portfolio.

Table 6. The Basic Information of ABCP Components

| Ticker | HAL | MO | HPQ |
|---|---|---|---|
| Industry sector | Oil and Gas | Tobacco | Computer and OA. |
| Long-run rating | BBB | BBB+ | A- |
| Short-run rating | A-3 | A-2 | A-1 |
| Issue date | 2005/4/15 | 2005/4/15 | 2005/4/15 |
| Maturity date | 2005/7/15 | 2005/7/15 | 2005/7/15 |

Table 7. Portfolio Spot and Forward PIS and ELGR

| **Portfolio Spot and Forward PIS & ELGR** | | | | |
|---|---|---|---|---|
| Year | Portfolio PIS | Portfolio ELGR (equally-weighted) | Portfolio ELGR (40%-40%-20%) | Portfolio ELGR (80%-10%-10%) |
| Panel A. Portfolio Spot PIS & ELGR | | | | |
| 0.25 | 0.0156% | 0.0058% | 0.0070% | 0.0051% |
| 0.5 | 0.3127% | 0.0077% | 0.0092% | 0.0184% |
| 1 | 1.3097% | 0.0481% | 0.0577% | 0.0938% |
| Panel B. Portfolio Forward PIS & ELGR(Yearly) | | | | |
| 2 | 1.6168% | 0.0612% | 0.0735% | 0.1003% |
| 3 | 1.8502% | 0.0723% | 0.0867% | 0.1166% |
| 4 | 1.7166% | 0.0655% | 0.0786% | 0.1040% |
| 5 | 1.8195% | 0.0706% | 0.0847% | 0.1091% |
| 6 | 1.7745% | 0.0691% | 0.0830% | 0.1112% |
| 7 | 1.7690% | 0.0687% | 0.0824% | 0.1077% |
| 8 | 1.8164% | 0.0707% | 0.0849% | 0.1109% |
| 9 | 1.6856% | 0.0651% | 0.0781% | 0.1056% |
| 10 | 1.7115% | 0.0649% | 0.0779% | 0.1044% |

*Spot PIS & ELGR: the cumulative value from the static time point (t=0).
*Forward PIS & ELGR: conditioning on the prior time point. In the table, 0.25 & 0.5 stands for quarterly and semi-annual simulation (dt=1, condition on the prior quarter; dt=2, condition on the prior semiannual); 1~10 represents for yearly simulation (dt=4, condition on the prior year )

Table 8. An Example of ABCP Pricing

| ABCP Pricing | | | | | | | |
|---|---|---|---|---|---|---|---|
| Portfolio weight | Maturity | Yield | d(0,t) | ELRGI | V(0,t) | Par | Theoretical Price |
| I. 3-month Commercial Paper Portfolio | | | | | | | |
| **33.3%HAL, 33.3%MO, 33.3%HPQ** | 2005/7/15 | 0.0297 | 0.9712 | 0.999942 | 0.971100 | 100 | **97.1100** |
| **40%HAL, 40%MO, 20%HPQ** | 2005/7/15 | 0.0297 | 0.9712 | 0.999930 | 0.971089 | 100 | **97.1089** |
| **80%HAL, 10%MO, 10%HPQ** | 2005/7/15 | 0.0297 | 0.9712 | 0.999949 | 0.971107 | 100 | **97.1107** |
| II. 6-month Commercial Paper Portfolio | | | | | | | |
| **33.3%HAL, 33.3%MO, 33.3%HPQ** | 2005/10/15 | 0.0333 | 0.9678 | 0.999923 | 0.967699 | 100 | **96.7699** |
| **40%HAL, 40%MO, 20%HPQ** | 2005/10/15 | 0.0333 | 0.9678 | 0.999908 | 0.967684 | 100 | **96.7684** |
| **80%HAL, 10%MO, 10%HPQ** | 2005/10/15 | 0.0333 | 0.9678 | 0.999816 | 0.967595 | 100 | **96.7595** |

d(0,t):discounted factor of ABCP without insolvency risk
V(0,t):the value of ABCP with insolvency risk/ per face value
Par: par value of ABCP
Theoretical Price: the portfolio value with insolvency risk / per face value

## V. Conclusions

In current short-term credit risk literature, liquidity crisis prediction model is the main research area. Within this research field, two major models can be classified--- "classical statistical models" and "stochastic intensity models". However few of them can obtain probability of insolvency and ratio of liquidity gap at the same time. In addition, within the above two frameworks, few studies apply stochastic solvency ratio models to predict corporate liquidity crisis.

"Solvency Based Multi-period Short-term Credit Risk Model" constructed in this study provides a systematic measuring process of corporate short-term credit risk assessments. It starts from determining a firm's future solvency ratio distributions by our "Time-dependent stochastic solvency ratio model" and then employing the distributions to obtain multi-period short-term credit information, probability of insolvency and expected liquidity gap ratio.

In addition, the new models can be used to do multi-period short-term credit assessment without knowing a firms' credit rating. They straightly consider the firm's future solvency ratio to do multi-period liquidity risk analyses rather than conducting a backward solution from firm's credit rating to forecast a corporate failure. For both outside investors and people inside a firm, our study provides a multi-period short-term credit model that needs only publicly available information of both corporate finance and the industrial economic state (i.e. the industrial cyclical information). From the empirical results of this study, the effectiveness of the new models seems preliminarily supported.

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## Appendix I. Goodness-of-fit tests and Dickey-Fuller tests for firms' lnSR

Through our observations for solvency ratio, we can initially suppose that it will comply with non-central positive value distributions such as lognormal, non-central chi-squared distributions and so on. However, in order to test the solvency ratio's actual distribution, we firstly implement goodness-of-fit tests on solvency ratio and then discover the log-normal distribution for solvency ratio is preliminarily supported by statistical results. The test results will show as the following table A1-1. Moreover, solvency ratio's "non-negative value" characteristic also matches up with lognormal distribution.

Furthermore, we also employ Dickey-Fuller tests to prove the other stochastic characteristic, mean-reverting. The results also show in table A1-1.

In table A1-1, the results show that **90.74%** sample companies significantly reject the null hypothesis of random walk when significant level is 0.1; namely mean-reverting, and **87.04%** sample companies don't significantly reject the null hypothesis of normality when significant level is 0.01.

**Analysis：**

The sample companies which not significantly reject the null hypothesis of random walk are A-1+ or A-1 rated excluding Time-Warner(A-2); on the other part, the sample companies which significantly reject the null hypothesis of normality are also A-1+ or A-1 rated. The result implies that the higher rating companies have higher and stable solvency ratio. So our model may cause these companies underrated. However, the underrating doesn't reduce model effectiveness when our model result matches these companies' actual ratings.

