Lecture 5

Greeks

一、Delta($\frac{\partial C}{\partial S}$)

它代表當股價變動一單位時，買權價值的變動:

$$\frac{\partial C}{\partial S} = N(d_1) > 0$$

證明:

$$\frac{\partial C}{\partial S} = N(d_1) + S\frac{\partial N(d_1)}{\partial d_1}\cdot\frac{\partial d_1}{\partial S} - Ke^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\cdot\frac{\partial d_2}{\partial S}$$

已知$\frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S}$，則

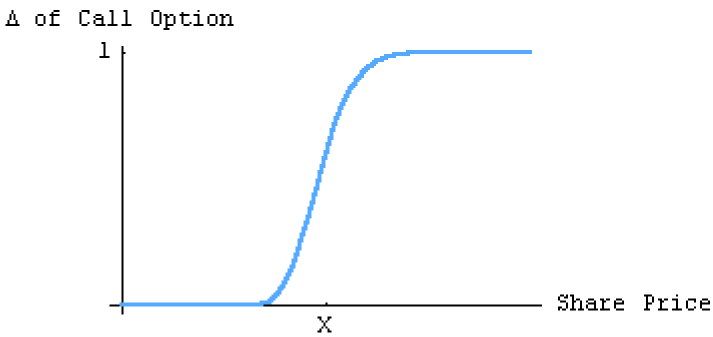
$$\begin{aligned}
Ke^{-r\tau}\frac{\partial N(d_2)}{\partial d_2} &= Ke^{-r\tau}\left((\frac{1}{\sqrt{2\pi}}e^{-d_2{}^2/2}) \right) \\
&= \frac{Ke^{-r\tau}}{\sqrt{2\pi}}[e^{-\frac{1}{2}(d_1{}^2 - 2\sigma\sqrt{\tau}d_1 + \sigma^2\tau)}] \\
&= Ke^{-r\tau}[\frac{1}{\sqrt{2\pi}}e^{-d_1{}^2/2}\cdot e^{d_1\sigma\sqrt{\tau} - \sigma^2\tau/2}] \\
&= Ke^{-r\tau}\cdot\frac{\partial N(d_1)}{\partial d_1}\cdot e^{\ln(S/K) + (r+\sigma^2/2)\tau - \sigma^2\tau/2} \\
&= Ke^{-r\tau}\cdot\frac{\partial N(d_1)}{\partial d_1}\cdot\frac{S}{K}e^{r\tau} = S\frac{\partial N(d_1)}{\partial d_1}
\end{aligned}$$

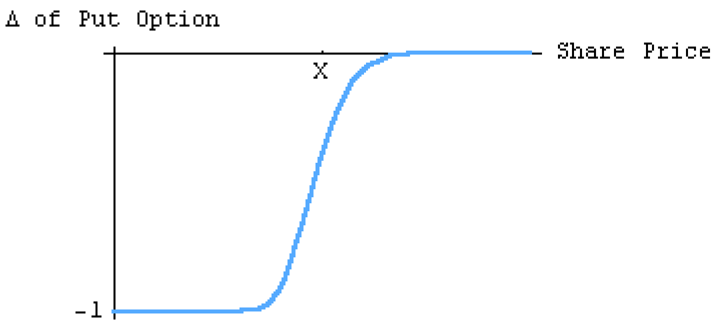
因此，

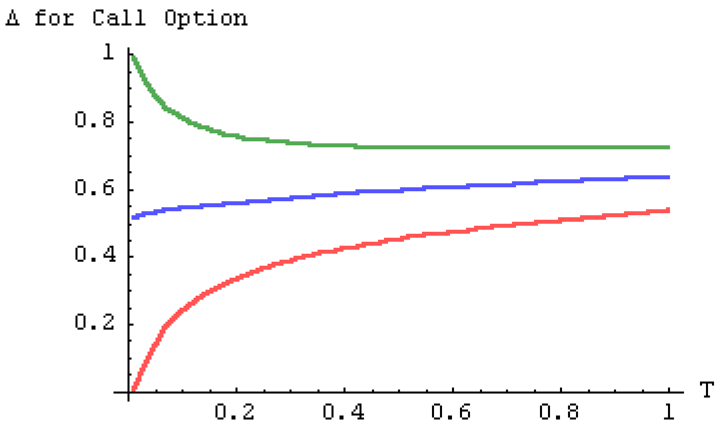
$$\frac{\partial C}{\partial S} = N(d_1)$$

