

Page 4

Relationships among Modes of Convergence

Counter examples:

(iv) Let $\omega \sim U(0,1)$, and $X_n(\omega) = n1_{(0, \frac{1}{n})}(\omega)$. $\forall \omega \in [0,1]$ and $\varepsilon > 0$, there

$\exists N_0$ s.t. $n \geq N_0$ with $\frac{1}{n} < \omega$ and $|X_n(\omega) - 0| < \varepsilon$. It implies that

$X_n \xrightarrow{a.s.} 0$. However, $E[|X_n - 0|^2] = n \not\rightarrow 0$. Thus, X_n does not converge to 0 in mean-squared sense.