Table A1-1. D-F tests and Normality tests for lnSR

| Short-term rating sample(Total=54) | | | | | |
|---|---|---|---|---|---|
| Ticker | **D-F test** | **Normality test** | Ticker | **D-F test** | **Normality test** |
| MMM | -3.944 | 0.71 | ROK | -3.516 | 0.33 |
| WY | -5.525 | 3.01 | CSC | -3.825 | 0.09 |
| IP | -4.347 | 2.81 | IBM | -5.118 | 11.12** |
| HAL | -4.236 | 0.86 | MSFT | -0.421* | 6.32 |
| SLB | -2.666 | 7.56 | INTC | -1.722* | 0.42 |
| XOM | -4.636 | 2.47 | TXN | -5.22 | 11.25** |
| SLE | -9.5 | 0.25 | BA | -1.955* | 7.75 |
| CPB | -7.182 | 4.72 | UTX | -3.008 | 9.6** |
| HNZ | -3.738 | 2.1 | HON | -3.781 | 2.76 |
| KO | -6.707 | 8.06 | GD | -2.791 | 4.24 |
| BUD | -6.29 | 16.09** | RTN | -3.136 | 4.79 |
| PEP | -4.481 | 0.65 | BAX | -7.82 | 4.01 |
| MO | -2.961 | 1.47 | MDT | -3.455 | 8.84 |
| DD | -3.196 | 3 | EK | -4.107 | 1.29 |
| DOW | -4.418 | 1.79 | BNI | -5.133 | 4.8 |
| BMY | -3.947 | 5.01 | NSC | -2.923 | 0.54 |
| JNJ | -5.231 | 2.81 | FDX | -6.238 | 5.09 |
| MRK | -4.242 | 4.19 | SBC | -3.805 | 2.79 |
| PFE | -2.66 | 2.06 | DIS | -5.87 | 9.21 |
| AMGN | -2.275* | 26.28** | HD | -2.86 | 1.57 |
| PG | -3.137 | 16.48** | MAY | -6.958 | 1.54 |
| AVP | -4.397 | 0.64 | WMT | -2.832 | 0.24 |
| CL | -4.331 | 5.5 | LTD | -3.361 | 4.32 |
| AA | -4.892 | 0.78 | RSH | -3.469 | 3.24 |
| G | -2.976 | 5.16 | MCD | -4.047 | 15.56** |
| BDK | -4.874 | 0.49 | TWX | -2.054* | 1.18 |
| HPQ | -4.034 | 2.02 | TYC | -4.599 | 2.09 |

*: Not significantly reject the null hypothesis of random walk when $\alpha$ =0.1.

**: Significantly reject the null hypothesis of Normality when $\alpha$ =0.01.

Note: In this study, we use Dickey-Fuller test (t-statistics) and Goodness of Fit test (chi-square statistics) to discover the characteristics of natural-log solvency ratio.

## Appendix II. The stochastic characteristics of industrial economic state

In this study, we use the change rate of seasonal-adjusted shipment of each industry to be the proxies for industrial economic state factors in the States market. The data Sources is <u>The Current Industrial Report, " Manufacturers' Shipments, Inventories and orders, 1992-2002</u>" by U.S. Department of Commence, 2003. The sample period is from 1992 to 2002 and the data-type is quarterly. Industries included are paper products, information & technology, petroleum, semiconductors, basic chemicals, computer and related products, and beverage & tobacco. Historical trend of each industrial economic state factor is illustrated in the following figures. From these figures, we can observe that there exists the phenomenon of mean-reversion in all industries. It is appropriate to use mean-reversion stochastic model to describe the behavior of the state of industrial economy.

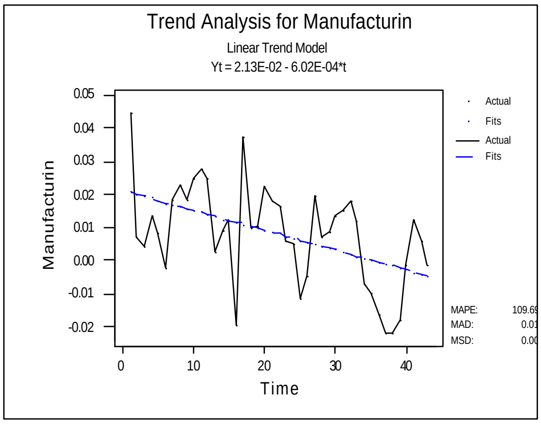



Figure A2-1. %Q Manufacturing shipments

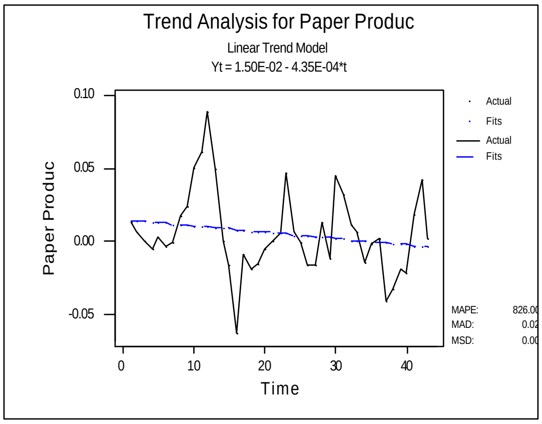



Figure A2-2. %Q Paper Products shipments

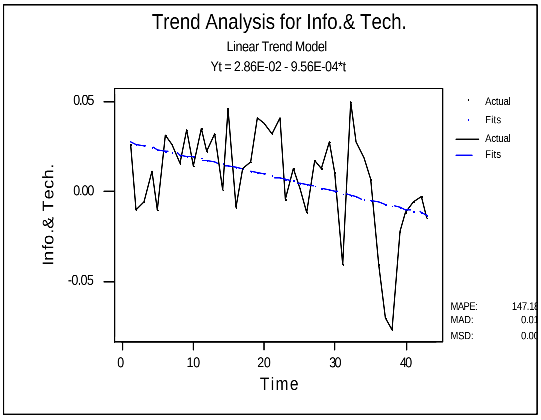



Figure A2-3. %Q Info.&Tech. shipments

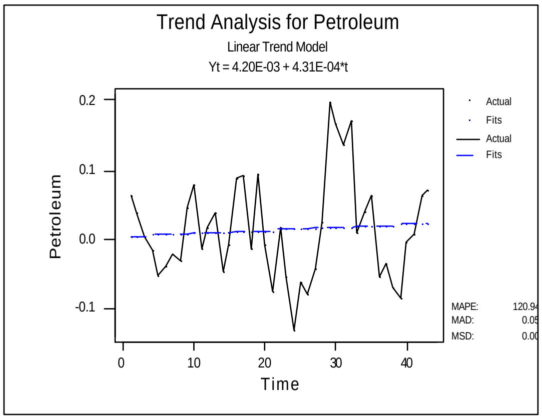
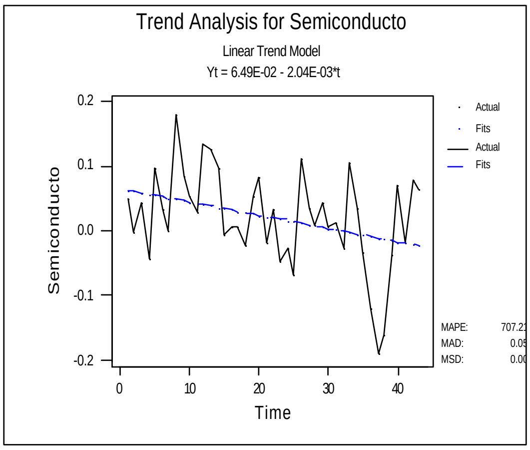