此外，

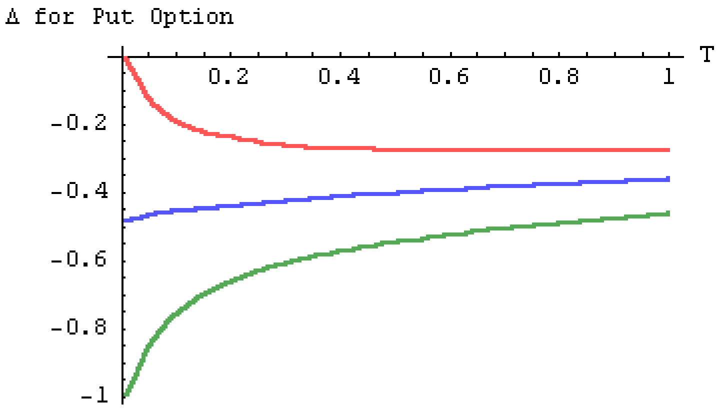
$$\begin{aligned}
\frac{\partial P}{\partial S} &= -N(-d_1) < 0 \\
&= -(1 - N(d_1)) = N(d_1) - 1 = \text{call delta-1<0}
\end{aligned}$$







由下至上分別爲價外、 價平、價內選擇權



由上至下分別爲價外、 價平、價內選擇權

二、Gamma($\dfrac{\partial^2 C}{\partial S^2}$)

Gamma 代表，當股價等於 S 的價位時，買權價值線的弧度(Curvature)。也是 Delta 變動的敏銳度，Gamma 愈大，代表當股價變動時，Delta 的變動愈大，避險愈困難。

$$\frac{\partial^2 C}{\partial S^2} = \frac{\partial}{\partial S} N(d_1) = \frac{n(d_1)}{S\sigma\sqrt{\tau}} = \frac{\partial^2 P}{\partial S^2}$$

