Dynamic Hedging with Futures: A Copula-based GARCH Model

Chih-Chiang Hsu, Chih-Ping Tseng, and Yaw-Huei Wang *†

ABSTRACT

In a number of prior studies it has been demonstrated that the traditional regression-based static approach is inappropriate for hedging with futures, with the result that a variety of alternative dynamic hedging strategies has emerged. In this paper we propose a class of new copula-based GARCH models for the estimation of the optimal hedge ratio and compare their effectiveness with that of other hedging models, including the conventional static, the constant conditional correlation (CCC) GARCH, and the dynamic conditional correlation (DCC) GARCH models. In regards to the reduction of variance in the returns of hedged portfolios, our empirical results show that in both the in-sample and out-of-sample tests, with full flexibility in the distribution specifications, the copula-based GARCH models perform more effectively than other dynamic hedging models.

* Chih-Chiang Hsu is an associate professor in the Department of Economics, Management School, National Central University, in Taoyuan, Taiwan.

Chih-Ping Tseng is a MBA graduate of National Central University, in Taoyuan, Taiwan.

Yaw-Huei Wang is an assistant professor in the Department of Finance, College of Management, National Taiwan University, in Taipei, Taiwan.

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† Author for correspondence: Chih-Chiang Hsu, Department of Economics, Management School, National Central University, No. 300 Jhongda Road, Jhongli City, Taoyuan County 32001, Taiwan. Tel.: +886 3422 7151 ext. 66308, Fax: +886 3422 2876, Email: cchsu@mgt.ncu.edu.tw.

1. INTRODUCTION

With the popularity of financial and commodity futures, how to determine the optimal hedging strategy has become an important issue in the field of risk management. While the early literature lays stress on estimating a static hedge ratio by means of the ordinary least squares technique,[1] more recent studies employ various bivariate conditional volatility models to estimate a time-varying hedge ratio and demonstrate that generally the dynamic hedging strategy can result in greater risk reduction than the static one.[2] The superiority of the time-varying hedge ratio essentially comes from taking account of the changing joint distribution of spot and futures returns.

In order to simplify the estimation of optimal hedge ratios, most of the preceding models are based on the constant conditional correlation (CCC) GARCH model of Bollerslev (1990). While the CCC GARCH model has clear computational advantages over the multivariate GARCH (BEKK) model of Engle and Kroner (1995), the correlation structure between the spot and futures markets is quite restricted. Engle and Sheppard (2001) and Engle (2002) subsequently propose the dynamic conditional correlation (DCC) GARCH model as a means of considering the flexible correlation structure and simplifying the estimation procedure with two steps. This innovation

[1] See, for example, Ederington (1979), Figlewski (1984), Lee et al. (1987), and Benet (1992), among many others.

[2] Cecchetti et al. (1988), Baillie and Myers (1991), Myers (1991), Kroner and Sultan (1991, 1993), Park and Switzer (1995), and Choudhry (2003) use different GARCH models to estimate a time-varying hedge ratio for various assets and support its superiority over the constant one. Moreover, Tong (1996) and Brooks and Chong (2001) argue that the advantages of dynamic hedging are more significant for the cross-hedge with currency futures.

provides an ideal alternative model for the construction of hedge portfolios.

However, most of these dynamic hedging models assume that the spot and futures returns follow a multivariate normal distribution with linear dependence. This assumption is at odds with numerous empirical studies, in which it has been shown that many financial asset returns are skewed, leptokurtic, and asymmetrically dependent.[3] Various explanations for the nature of these empirical facts have been provided, such as leverage effects and asymmetric responses to uncertainty. Hence, these characteristics should be considered in the specifications of any effective hedging model.