Figure A2-4. %Q Petroleum shipments



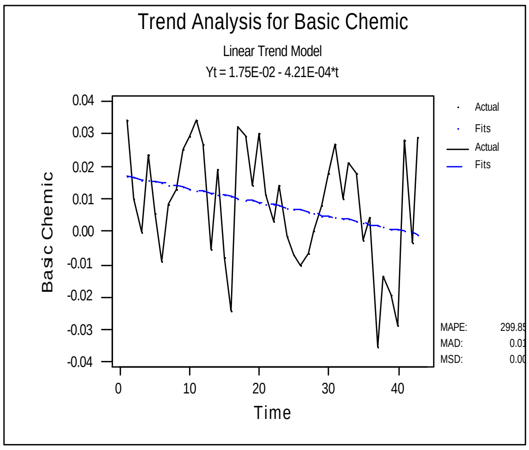
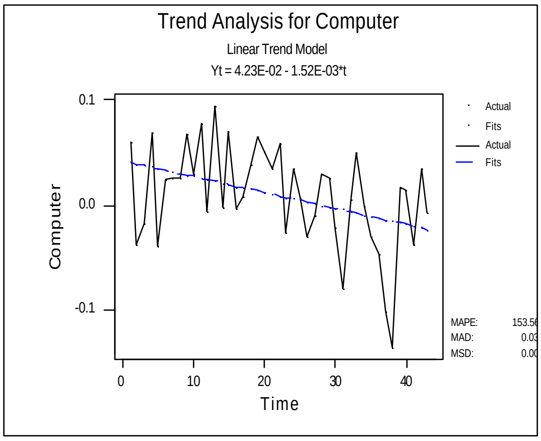




Figure A2-5. %Q Semiconductor shipments

Figure A2-6. %Q Basic Chemical shipments



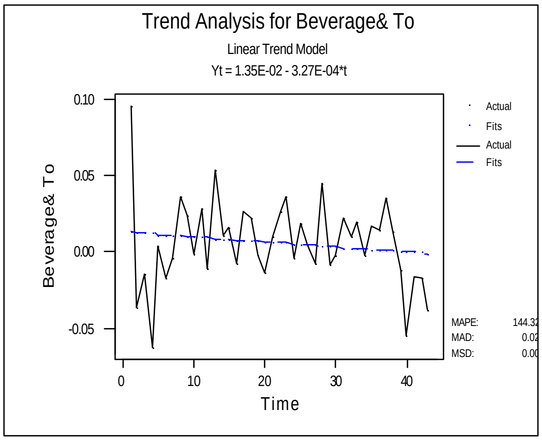




Figure A2-7. %Q Computer shipments

Figure A2-8. %Q Beverage & Tobacco shipments

## Appendix III. The method to estimate parameters of time-dependent stochastic solvency ratio model

In this study, our stochastic solvency ratio model can be showed as equation (A3-1):

$$d(\ln SR_t) = a(t)\cdot[b(t) - \ln SR_{t-1}]\cdot dt + s(t)\cdot dz\,,\quad dz = e\sqrt{dt}\,,\quad e \sim N(0,1) \qquad \text{(A3-1)}$$

where,

$d(\ln SR_t)$: lnSR's term variation (or instantaneous changes in continuous time)

*a(t)*: lnSR's mean-reversion speed.

*b(t)*: lnSR's long-term average level

$s(t)$: standard deviation of lnSR's term variation, namely $\sqrt{Var(d(\ln SR_t))}$.

To adjust the parameters of the cash flow model (b(t), σ(t)), we first estimate an initial parameter values through AR(1) method (Chen, 1996) and then employ Maximum likelihood estimation from historical data.

Let lnSR denote the natural value of solvency ratio and $w$ indicate the industrial economic state factor. The relationship between lnSR and industrial economic state factor ($w$) by the regression is shown in equation (A3-2):

$$\ln SR(t) = a_0 + a_1\cdot w(t) \qquad \text{(A3-2)}$$

We can then make time-varying adjustments on the long-term average of lnSR (b(t)) based on the future lnSR's growth rate relative to the initial value of b. According to equation (A3-3), we can further transfer the future lnSR's growth rate to the future industrial economic state indictor's growth rate:

$$\begin{aligned} b(t) &= b\cdot\left(1+[(\frac{\ln SR(t)}{\bar{c}_{long-term}})^{\frac{1}{t}}-1\right) \\ &= b\cdot\left(1+[(\frac{a_0+a_1\cdot w(t)}{b})^{\frac{1}{t}}-1]\right) \end{aligned} \qquad \text{(A3-3)}$$

In equation (A4-3), it has to be noticed that the parameter's, b, time-varying adjustment is quarter-based. We therefore use geometric mean method to make it.

To adjust the variances of lnSR (σ), first, we difference on the both sides of equation (A3-2) and then take variances as shown in equation (A3-4) and (A3-5).

$$\Delta\ln SR(t) = a_1\cdot\Delta w(t) \qquad \text{(A3-4)}$$

$$Var(\Delta \ln SR(t)) = a_1^2 \cdot Var(\Delta w(t)) \Rightarrow s_{\Delta(\ln SR_t)} = |a_1| \cdot s_{\Delta(w_t)} \qquad \text{(A3-5)}$$

According to equation (A3-4) and (A3-5), we can obtain equation (A3-6) and (A3-7) When $a_1$ is positive or negative respectively.

$$a_1 = \frac{\Delta \ln SR(t)}{\Delta w(t)} = \frac{s_{\Delta(\ln SR_t)}}{s_{\Delta(w_t)}} \qquad \text{(A3-6)}$$

$$-a_1 = -\frac{\Delta \ln SR(t)}{\Delta w(t)} = \frac{s_{\Delta(\ln SR_t)}}{s_{\Delta(w_t)}} \qquad \text{(A3-7)}$$

We can therefore conclude that the size of "effect on the changes of $\Delta \ln SR(t)$ caused by the changes of $\Delta w(t)$"(called *A event*) will be the same with the size of "effect on the $s_{\Delta(\ln SR_t)}$ caused by the changes of $s_{\Delta(w_t)}$"(called *B event*) when industrial economic state changes in the future.