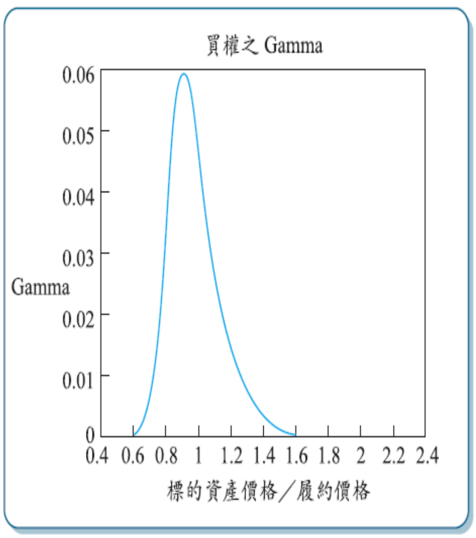


圖 15-5 歐式買權 Gamma 值在價內、價平、價外的表現

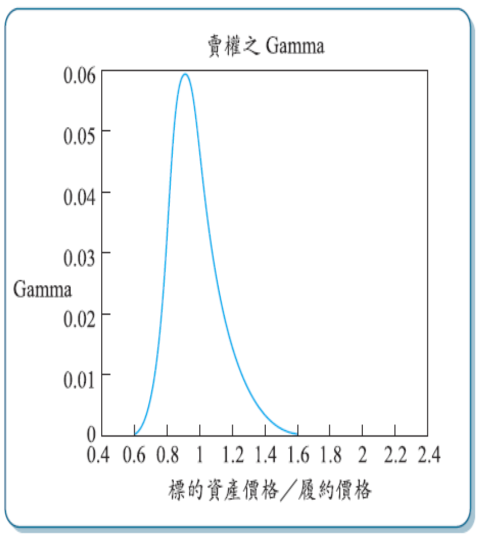


圖 15-6 歐式賣權 Gamma 值在價內、價平、價外的表現

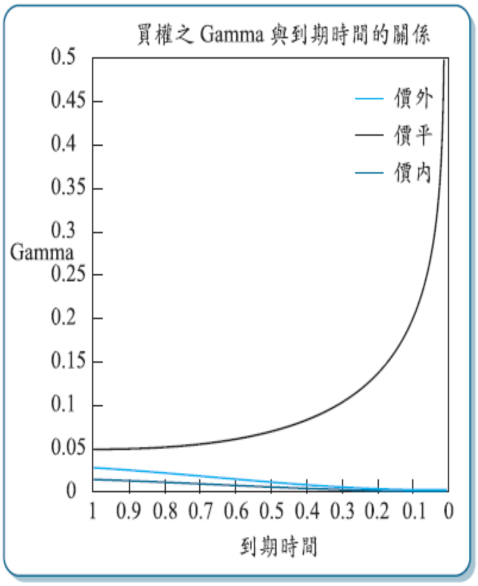


圖 15-7　歐式買權 Gamma 值與到期時間的關係

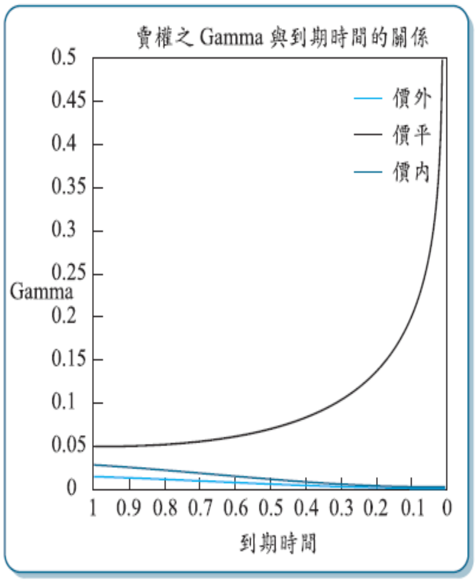


圖 15-8　歐式賣權 Gamma 值與到期時間的關係

三、Vega$\left(\dfrac{\partial C}{\partial \sigma}\right)$

$$\frac{\partial C}{\partial \sigma} = S\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \sigma} - Ke^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial \sigma} = S\sqrt{\tau}n(d_1) > 0$$

$$\frac{\partial P}{\partial \sigma} = \frac{\partial C}{\partial \sigma} = S\sqrt{\tau}n(d_1) > 0$$