This paper attempts to improve the effectiveness of dynamic hedging by specifying the joint distribution of spot and futures returns more realistically. We introduce a class of new copula-based GARCH models for the estimation of the optimal hedge ratio. Without the assumption of multivariate normality, the joint distribution can be decomposed into its marginal distributions and a copula, which can then be considered both separately and simultaneously. The marginal distributions can be any non-elliptical distributions, while the copula function describes the dependence structure between the spot and futures returns.[4]

Specifically, the proposed hedging model uses the GJR-skewed-*t* specification for the marginal distributions and three different copulas (Gaussian, Gumbel, and Clayton)

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[3] See, for example, Longin and Solnik (2001), Ang and Chen (2002), and Patton (2006a).
[4] Explanations of the copula theory can be found in Joe (1997) and Nelsen (1999); Patton (2006a,b) and Bartram et al. (2007) describe in detail the theory and application of the conditional copula .

for the joint distribution to permit a wide range of possible dependence structures.[5] The dependence parameters in these copulas are modeled as time-varying processes in order to capture possible dynamic and non-linear relationships between the spot and futures returns. Following the multi-stage maximum likelihood method of Patton (2006b) and Bartram et al. (2007), we estimate the model parameters and generate the dynamic hedge ratio with covariance, which is computed by a numerical integration on the copula-based joint density.

We demonstrate the usefulness of the copula-based GARCH model for two types of hedging strategies. The first empirical comparison is conducted for the S&P 500 and FTSE 100 indices directly hedged with their own futures. Tong (1996) indicates that the optimal hedge ratio determined by the fluctuations of spot and futures prices is unlikely to change much through time, because these two prices are tied closely by the arbitrage forces. Hence, in our second application we use foreign currency (the Swiss Franc) futures to cross-hedge the currency exposure of holding foreign equity (the MSCI Switzerland index). We expect that the copula-based GARCH model, which permits non-linear and asymmetric dependence between the two assets in the cross-hedge portfolio, can result in greater risk reduction.[6] As compared to other hedging methods, including the conventional, CCC GARCH, and DCC GARCH models, the copula-based GARCH models on average provide more effective hedging

[5] The selection of the optimal copula in dynamic hedging requires further empirical and theoretical works. For the purposes of comparison, it might be useful to consider several copulas which exhibit different patterns of dependence.

[6] We thank an anonymous referee for drawing our attention to this issue.

performance. The Gaussian copula-based model nearly dominates all of the other models for the direct hedge, while the Gumbel copula-based model provides the best hedging effectiveness for the cross hedge.

This paper contributes to the literature by proposing and demonstrating a class of new models for effective futures hedging. The remainder of this paper is organized as follows. Section 2 describes the hedging models, including the conventional, CCC GARCH, and DCC GARCH models. Section 3 presents the copula-based GARCH models for futures hedging. Section 4 provides details of the data used in this study and the empirical results of the different hedging models. The conclusions drawn from this study are presented in Section 5.

2. THE HEDGING MODELS

The optimal hedge ratio is defined as the ratio of futures holdings to a spot position that minimizes the risk of the hedged portfolio. Let s_t and f_t be the respective changes in the spot and futures prices at time *t*. If the joint distribution of spot and futures returns remains the same over time, then the conventional risk-minimizing hedge ratio δ^* will be:[7]

$$\delta^* = \frac{\text{cov}(s_t, f_t)}{\text{var}(f_t)}. \tag{1}$$

An estimation of this static hedge ratio is easily undertaken from the least-squares

[7] Following Kroner and Sultan (1993), the optimal hedge ratio is derived by maximizing the mean-variance expected utility of the hedged portfolio without the assumption of marginal and joint distributions.

regression of s_t on f_t. However, with the arrival of new information, the joint distribution of these assets may be time-varying, in which case the static hedging strategy is not suitable for an extension to multi-period futures hedging. Conditional on the information set at time t-1, we obtain the optimal time-varying hedge ratio by minimizing the risk of the hedged return $s_t - \delta_{t-1} f_t$, or:

$$\delta_{t-1}^{*} = \frac{\text{cov}_{t-1}(s_t, f_t)}{\text{var}_{t-1}(f_t)}. \tag{2}$$

Obviously, the dynamic hedge ratio depends on the way in which the conditional variances and covariances are specified.