From equations (A3-6) and (A3-6), we know that $s_{\Delta(\ln SR_t)}$ is a function of $s_{\Delta(w_t)}$ and $\Delta \ln SR(t)$ is a function of $\Delta w(t)$. And both two functions are related to the same base, namely $a_1$, which is the regressive coefficient in $\Delta \ln SR(t) = a_1 \cdot \Delta w(t)$. Therefore according to the concept of ***varying coefficient model***, the effects on *A event* and *B event* will be the same by $a_1$ when the industrial economic state changes in the future ($\Delta w(t)$, $s_{\Delta(w_t)}$). As a result, we can make adjustments on the variances of cash flow model ($\sigma$) by using *A event* instead of *B event*. In the following, we will infer the *A event*'s effect firstly, then apply the result to *B event* and at last we can conclude the adjustment methods of the variances of lnSR($\sigma$):

**Inferences:**

When the industrial economic state indictor is $w(t-1)$ in the future time t-1, we can obtain the adjustment effect reflecting in the change in the long-term average lnSR (b) according to equation (A3-3):

$$\frac{b(t-1)-b}{b}=\left(\frac{a_0+a_1\cdot w(t-1)}{b}\right)^{\frac{1}{t-1}}-1 \tag{A3-8}$$

When the industrial economic state factor is $w(t)$ in the future time t, we can get the adjustment effect of reflecting on the long-term average lnSR (b) according to equation (A3-3):

$$\frac{b(t)-b}{b}=\left(\frac{a_0+a_1\cdot w(t)}{b}\right)^{\frac{1}{t}}-1 \tag{A3-9}$$

We let equation (A3-9) minus equation (A3-8) and then get the influencing amount of *A event*:

$$b\cdot\left(\left(\frac{a_0+a_1\cdot w(t)}{b}\right)^{\frac{1}{t}}-\left(\frac{a_0+a_1\cdot w(t-1)}{b}\right)^{\frac{1}{t-1}}\right) \tag{A3-10}$$

Therefore the percentage influencing size of *A event* from time t-1 to time t can be shown as the equation (A3-11) :

$$\left(\frac{a_0+a_1\cdot w(t)}{b}\right)^{\frac{1}{t}}-\left(\frac{a_0+a_1\cdot w(t-1)}{b}\right)^{\frac{1}{t-1}} \tag{A3-11}$$

Because the influencing effects for A event and B event are the same when applying in the parameter's, $s$ , adjustments. We therefore illustrate the time-varying $s$ according to equation (A3-11) in the following.

$$s_{dC_t}(t)=s_{dC}\cdot\left(1+\left(\frac{a_0+a_1\cdot w(t)}{b}\right)^{\frac{1}{t}}-\left(\frac{a_0+a_1\cdot w(t-1)}{b}\right)^{\frac{1}{t-1}}\right) \tag{A3-12}$$

In equation (A3-12), t must be larger than one.

