Solution to the selected problems in the text book

2.15

. The clearinghouse member is required to provide $20 \times \$2,000 = \$40,000$ as initial margin for the new contracts. There is a gain of $(50,200 - 50,000) \times 100 = \$20,000$ on the existing contracts. There is also a loss of $(51,000 - 50,200) \times 20 = \$16,000$ on the new contracts. The member must therefore add

$$40,000 - 20,000 + 16,000 = \$36,000$$

to the margin account.

3.6

The optimal hedge ratio is

$$0.8 \times \frac{0.65}{0.81} = 0.642$$

This means that the size of the futures position should be 64.2% of the size of the company's exposure in a 3-month hedge.

3.7 One S&P contract is on 250 times the index. The formula for the number of contracts that should be shorted gives

$$1.2 \times \frac{20{,}000{,}000}{1080 \times 250} = 88.89$$

Rounding to the nearest whole number, 89 contracts should be shorted. To reduce the beta to 0.6, $88.89 \times 0.4 = 35.56$ contracts should be shorted.

4.14

$${}_1f_2 = \frac{1.03^2}{1.02} - 1 = 0.040$$

$${}_2f_3 = \frac{1.037^3}{1.03^2} - 1 = 0.051$$

$${}_3f_4 = \frac{1.042^4}{1.037^3} - 1 = 0.057$$

$${}_4f_5 = \frac{1.045^5}{1.042^4} - 1 = 0.057$$

4.16

A long position in two of the 4% coupon bonds combined with a short position in one of the 8% coupon bonds leads to the following cash flows

$$\text{Year } 0: 90 - 2 \times 80 = -70$$

$$\text{Year } 10: 200 - 100 = 100$$

since the coupons cancel out. The 10-year spot rate is therefore

$$\frac{1}{10}\ln\frac{100}{70} = 0.0357$$

or 3.57% per annum.

4.22

(a) The bond's price is

$$8e^{-0.11} + 8e^{-0.11\times2} + 8e^{-0.11\times3} + 8e^{-0.11\times4} + 108e^{-0.11\times5} = 86.80$$

(b) The bond's duration is

$$\frac{1}{86.80}\left[8e^{-0.11} + 2\times8e^{-0.11\times2} + 3\times8e^{-0.11\times3} + 4\times8e^{-0.11\times4} + 5\times108e^{-0.11\times5}\right]$$

$$= 4.256 \text{ years}$$

(c) Since, with the notation in the chapter

$$\Delta B = -BD\Delta y$$

the effect on the bond's price of a 0.2% decrease in its yield is

$$86.80\times4.256\times0.002 = 0.74$$

The bond's price should increase from 86.80 to 87.54.

(d) With a 10.8% yield the bond's price is

$$8e^{-0.108} + 8e^{-0.108\times2} + 8e^{-0.108\times3} + 8e^{-0.108\times4} + 108e^{-0.108\times5} = 87.54$$

This is consistent with the answer in (c).

5.11 The futures contract lasts for 11 months. The dividend yield is 5% for four of the months and 2% for 7 months. The average dividend yield is therefore

$$\frac{1}{11}(5\%\times4 + 2\%\times7) = 3.091\%$$

The futures price if therefore

$$1300\mathrm{e}^{(0.09-0.03091)\times\frac{11}{12}} = 1372.36$$

5.16

If

$$F_2 > (F_1)e^{r(t_2-t_1)}$$

an investor could make a riskless profit by
(a) taking a long position in a futures contract which matures at time t_1
(b) taking a short position in a futures contract which matures at time t_2
When the first futures contract matures, an amount F_1 is borrowed at rate r for time $t_2 - t_1$. The funds are used to purchase the asset for F_1 and store it until time t_2. At time t_2 it is exchanged for F_2 under the second contract. An amount $(F_1)e^{r(t_2-t_1)}$ is required to repay the loan. A positive profit of

$$F_2 - (F_1)e^{r(t_2-t_1)}$$

is, therefore, realized at time t_2. This type of arbitrage opportunity cannot exist for long. Hence:

$$F_2 \leq (F_1)e^{r(t_2-t_1)}$$

5.22

When the geometric average of the price relatives is used, the changes in the value of the index do not correspond to changes in the value of a portfolio that is traded. Equation (5.8) is therefore no longer correct. The changes in the value of the portfolio is monitored by an index calculated from the arithmetic average of the prices of the stocks in the portfolio. Since the geometric average of a set of numbers is always less than the arithmetic average, equation (5.8) overstates the futures price. It is rumored that at one time (prior to 1988), equation (5.8) did hold for the Value Line Index. A major Wall Street firm was the first to recognize that this represented a trading opportunity. It made a financial killing by buying the stocks underlying the index and shorting the futures.

7.14 In an interest-rate swap a financial institution's exposure is the difference between a fixed-rate of interest and a floating-rate of interest. It has no exposure as far as the principal amount of the loan is concerned.

8.12 Draw it yourself.

8.15 Writing a put gives a payoff of $\min\{S_T\text{-}X,0\}$. Buying a call gives a payoff of $\max\{S_T\text{-}X,0\}$. In both cases the potential payoff is S_T-X. The difference is that for written put the counterparty chooses whether you get the payoff (and will allow you to get it only when it is negative). For a long put you decide whether you get the payoff and you choose to get it when it is positive.