標的股波動度增加時，買賣權價值皆上升。

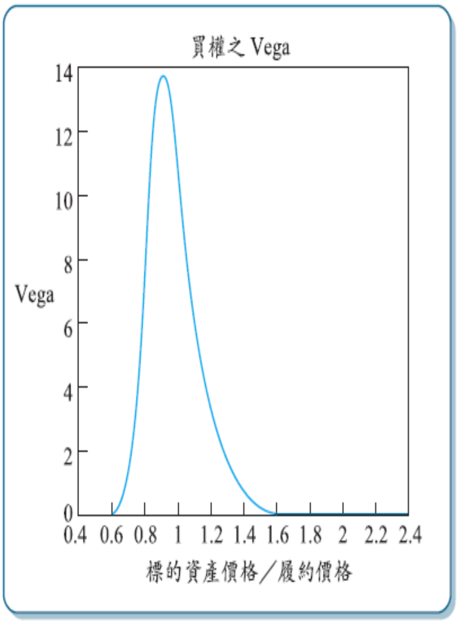


圖 15-13　歐式買權 Vega 值在價內、價平、價外的表現

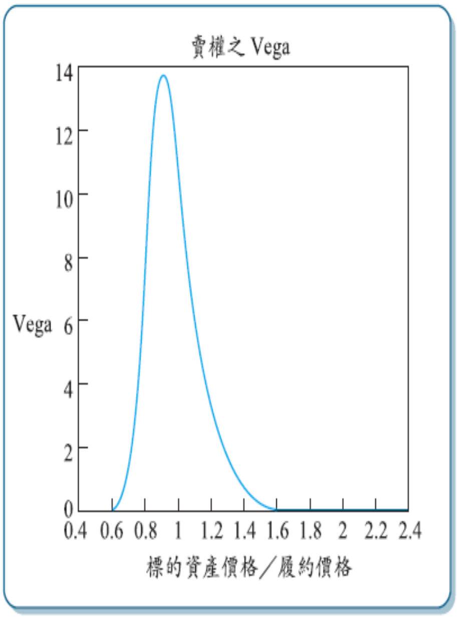


圖 15-14　歐式賣權 Vega 值在價內、價平、價外的表現

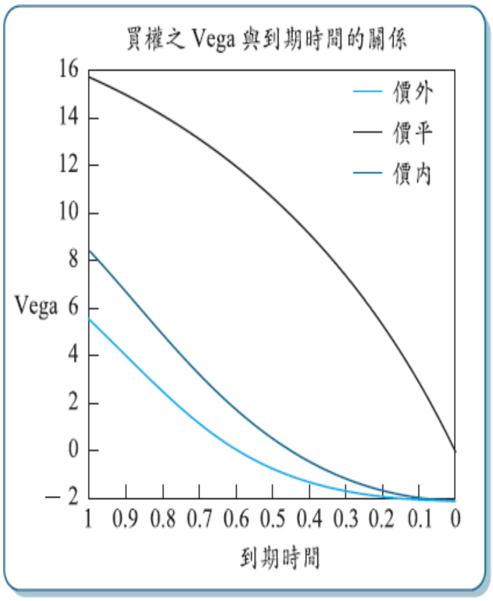


圖 15-15　歐式買權 Vega 值與到期時間的關係

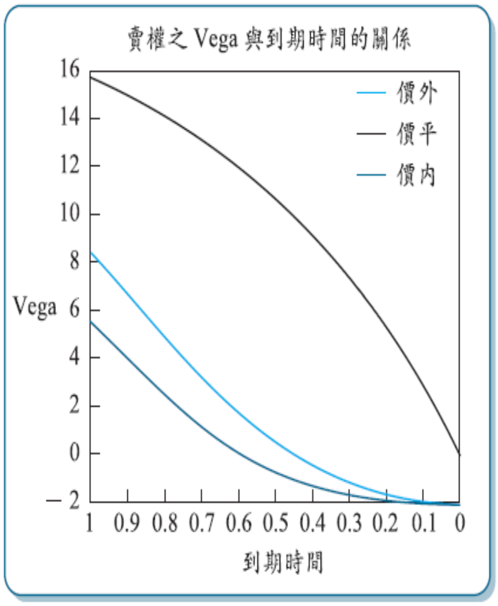


圖 15-16　歐式賣權 Vega 值與到期時間的關係

四、Rho($\dfrac{\partial C}{\partial r}$)

Rho 代表，當利率變動時，對選擇權價格的影響。

$$\frac{\partial C}{\partial r}=S\frac{\partial N(d_1)}{\partial d_1}\cdot\frac{\partial d_1}{\partial r}+K\tau e^{-r\tau}N(d_2)-Ke^{-r\tau}\frac{\partial N(d_2)}{\partial d_2}\cdot\frac{\partial d_2}{\partial r}$$

因此，買權的價值隨利率水準上升而上升。

此外，根據 put-call parity

$$\frac{\partial P}{\partial r}=\frac{\partial C}{\partial r}-K\tau e^{-r\tau}=-\tau Ke^{-r\tau}N(-d_2)<0$$

