

## 財務工程入門小考解答

1. Current value =  $\frac{100}{1.20} + \frac{100}{(1.20)^2} + \frac{1100}{(1.20)^3} = 789.35$

$$\text{Duration} = \frac{1 \cdot \left[ \frac{100}{1.20} \right] + 2 \cdot \left[ \frac{100}{(1.20)^2} \right] + 3 \cdot \left[ \frac{1100}{(1.20)^3} \right]}{789.35} = 2.70$$

2. Refer to lecture note 1.

3. If the index exceeds 1025, you will receive  $x - 1025$ . After buying the index for  $x$  you will have spent 1025. If the index is below 1025, you will pay  $1025 - x$  and after buying the index for  $x$  you will have spent 1025. One way to get the cost is to note that the forward price is  $1000(1.05) = 1050$ . You want to pay 25 less and so must spend  $25/1.05 = 23.81$  today.

4. Duration is defined as  $\frac{\sum_{t=1}^n t v^t R_t}{\sum_{t=1}^n v^t R_t}$ , where  $v = \frac{1}{1+8\%}$  in this problem.

The current price of the bond is  $\sum_{t=1}^n v^t R_t$ , the denominator of the duration expression, and is given as 100. The

derivative of price with respect to the yield to maturity is  $-\sum_{t=1}^n t v^{t+1} R_t = -v$  times the numerator of the duration expression. Thus, the numerator of the duration expression is  $-(1.08)$  times the derivative. But the derivative is given as  $-700$ . So the numerator of the duration expression is 756. Thus, the duration =  $756/100 = 7.56$ .

5. 無限

6. 5000

7. 65

8. 60

9. Long straddle

10. Strangle/Straddle

11. a. 買 call 獲利無限，賣 put 獲利有限

b. 買 call 損失有限(權利金)，賣 put 損失無限大

12.  $dx = ax dt + bx dz$ , where  $a, b$  are constant

$$G(x, t) = \ln x$$

$$G_x = \frac{1}{x}$$

$$G_{xx} = -\frac{1}{x^2}$$

$$G_t = 0$$

$$\begin{aligned} dG &= \left[ ax \frac{1}{x} + 0 + \frac{1}{2} (bx)^2 \left( -\frac{1}{x^2} \right) \right] \cdot dt + bx \cdot \frac{1}{x} dz \\ &= \left( a - \frac{1}{2} b^2 \right) \cdot dt + b \cdot dz \end{aligned}$$

13.  $E(X) = 0$

14. Refer to lecture note 3.

15. Refer to lecture note 4.