

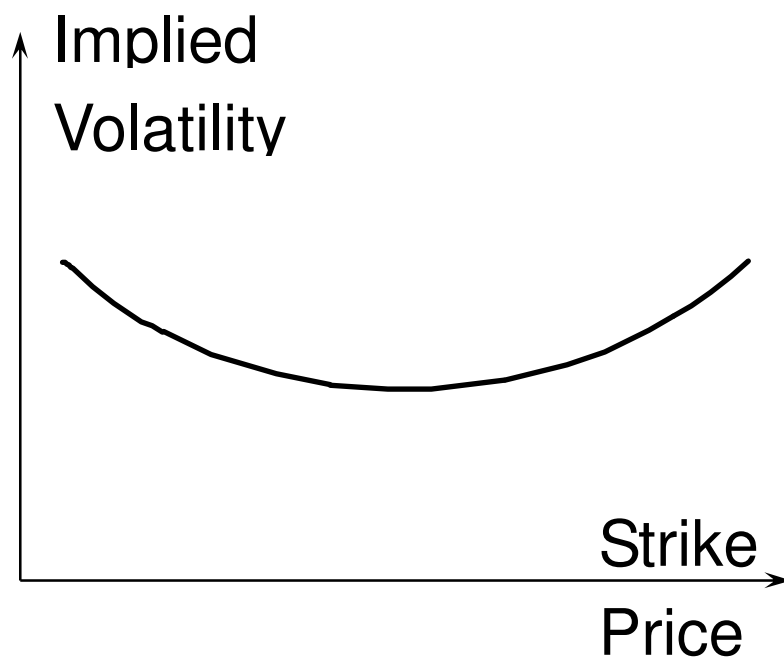
What is a Volatility Smile?

- It is the relationship between implied volatility and strike price for options with a certain maturity
- The volatility smile for European call options should be exactly the same as that for European put options
- The same is at least approximately true for American options

Why the Volatility Smile is the Same for Calls and Put

- Put-call parity  $p + S_0 \exp(-qT) = c + K \exp(-rT)$  holds for market prices ( $P_{mkt}$  and  $C_{mkt}$ ) and for Black-Scholes prices ( $P_{BS}$  and  $C_{BS}$ )
- It follows that  $P_{mkt} - P_{BS} = C_{mkt} - C_{BS}$
- When  $P_{BS} = P_{mkt}$ , it must be true that  $C_{BS} = C_{mkt}$
- It follows that the implied volatility calculated from a European call option should be the same as that calculated from a European put option when both have the same strike price and maturity

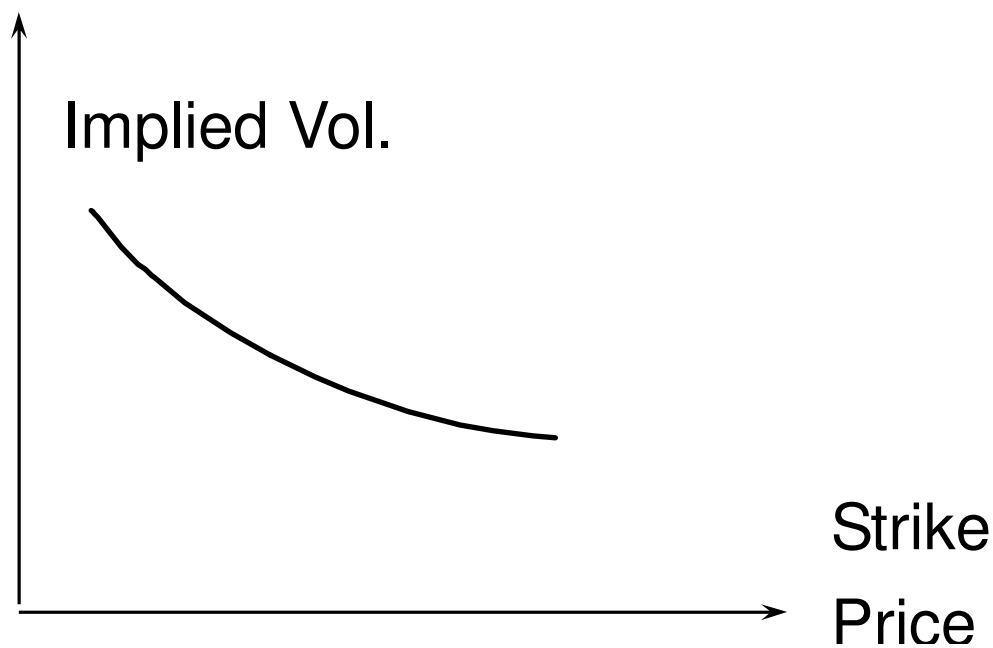
The Volatility Smile for Foreign Currency Options



Implied Distribution for Foreign Currency Options

- Both tails are heavier than the lognormal distribution
- It is also “more peaked” than the lognormal distribution

The Volatility Smile for Equity Options



#### Implied Distribution for Equity Options

- The left tail is heavier and the right tail is less heavy than the lognormal distribution

#### Other Volatility Smiles?

What is the volatility smile if

- True distribution has a less heavy left tail and heavier right tail
- True distribution has both a less heavy left tail and a less heavy right tail

#### Ways of Characterizing the Volatility Smiles

- Plot implied volatility against  $K/S_0$  (The volatility smile is then more stable)
- Plot implied volatility against  $K/F_0$  (Traders usually define an option as at-the-money when  $K$  equals the forward price,  $F_0$ , not when it equals the spot price  $S_0$ )
- Plot implied volatility against delta of the option (This approach allows the volatility smile to be applied to some non-standard options. At-the money is defined as a call with a delta of 0.5 or a put with a delta of  $-0.5$ . These are referred to as 50-delta options)

#### Possible Causes of Volatility Smile

- Asset price exhibits jumps rather than continuous changes
- Volatility for asset price is stochastic
  - In the case of an exchange rate volatility is not heavily correlated with

the exchange rate. The effect of a stochastic volatility is to create a symmetrical smile

- In the case of equities volatility is negatively related to stock prices because of the impact of leverage. This is consistent with the skew that is observed in practice

#### Volatility Term Structure

- In addition to calculating a volatility smile, traders also calculate a volatility term structure
- This shows the variation of implied volatility with the time to maturity of the option

#### Volatility Term Structure

The volatility term structure tends to be downward sloping when volatility is high and upward sloping when it is low

#### Example of a Volatility Surface

	$K/S_0$				
	0.90	0.95	1.00	1.05	1.10
1 mnth	14.2	13.0	12.0	13.1	14.5
3 mnth	14.0	13.0	12.0	13.1	14.2
6 mnth	14.1	13.3	12.5	13.4	14.3
1 year	14.7	14.0	13.5	14.0	14.8
2 year	15.0	14.4	14.0	14.5	15.1
5 year	14.8	14.6	14.4	14.7	15.0