## Appendix IV. Parameter estimation of solvency ratio model by MLE optimization

Table A4-1. The Parameters' estimation of solvency ratio model

| Parameters Estimation for Solvency Ratio Model | | | | | | | |
|---|---|---|---|---|---|---|---|
| Item | Ticker | *a* | *b* | **σ** | Fval. | $\alpha_0$ | $\alpha_1$ |
| | | | I. Paper & Wood Industry | | | | |
| 1 | MMM | **0.8156** | **1.5388** | **0.3580** | -2.025 | 3.3244 | -0.0147 |
| | | (0.0286) | (0.0004) | (0.0068) | | | |
| 2 | WY* | **1.5090** | **0.9830** | **0.6770** | -15.877 | 0.7871 | 0.0007 |
| | | (0.7057) | (0.0993) | (0.1153) | | | |
| 3 | IP | **0.8274** | **1.3894** | **0.7255** | -30.799 | -1.4836 | 0.0186 |
| | | (0.0325) | (0.0013) | (0.0148) | | | |
| | | | II. Oil & Gas Industry | | | | |
| 4 | HAL | **0.8340** | **1.5137** | **0.8223** | -35.830 | 2.7356 | -0.0094 |
| | | (0.0307) | (0.0011) | (0.0162) | | | |
| 5 | SLB | **0.2618** | **1.6183** | **0.3721** | -13.302 | 2.5385 | -0.0054 |
| | | (0.0060) | (0.0016) | (0.0042) | | | |
| 6 | XOM | **1.2573** | **1.2394** | **0.3951** | 0.143 | 1.3068 | -0.0007 |
| | | (0.0675) | (0.0002) | (0.0115) | | | |
| | | | III. Foods & Tobacco Industry | | | | |
| 7 | SLE* | **1.9065** | **1.2272** | **0.8356** | -23.139 | 0.6010 | 0.0045 |
| | | (0.7522) | (0.0327) | (0.1319) | | | |
| 8 | CPB* | **1.9489** | **0.9649** | **0.5896** | -7.164 | 2.7984 | -0.0153 |
| | | (0.5866) | (0.0140) | (0.0965) | | | |
| 9 | HNZ | **0.9618** | **1.0197** | **0.5469** | -16.798 | 0.8653 | 0.0006 |
| | | (0.0403) | (0.0007) | (0.0123) | | | |
| 10 | KO | **1.4067** | **1.4962** | **0.4665** | -4.821 | 2.7436 | -0.0107 |
| | | (0.1194) | (0.0011) | (0.0201) | | | |
| 11 | BUD* | **1.5226** | **1.0316** | **0.4566** | -1.612 | 0.8205 | 0.0019 |
| | | (0.7073) | (0.0485) | (0.0612) | | | |
| 12 | PEP | **1.1471** | **1.2421** | **0.6861** | -23.927 | 4.2765 | -0.0247 |
| | | (0.0539) | (0.0001) | (0.0177) | | | |
| 13 | MO | **0.5102** | **0.9912** | **0.4325** | -14.805 | -0.2291 | 0.0097 |
| | | (0.0132) | (0.0006) | (0.0060) | | | |
| | | | IV. Chemicals Industry | | | | |
| 14 | DD | **1.2329** | **1.5137** | **0.7524** | -22.748 | 0.7492 | 0.0062 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | (0.0658) | (0.0006) | (0.0218) | | | |
| 15 | DOW | **0.9228** | **1.5188** | **0.7030** | -28.073 | 2.4454 | -0.0080 |
| | | (0.0387) | (0.0011) | (0.0156) | | | |
| 16 | BMY | **0.2970** | **2.4823** | **0.3059** | -4.576 | 4.1994 | -0.0131 |
| | | (0.0061) | (0.0003) | (0.0034) | | | |
| 17 | JNJ | **1.7221** | **2.0783** | **0.7221** | -19.171 | -0.7602 | 0.0213 |
| | | (0.1567) | (0.0000) | (0.0351) | | | |
| 18 | MRK | **1.0411** | **1.8735** | **0.4851** | -11.169 | 2.3502 | -0.0033 |
| | | (0.0435) | (0.0000) | (0.0112) | | | |
| 19 | PFE | **0.4226** | **2.4506** | **0.3754** | -10.331 | 0.6444 | 0.0131 |
| | | (0.0097) | (0.0003) | (0.0047) | | | |
| 20 | AMGN | **0.6888** | **2.7307** | **0.5046** | -18.096 | 1.5724 | 0.0091 |
| | | (0.0470) | (0.0041) | (0.0136) | | | |
| 21 | MEDI | **1.0927** | **3.5203** | **1.1617** | -46.255 | 5.8592 | -0.0165 |
| | | (0.0595) | (0.0022) | (0.0326) | | | |
| 22 | PG | **0.3729** | **1.2199** | **0.3567** | -9.403 | 2.4045 | -0.0101 |
| | | (0.0106) | (0.0016) | (0.0047) | | | |
| 23 | AVP* | **1.2916** | **1.2916** | **0.6920** | -25.636 | 2.4153 | -0.0092 |
| | | (0.4801) | (0.0431) | (0.1248) | | | |
| 24 | CL | **1.3792** | **1.2771** | **0.4511** | -3.772 | 2.3239 | -0.0077 |
| | | (0.0859) | (0.0003) | (0.0152) | | | |
| V. Primary Metal Industry | | | | | | | |
| 25 | ATI | **0.5763** | **1.2933** | **0.7431** | -35.851 | 4.7593 | -0.0285 |
| | | (0.0153) | (0.0005) | (0.01090) | | | |
| 26 | AA | **1.3999** | **1.5410** | **0.9220** | -32.831 | 0.5595 | 0.0083 |
| | | (0.0897) | (0.0006) | (0.0319) | | | |
| 27 | G | **0.5272** | **1.1875** | **0.5231** | -22.303 | -0.8536 | 0.0141 |
| | | (0.0340) | (0.0002) | (0.0142) | | | |
| 28 | BDK* | **1.2298** | **1.1101** | **1.0630** | -41.184 | -0.4891 | 0.0100 |
| | | (0.4448) | (0.0313) | (0.1396) | | | |
| VI. Computer Related Industry | | | | | | | |
| 29 | HPQ | **0.9418** | **2.3327** | **0.9206** | -38.853 | 1.5439 | 0.0043 |
| | | (0.0360) | (0.0004) | (0.0194) | | | |
| 30 | ROK | **0.7071** | **1.4717** | **0.5319** | -19.965 | 2.1862 | -0.0051 |
| | | (0.0224) | (0.0007) | (0.0091) | | | |
| 31 | CSC | **0.8933** | **1.5433** | **0.6877** | -26.946 | -0.3386 | 0.0122 |
| | | (0.0324) | (0.0003) | (0.0138) | | | |
| 32 | IBM | **0.8668** | **1.1458** | **0.2998** | 6.029 | 2.4073 | -0.0080 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | (0.0307) | (0.0001) | (0.0059) | | | |
| 33 | LU | **0.8928** | **1.5887** | **1.0704** | -29.176 | 2.3959 | -0.0064 |
| | | (0.0397) | (0.0031) | (0.0244) | | | |
| 34 | MSFT | **0.0409** | **2.8312** | **0.1413** | 27.037 | 4.1831 | -0.0057 |
| | | (0.0404) | (0.3145) | (0.0563) | | | |
| 35 | ORCL | **0.3156** | **2.7033** | **0.3007** | -3.371 | 3.4191 | -0.0050 |
| | | (0.0066) | (0.0003) | (0.0034) | | | |
| 36 | UIS | **1.0561** | **1.6960** | **0.8223** | -32.595 | 3.2232 | -0.0101 |
| | | (0.0491) | (0.0010) | (0.0205) | | | |
| VII. Semiconductor Industry | | | | | | | |
| 37 | INTC | **0.1967** | **2.7165** | **0.3720** | -14.644 | 2.6737 | -0.0008 |
| | | (0.0041) | (0.0018) | (0.0039) | | | |
| 38 | TXN | **1.6944** | **2.5987** | **0.6201** | -13.222 | 2.9838 | -0.0017 |
| | | (0.1567) | (0.0005) | (0.0304) | | | |
| VIII. Aircraft & Transportation Industry | | | | | | | |
| 39 | BA | **0.2315** | **2.2561** | **0.5298** | -28.414 | 3.4609 | -0.0101 |
| | | (0.0043) | (0.0002) | (0.0056) | | | |
| 40 | UTX | **0.6515** | **1.7058** | **0.4304** | -12.196 | 1.1723 | 0.0046 |
| | | (0.0185) | (0.0002) | (0.0068) | | | |
| 41 | HON | **0.7352** | **1.9796** | **0.7677** | -34.556 | 2.2137 | -0.0036 |
| | | (0.0246) | (0.0012) | (0.0137) | | | |
| 42 | GD | **0.1379** | **2.0024** | **0.3954** | -18.429 | 4.6119 | -0.0212 |
| | | (0.0030) | (0.0037) | (0.0039) | | | |
| IX. Surgical, Medical, Detection Electron. Industry | | | | | | | |
| 43 | RTN | **0.5516** | **0.8199** | **0.6411** | -30.219 | 4.3296 | -0.0259 |
| | | (0.0141) | (0.0001) | (0.0091) | | | |
| 44 | BAX* | **1.9780** | **0.9901** | **0.6023** | -9.087 | 0.7594 | 0.0016 |
| | | (0.8504) | (0.0297) | (0.0998) | | | |
| 45 | MDT | **0.7478** | **1.9998** | **0.4544** | -9.839 | 4.3918 | -0.0175 |
| | | (0.0232) | (0.0003) | (0.0079) | | | |
| 46 | EK | **0.9171** | **0.8279** | **0.4240** | -7.427 | 3.6449 | -0.0208 |
| | | (0.0336) | (0.0000) | (0.0086) | | | |
| X. Railroad & Transportation Industry | | | | | | | |
| 47 | BNI | **1.6937** | **0.6255** | **0.3308** | 12.532 | 1.3889 | -0.0053 |
| | | (0.1490) | (0.0000) | (0.0156) | | | |
| 48 | NSC | **0.4447** | **0.9198** | **0.3260** | -4.382 | 3.5776 | -0.0183 |
| | | (0.0108) | (0.0005) | (0.0043) | | | |
| 49 | FDX* | **1.9034** | **1.4403** | **0.9915** | -29.542 | 1.7067 | -0.0023 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | (0.7497) | (0.0130) | (0.1756) | | | |
| XI. Communication Industry | | | | | | | |
| 50 | NXTL | **0.4745** | **1.6846** | **0.6478** | -25.872 | 0.1377 | 0.0058 |
| | | (0.0113) | (0.0004) | (0.0086) | | | |
| 51 | T | **1.8623** | **1.1588** | **1.0893** | -34.587 | 0.9963 | 0.0006 |
| | | (0.2029) | (0.0003) | (0.0629) | | | |
| 52 | SBC | **0.4775** | **0.9562** | **0.3203** | -3.076 | 1.1664 | -0.0011 |
| | | (0.0134) | (0.0009) | (0.0045) | | | |
| 53 | VZ* | **1.7325** | **0.9065** | **0.5097** | -5.588 | 0.7572 | 0.0006 |
| | | (0.3643) | (0.0053) | (0.0582) | | | |
| 54 | CCU* | **1.9246** | **0.9005** | **1.0179** | -31.541 | 1.1970 | -0.0009 |
| | | (0.8440) | (0.0055) | (0.1773) | | | |
| 55 | DIS | **1.3030** | **1.0610** | **0.4816** | -7.398 | 1.6028 | -0.0023 |
| | | (0.0778) | (0.0004) | (0.0154) | | | |
| XII. Retails Industry | | | | | | | |
| 56 | WMB | **1.3247** | **1.1650** | **0.9757** | -36.075 | 0.1907 | 0.0064 |
| | | (0.0759) | (0.0003) | (0.0305) | | | |
| 57 | AES | **0.3313** | **1.5187** | **0.3427** | -8.561 | 3.4091 | -0.0119 |
| | | (0.0077) | (0.0009) | (0.0041) | | | |
| 58 | HD | **0.4023** | **1.4595** | **0.4397** | -13.747 | -0.3396 | 0.0107 |
| | | (0.0089) | (0.0003) | (0.0054) | | | |
| 59 | MAY* | **1.6269** | **0.7627** | **0.4706** | 1.878 | 0.7010 | 0.0000 |
| | | (0.6131) | (0.0292) | (0.0020) | | | |
| 60 | S | **0.6636** | **1.0913** | **0.6335** | -27.838 | -0.6497 | 0.0116 |
| | | (0.0189) | (0.0000) | (0.0100) | | | |
| 61 | WMT | **0.5640** | **0.9622** | **0.4825** | -12.482 | -0.5503 | 0.0099 |
| | | (0.0159) | (0.0008) | (0.0072) | | | |
| 62 | LTD | **0.5014** | **1.8147** | **0.8269** | -43.531 | 0.3187 | 0.0079 |
| | | (0.0146) | (0.0022) | (0.0120) | | | |
| 63 | RSH | **0.5350** | **1.3840** | **0.4284** | -13.980 | 0.7826 | 0.0033 |
| | | (0.0164) | (0.0012) | (0.0065) | | | |
| 64 | MCD | **1.2791** | **1.3239** | **0.3440** | 9.358 | 1.1411 | 0.0012 |
| | | (0.5069) | (0.0096) | (0.0161) | | | |
| 65 | TOY* | **1.8980** | **0.7602** | **1.3347** | -43.267 | 0.3972 | 0.0017 |
| | | (0.7316) | (0.0420) | (0.2290) | | | |
| XIII. Services Industry | | | | | | | |
| 66 | TWX | **0.2342** | **2.8944** | **0.5800** | -22.925 | 7.5983 | -0.0341 |
| | | (0.0051) | (0.0032) | (0.0064) | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 67 | HET | **1.9061** | **1.5073** | **0.7166** | -16.520 | 0.5770 | 0.0061 |
| | | (0.9878) | (0.1058) | (0.1229) | | | |
| 68 | HCA | **1.4698** | **0.9139** | **0.6656** | -17.752 | 1.0220 | -0.0011 |
| | | (0.0994) | (0.0002) | (0.0244) | | | |
| 69 | TYC | **1.3418** | **1.4356** | **1.0126** | -36.483 | -0.8404 | 0.0159 |
| | | (0.0788) | (0.0003) | (0.0323) | | | |