因此賣權價值隨利率水準上升而上升。

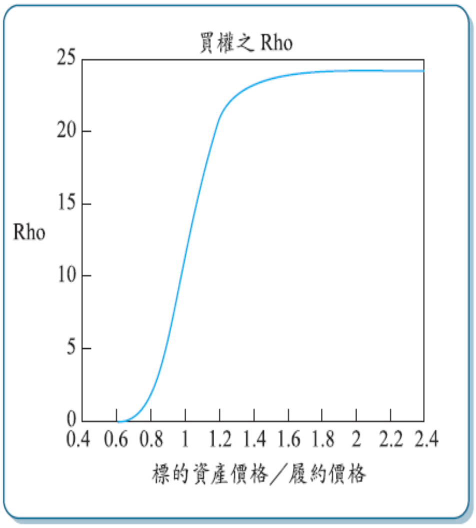


圖 15-17　歐式買權 Rho 值在價內、價平、價外的表現

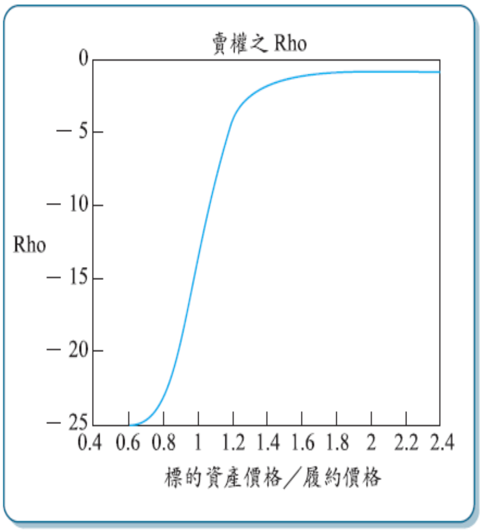


圖 15-18　歐式賣權 Rho 值在價內、價平、價外的表現

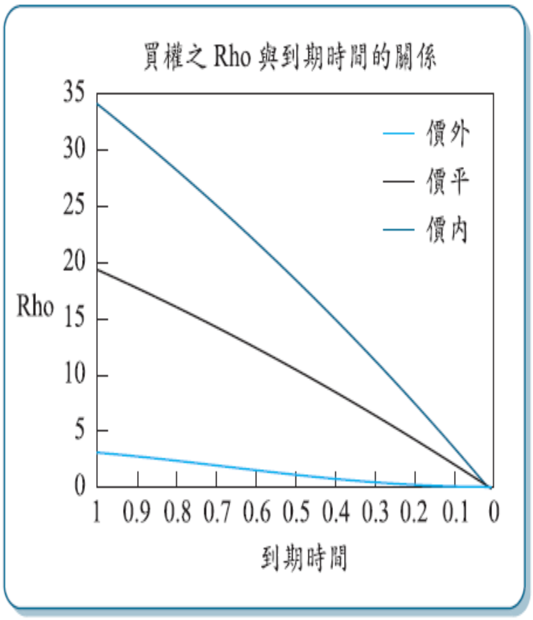


圖 15-19 歐式買權 Rho 值與到期時間的關係

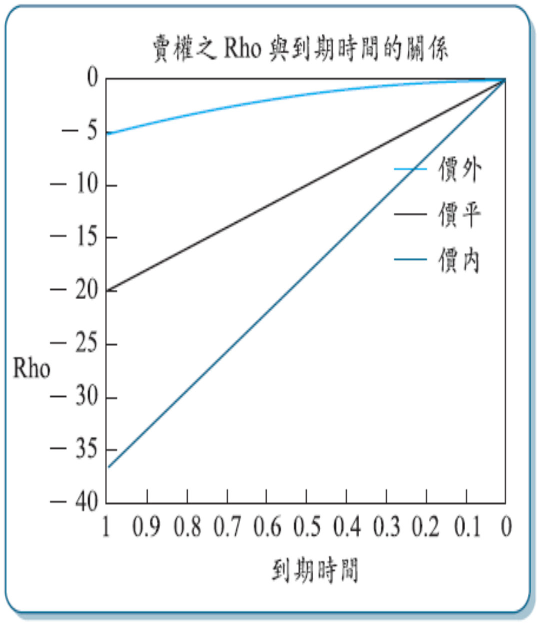


圖 15-20 歐式賣權 Rho 值與到期時間的關係

五、Theta($\dfrac{\partial C}{\partial t}$)

它代表買權價值隨著時間流逝而消失的價值。

$$\frac{\partial C}{\partial t} = -\frac{\partial C}{\partial \tau} = -[S\frac{\partial N(d_1)}{\partial d_1}\cdot\frac{\partial d_1}{\partial \tau} + Kre^{-r\tau}N(d_2) - Ke^{-rT}\frac{\partial N(d_2)}{\partial d_2}\cdot\frac{\partial d_2}{\partial \tau}],\quad r = \tau - t$$

$$= -[S\frac{1}{\sqrt{2\pi}}e^{-d_1^2/2}\frac{\partial d_1}{\partial \tau} + Kre^{-r\tau}N(d_2) - Ke^{-rT}\frac{1}{\sqrt{2\pi}}e^{-d_2^2/2}\frac{\partial d_2}{\partial \tau}]$$

$$= -[\frac{S\sigma n(d_1)}{2\sqrt{\tau}} + Kre^{-r\tau}N(d_2)] < 0$$

此外，

$$\frac{\partial P}{\partial t} = -\frac{\partial P}{\partial \tau} = -\frac{\partial}{\partial \tau}(C - S + Ke^{-r\tau})$$