8.17
(a) The option contract becomes one to by 500×1.1 = 550 shares with an exercise price 40/1.1 = 36.36.
(b) There is no effect. The terms of an options contract are not normally adjusted for

cash dividends.

(c) The option contract becomes one to buy 500×4 = 2,000 shares with an exercise price 40/4 = 10.

9.3 The lower bound is $15e^{-0.06\times\frac{1}{12}} - 12 = 2.93$

9.12

In this case the present value of the strike price is $50e^{-0.08333\times0.06} = 49.75$ Since

$$2.5 < 49.75 - 47.00$$

the condition in equation (9.2) is violated. An arbitrageur can lock in a profit of at least \$0.25 by buying the put option and buying the stock.

9.18

As in the text we use c and p to denote the European call and put option price, and C and P to denote the American call and put option prices. Since $P > p$, it follows from put–call parity that

$$P > c + Xe^{-rT} - S_0$$

and since $c = C$,

$$P > C + Xe^{-rT} - S_0$$

or

$$C - P < S_0 - Xe^{-rT}.$$

For a further relationship between C and P, consider

Portfolio I: One European call option plus an amount of cash equal to X.

Portfolio J: One American put option plus one share.

Both options have the same exercise price and expiration date. Assume that the cash in portfolio I is invested at the risk-free interest rate. If the put option is not exercised early portfolio J is worth

$$\max(S_T, X)$$

at time T. Portfolio I is worth

$$\max(S_T - X, 0) + Xe^{rT} = \max(S_T, X) - X + Xe^{rT}$$

at this time. Portfolio I is therefore worth more than portfolio J. Suppose next that the put option in portfolio J is exercised early, say, at time τ. This means that portfolio J is worth X at time τ. However, even if the call option were worthless, portfolio I would be worth $Xe^{r\tau}$ at time τ. It follows that portfolio I is worth more than portfolio J in all circumstances. Hence

$$c + X > P + S_0$$

Since $c = C$,

$$C + X > P + S_0$$

or

$$C - P > S_0 - X$$

Combining this with the other inequality derived above for $C - P$, we obtain

$$S_0 - X < C - P < S_0 - Xe^{-rT}$$

9.19

As in the text we use c and p to denote the European call and put option price, and C and P to denote the American call and put option prices. The present value of the dividends will be denoted by D. As shown in the answer to Problem 7.19, when there are no dividends

$$C - P < S_0 - Xe^{-rT}$$

Dividends reduce C and increase P. Hence this relationship must also be true when there are dividends.

For a further relationship between C and P, consider

Portfolio I: one European call option plus an amount of cash equal to $D + X$

Portfolio J: one American put option plus one share

Both options have the same exercise price and expiration date. Assume that the cash in portfolio I is invested at the risk-free interest rate. If the put option is not exercised early, portfolio J is worth

$$\max(S_T, X) + De^{rT}$$

at time T. Portfolio I is worth

$$\max(S_T, 0) + (D + X)e^{rT} = \max(S_T, X) + De^{rT} + X(e^{rT} - X)$$

at this time. Portfolio I is therefore worth more than portfolio J. Suppose next that the put option in portfolio J is exercised early, say, at time τ. This means that portfolio J is worth at most $X + De^{r\tau}$ at time τ. However, even if the call option were worthless, portfolio I would be worth $(D + X)e^{r\tau}$ at time τ. It follows that portfolio I is worth more than portfolio J in all circumstances. Hence

$$c + D + X > P + S_0$$

9.23 Consider a portfolio that is long one option with strike price K_1, long one option with strike price K_3, and short two options with strike price K_2. The value of this portfolio in different situations is given as follows:

	Portfolio Value
$S_T \le K_1$	0
$K_1 \le S_T \le K_2$	$S_T - K_1 > 0$
$K_2 \le S_T \le K_3$	$S_T - K_1 - 2(S_T - K_2) = (K_2 - K_1) - (S_T - K_2) \ge 0$
$S_T \ge K_3$	$(S_T - K_3) - 2(S_T - K_2) + (S_T - K_1) = 0$

Since the portfolio value is always zero or positive at some future time the same must be true today. Hence

$$c_1 + c_3 - 2c_2 \ge 0 \quad \text{or} \quad c_2 \le 0.5(c_1 + c_3)$$

10.2 A bear spread can be created using 2 call options with the same maturity and different strike prices. The investor shorts the call option with the lower strike price and buys the call option with the higher strike price. A bear spread can also be created using 2 put options with the same maturity and different strike prices. In this case the investor shorts the put option with the lower strike price and buys the put option with the higher strike price.

10.10

A bull spread is created by buying the $30 put and selling the $35 put. This strategy gives rise to an initial cash inflow of $3. The outcome is as follows:

Stock Price	Payoff	Profit
$S_T \ge 35$	0	3
$30 \le S_T < 35$	$S_T - 35$	$S_T - 32$
$S_T < 30$	-5	-2

A bear spread is created by selling the $30 put and buying the $35 put. This strategy costs $3 initially. The outcome is as follows:

Stock Price	Payoff	Profit
$S_T \ge 35$	0	-3
$30 \le S_T < 35$	$35 - S_T$	$32 - S_T$
$S_T < 30$	5	2