1. The value in () is standard deviation of model's parameter.
2. In this study, we use maximum likelihood estimation (MLE) method, genetic algorithm, and optimization technique to implement parameters estimation for conditioning on mean-reverting stochastic model.
3. The sign "*" stands for using genetic algorithm and optimization technique to estimate model's parameters. This is because that parameter value is limited by designation of mean-reverting model
4. Fval stands for the maximum value of likelihood function.

## Appendix V. Parameter estimation of stochastic industrial economic state model by MLE optimization

Table A5-1. Parameters' estimation of stochastic industrial economic state model

| Parameters' estimation of stochastic industrial economic state model | | | | | |
|---|---|---|---|---|---|
| Industry | Simulation target | $a_h$ | $b_h$ | $s_h$ | Fval. |
| Paper | Change rate of Paper products Shipments | **0.5272** (1.32E-02) | **0.0052** (4.77E-06) | **0.0285** (3.97E-04) | 101.740 |
| Wood* | Change rate of Wood Shipments | **1.6180** (6.70E-01) | **0.0076** (2.90E-03) | **0.0743** (1.07E-02) | 76.975 |
| Oil* | Change rate of Oil Shipments | **1.4053** (5.16E-01) | **0.0067** (4.52E-03) | **0.0860** (1.91E-02) | 59.872 |
| Petroleum | Change rate of Petroleum Shipments | **0.5799** (1.54E-02) | **0.0147** (4.91E-05) | **0.0773** (1.14E-03) | 59.810 |
| Food(1)* | Change rate of Food Shipments | **1.9924** (8.97E-01) | **0.0057** (3.64E-04) | **0.0327** (3.37E-03) | 118.490 |
| Food(2)* | Change rate of Food services Shipments | **1.7678** (6.68E+00) | **0.0339** (1.28E-01) | **0.0481** (3.23E-01) | 121.359 |
| Tobacco* | Change rate of Tobacco Shipments | **1.8139** (5.92E-01) | **0.0083** (6.09E-03) | **0.1128** (1.81E-02) | 59.523 |
| Food* | Change rate of Food Shipments | **1.9924** (8.97E-01) | **0.0057** (3.64E-04) | **0.0327** (3.37E-03) | 118.490 |
| Chemical | Change rate of Chemicals Shipments | **1.1996** (6.19E-02) | **0.0081** (1.89E-05) | **0.0271** (7.59E-04) | 114.530 |
| Primary Metal | Change rate of Primary Metal Shipments | **0.6483** (1.85E-02) | **0.0022** (1.75E-05) | **0.0320** (5.01E-04) | 98.984 |
| Semi-conductor | Change rate of Semi-conductor Shipments | **0.8554** (2.98E-02) | **0.0204** (4.30E-05) | **0.0952** (1.83E-03) | 55.480 |
| Computer | Change rate of Computer Shipments | **0.4979** (1.22E-02) | **0.0087** (1.77E-05) | **0.0317** (4.29E-04) | 96.599 |
| Aircraft* | Change rate of Aircraft Shipments | **1.4463** (7.48E-01) | **-0.0046** (1.65E-02) | **0.1507** (2.59E-02) | 43.307 |
| Railroads* | Change rate of Railroads Shipments | **1.6123** (8.34E-01) | **0.0025** (9.90E-03) | **0.0658** (1.23E-02) | 80.959 |
| Communication | Change rate of | **0.8376** | **0.0186** | **0.0717** | 67.421 |