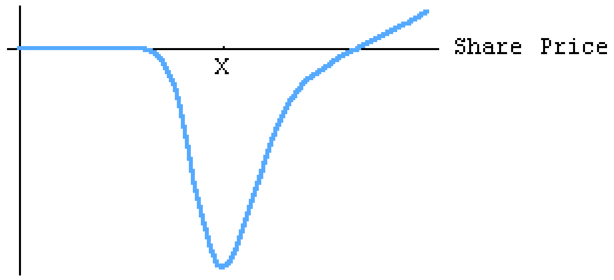
$$= -[\frac{\partial C}{\partial \tau} - rKe^{-r\tau}]$$

$$= -\frac{S\sigma n(d_1)}{2\sqrt{\tau}} + Kre^{-r\tau}(1 - N(d_2))$$

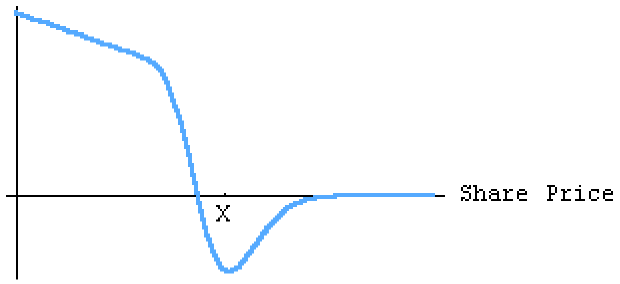
$$= -\frac{S\sigma n(d_1)}{2\sqrt{\tau}} + Kre^{-r\tau}N(-d_2)$$

當股價 S 下跌，且很低於 K 時，即成為正值，但若股價上升，且很高於 K 時，即成為負值。

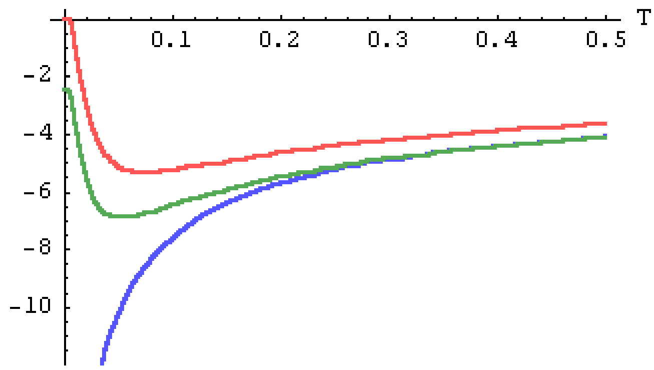
Θ of Call Option

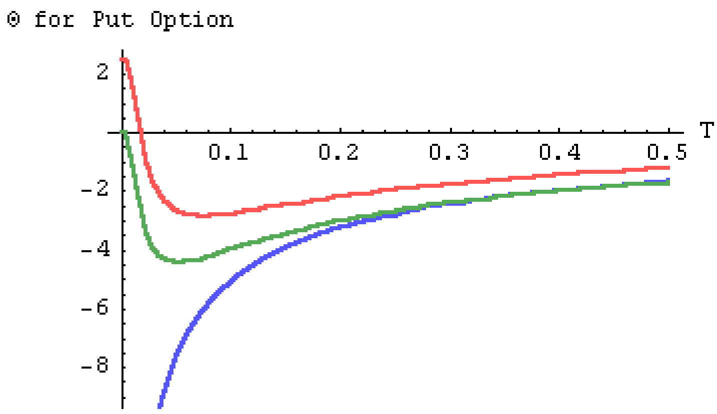


Θ of Put Option



Θ for Call Option





由上至下分別爲價外、價平、價內選擇權