| | | | | | |
|---|---|---|---|---|---|
| | Communication Shipments | (2.97E-02) | (6.27E-05) | (1.38E-03) | |
| Manufacturing | Change rate of Manufacturing Shipments | **0.8597** | **0.0074** | **0.0188** | 125.290 |
| | | (3.01E-02) | (8.01E-06) | (3.63E-04) | |
| Retails* | Change rate of Retails & Food Sales | **1.5703** | **0.0126** | **0.0275** | 120.433 |
| | | (1.00E+00) | (2.53E-03) | (3.46E-03) | |
| Services | Change rate of GDP | **1.6659** | **0.0130** | **0.0080** | 205.330 |
| | | (1.42E-01) | (2.90E-07) | (3.64E-04) | |

1. The value in () is standard deviation of model's parameter.
2. In this study, we use maximum likelihood estimation (MLE) method, genetic algorithm, and optimization technique to implement parameters estimation for conditioning on mean-reverting stochastic model.
3. The sign "*" stands for using genetic algorithm and optimization technique to estimate model's parameters. This is because that parameter value is limited by designation of mean-reverting model

**Appendix VI. Table of probability of Insolvency and Correlation of Long- and Short-term ratings**

**Table A-6 One -Year PIS & Correlation of Long- and Short-term ratings**

| **One -Year PIS & Correlation of Long- and Short-term ratings** | | | | |
|---|---|---|---|---|
| **Panel A. The corresponding PIS for each Long- and Short-term ratings (consideration of overlapping rating)** | | | | |
| Long-term rating | A+ ~ AAA | A- ~ A+ | BBB ~ A- | BBB- ~ BBB |
| Short-term rating | A-1+ | A-1 | A-2 | A-3 |
| **One-Year PIS(Lower Bound)** | **0.04%** | **0.18%** | **0.50%** | **1.00%** |
| The Difference of Groups | 0.04% | 0.14% | 0.32% | 0.50% |
| **Panel B. The corresponding PIS for each Long- and Short-term ratings (No consideration of overlapping rating)** | | | | |
| Long-term rating | AA- ~ AAA | A ~ A+ | BBB+ ~ A- | BBB- ~ BBB |
| Short-term rating | A-1+ | A-1 | A-2 | A-3 |
| **One-Year PIS(Lower Bound)** | **0.03%** | **0.06%** | **0.34%** | **1.00%** |
| The Difference of Groups | 0.03% | 0.03% | 0.28% | 0.66% |

* The difference between group A-1+ and A-1 is very lower (0.14%; 0.03%) in both panel A and panel B relative to other groups differences. Moreover, the one year PIS of group A-1+ is very tiny (0.00%~0.04%) so that it is hard to separate it from group A-1 (0.04%~0.18%). Based upon the two above reasons, we can reasonably view groups A-1+ and A-1 as the same group (0.00%~0.18%).

* Data sources: S&P websites.

## Appendix VII. Empirical results of solvency ratio model

The detailed empirical credit analyses results are illustrated in table A7-1. The fourth column of table A7-1 stands for probability of liquidity crisis and of each sample firm calculated by our solvency ratio model (denoted as "model's PIS") during the future one year. The fifth column of table A7-1 represents expected ratio of insufficient liquidity of each sample firm calculated by our solvency ratio model (denoted as "model's ELGR") during the future one year. The sixth column of table A7-1 represents for each firm's theoretical long-term rating and its corresponding short-term rating (denoted as "model's rating"). The model's ratings are assigned to each firm by comparing model's PIS` to the one-year default rates curve in American market[16]. And then we can get the firm's theoretical short-term rating by utilizing the correlation of long- and short-term ratings[17].

Table A7-1. Empirical results of solvency ratio model

| **Empirical results of solvency ratio model for companies with short-term & long-term rating** | | | | | | |
|---|---|---|---|---|---|---|
| Item | Ticker | Rating Date | Model's PIS (One-Year) | Model's ELGR (One-Year) | Model's Rating (ST / LT) | Actual Rating (ST / LT) |
| I. Paper & Wood Industry | | | | | | |
| 1 | MMM* | 1998/2/10 | 0.00% | 0.0000% | A-1+ / AAA | A-1+ / AA |
| 2 | WY**** | 2002/2/15 | 0.46% | 0.0482% | A-2 or A-3 / BBB | A-2 / BBB |
| 3 | IP**** | 2001/6/12 | 0.58% | 0.0857% | A-2 or A-3 / BBB | A-3 / BBB |
| II. Oil & Gas Industry | | | | | | |
| 4 | HAL* | 2002/12/18 | 0.70% | 0.1250% | A-3 / BBB- | A-3 / BBB |
| 5 | SLB* | 2002/12/11 | 0.06% | 0.0083% | A-1 / A | A-1 / A+ |
| 6 | XOM* | 1999/12/6 | 0.00% | 0.0000% | A-1+ / AAA | A-1+ / AAA |
| III. Foods & Tobacco Industry | | | | | | |
| 7 | SLE* | 2005/1/19 | 0.06% | 0.0084% | A-1 / A | A-1 / A |
| 8 | CPB* | 2001/2/13 | 0.08% | 0.0085% | A-1 / A | A-1 / A |
| 9 | HNZ** | 2001/6/8 | 0.32% | 0.0422% | A-2 / BBB+ | A-1 / A |

[16] The average forward default rates are provided by Standard and Poor's (1981~2003).

[17] The correlation of long- and short-term ratings is in the following: For the ratings higher than A+, their short-term ratings will be equivalent to A-1+; for the ratings between A+ and A-, their short-term ratings will be equivalent to A-1; for the ratings between A- and BBB, their short-term ratings will be equivalent to A-2; for the ratings between BBB and BBB-, their short-term ratings will be equivalent to A-3; for the ratings between BB+ and BB-, their short-term ratings will be equivalent to B; for the ratings between B+ and C, their short-term ratings will be equivalent to C; and so on. (from the S&P's rating tables)

| | | | | | | |
|---|---|---|---|---|---|---|
| 10 | KO*** | 1999/12/21 | 0.00% | 0.0000% | A-1+ / AAA | A-1 / A+ |
| 11 | BUD* | 1997/5/22 | 0.04% | 0.0024% | A-1 / A | A-1 / A+ |
| 12 | PEP** | 2003/10/30 | 0.34% | 0.0295% | A-2 / BBB+ | A-1 / A+ |
| 13 | MO**** | 2003/4/9 | 0.36% | 0.0613% | A-2 or A-3 / BBB | A-2 / BBB+ |
| IV. Chemicals Industry | | | | | | |
| 14 | DD* | 1995/4/7 | 0.02% | 0.0018% | A-1+ / AA- | A-1+ / AA- |
| 15 | DOW**** | 2003/3/13 | 0.16% | 0.0202% | A-1 or A-2 / A- | A-2 / A- |
| 16 | BMY*** | 2004/8/16 | 0.00% | 0.0000% | A-1+ / AAA | A-1 / A+ |
| 17 | JNJ* | 1987/8/17 | 0.00% | 0.0000% | A-1+ / AAA | A-1+ / AAA |
| 18 | MRK* | 2004/11/16 | 0.00% | 0.0000% | A-1+ / AAA | A-1+ / AA- |
| 19 | PFE* | 1986/5/14 | 0.00% | 0.0000% | A-1+ / AAA | A-1+ / AAA |
| 20 | AMGN*** | 2002/7/18 | 0.00% | 0.0000% | A-1+ / AAA | A-1 / A+ |
| 21 | PG* | 2001/11/16 | 0.02% | 0.0006% | A-1+ / AA- | A-1+ / AA- |
| 22 | AVP**** | 1996/3/25 | 0.14% | 0.0195% | A-1 or A-2 / A- | A-1 / A |
| 23 | CL* | 2001/5/4 | 0.00% | 0.0000% | A-1+ / AAA | A-1+ / AA- |
| V. Primary Metal Industry | | | | | | |
| 24 | AA**** | 2003/6/20 | 0.18% | 0.0387% | A-1 or A-2 / A- | A-2 / A- |
| 25 | G** | 2001/5/17 | 0.24% | 0.0262% | A-2 / BBB+ | A-1+ / AA- |
| 26 | BDK** | 1999/4/16 | 3.76% | 0.7563% | B / BB- | A-2 / BBB |
| VI. Computer Related Industry | | | | | | |
| 27 | HPQ* | 2002/5/7 | 0.06% | 0.0133% | A-1 / A | A-1 / A- |
| 28 | ROK* | 2001/6/29 | 0.04% | 0.0025% | A-1 / A | A-1 / A |
| 29 | CSC**** | 1996/11/5 | 0.12% | 0.0112% | A-1 or A-2 / A- | A-1 / A |
| 30 | IBM*** | 1998/2/26 | 0.00% | 0.0000% | A-1+ / AAA | A-1 / A+ |
| 31 | MSFT* | 1997/3/12 | 0.00% | 0.0000% | A-1+ / AAA | A-1+ / AAA |
| VII. Semiconductor Industry | | | | | | |
| 32 | INTC* | 1993/2/17 | 0.00% | 0.0000% | A-1+ / AAA | A-1+ / A+ |
| 33 | TXN*** | 1991/3/8 | 0.00% | 0.0000% | A-1+ / AAA | A-1 / A |
| VIII. Aircraft & Transportation Industry | | | | | | |
| 34 | BA* | 2003/5/15 | 0.06% | 0.0107% | A-1 / A | A-1 / A |
| 35 | UTX*** | 2003/10/2 | 0.00% | 0.0000% | A-1+ / AAA | A-1 / A |
| 36 | HON* | 1992/12/29 | 0.04% | 0.0069% | A-1 / A | A-1 / A |
| 37 | GD* | 1999/7/9 | 0.04% | 0.0043% | A-1 / A | A-1 / A |
| IX. Surgical, Medical, Detection Electron. Industry | | | | | | |
| 38 | RTN** | 1999/10/29 | 7.30% | 1.6070% | C / B | A-3 / BBB- |
| 39 | BAX*** | 2004/1/13 | 0.08% | 0.0037% | A-1 / A | A-2 / A- |
| 40 | MDT* | 2001/8/30 | 0.00% | 0.0000% | A-1+ / AAA | A-1+ / AA- |

| | | | | | | |
|---|---|---|---|---|---|---|
| 41 | EK* | 2003/9/25 | 0.54% | 0.0365% | A-3 / BBB- | A-3 / BBB- |
| X. Railroad & Transportation Industry | | | | | | |
| 42 | BNI**** | 1998/12/23 | 0.08% | 0.0015% | A-1 or A-2 / A- | A-2 / BBB+ |
| 43 | NSC**** | 2000/5/3 | 0.40% | 0.0431% | A-2 or A-3 / BBB | A-2 / BBB |
| 44 | FDX* | 1998/2/2 | 0.28% | 0.0321% | A-2 / BBB+ | A-2 / BBB |
| XI. Communication Industry | | | | | | |
| 45 | SBC* | 2004/9/28 | 0.06% | 0.0066% | A-1 / A | A-1 / A |
| 46 | DIS*** | 2002/10/4 | 0.06% | 0.0049% | A-1 / A | A-2 / BBB+ |
| XII. Retails Industry | | | | | | |
| 47 | HD**** | 2000/12/12 | 0.03% | 0.0008% | A-1+ or A-1 / A+ | A-1+ / AA |
| 48 | MAY* | 2004/7/13 | 0.26% | 0.0160% | A-2 / BBB+ | A-2 / BBB |
| 49 | WMT* | 1983/7/25 | 0.02% | 0.0019% | A-1+ / AA- | A-1+ / AA |
| 50 | LTD**** | 2004/10/7 | 0.44% | 0.0916% | A-2 or A-3 / BBB | A-2 / BBB |
| 51 | RSH* | 1993/2/24 | 0.06% | 0.0023% | A-1 / A | A-2 / A- |
| 52 | MCD*** | 2003/5/8 | 0.00% | 0.0000% | A-1+ / AAA | A-1 / A |
| XIII. Services Industry | | | | | | |
| 53 | TWX* | 2001/1/12 | 0.32% | 0.0584% | A-2 / BBB+ | A-2 / BBB+ |
| 54 | TYC**** | 2004/5/26 | 0.36% | 0.0558% | A-2 or A-3 / BBB | A-2 / BBB |

.*：model's rating is as same as actual rating； **：model's rating is lower than actual rating；

***：model's rating is higher than actual rating；

****：the short-term rating interval transformed by the same long-term rating

1. Each firm's actual rating is acquired from Standard and Poor's website.
2. Model's PIS and ELGR: The results are from 10000 times simulation of solvency ratio model for each sampled firm. And then we can calculate the one-year probability of liquidity crisis (hazard rate) and one-year expected ratio of insufficient liquidity through "A Solvency-based Multi-period Short-term Credit Risk Model".
3. Model's rating: Let model's PLC and ERIC correspond to American one-year average forward default rates provided by S&P(1981~2003) and further we can decide the credit rating for each firm.
4. Except for MDT, WY, and CPB, the estimation periods for all other companies are 1994~2004Q1.(the estimation period of MDT, WY, CPB is individually 1996~2004Q1, 1994~2002Q1, and 1994~2001Q1), ; this is because the limitation of missing data.)