Kroner and Sultan (1993) propose the following bivariate error correction model of s_t and f_t with a constant correlation GARCH (1,1) structure for the estimation of δ_t^{*}:

$$\begin{aligned} s_t &= \alpha_{0s} + \alpha_{1s}(S_{t-1} - \lambda F_{t-1}) + \varepsilon_{st} \\ f_t &= \alpha_{0f} + \alpha_{1f}(S_{t-1} - \lambda F_{t-1}) + \varepsilon_{ft}, \end{aligned} \tag{3}$$

$$\begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{bmatrix} | \Psi_{t-1} \sim N(0, H_t), \tag{4}$$

$$H_t = \begin{bmatrix} h_{s,t}^2 & h_{sf,t} \\ h_{sf,t} & h_{f,t}^2 \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} = D_t R D_t, \tag{5}$$

$$\begin{aligned} h_{s,t}^2 &= c_s + a_s \varepsilon_{s,t-1}^2 + b_s h_{s,t-1}^2 \\ h_{f,t}^2 &= c_f + a_f \varepsilon_{f,t-1}^2 + b_f h_{f,t-1}^2, \end{aligned} \tag{6}$$

where S_{t-1} and F_{t-1} are the spot and futures prices, respectively, S_{t-1}-λ F_{t-1} is the error correction term, Ψ_{t-1} is the information set at time t-1, and the disturbance term $\varepsilon_t = (\varepsilon_{st}, \varepsilon_{ft})'$ follows a bivariate normal distribution with zero mean and a conditional covariance matrix H_t with a constant correlation ρ. The GARCH term allows the hedge ratio to be time-varying, while the error correction term characterizes the long-run relationship between the spot and futures prices.

Since the assumption of constant correlation may be too restrictive to fit in with reality, we adopt the DCC GARCH model proposed by Engle and Sheppard (2001) and Engle (2002) to release this restriction and improve the flexibility of the hedging models. In contrast with the CCC GARCH model, the DCC GARCH model allows the correlation R to be time-varying:

$$H_t = D_t R_t D_t = D_t J_t Q_t J_t D_t, \tag{7}$$

where D_t is the diagonal matrix of conditional standard deviations from univariate GARCH models, $Q_t=(q_{ij,t})_{2x2}$ is a positive definite matrix, $J_t = \mathrm{diag}\{q_{s,t}^{-1/2}, q_{f,t}^{-1/2}\}$, and Q_t satisfies:

$$Q_t = (1-\theta_1-\theta_2)\bar{Q} + \theta_1 \zeta_{t-1}\zeta_{t-1}^{'} + \theta_2 Q_{t-1}, \tag{8}$$

where ζ_t is the standardized disturbance vector, such that $\zeta_t = D_t^{-1}\varepsilon_t$, $\bar{Q}$ is the unconditional correlation matrix of ζ_t, and θ_1 and θ_2 are non-negative parameters satisfying $\theta_1 + \theta_2 < 1$.

Engle (2002) proposes the use of the two-stage maximum likelihood method for the estimation of the parameters for the DCC GARCH model. Let Ξ denote the parameters of the univariate GARCH model in D_t and Θ denote the other parameters in R_t. Under the assumption of normality, we decompose the likelihood function into a volatility component, $L_V(\Xi)$, and a correlation component, $L_C(\Xi,\Theta)$; i.e.:

$$L(\Xi,\Theta) = -\frac{1}{2}\sum_{t=1}^{T}(2\log(2\pi) + \log|H_t| + \hat{\varepsilon}_t' H_t^{-1}\hat{\varepsilon}_t) = L_V(\Xi) + L_C(\Xi,\Theta), \tag{9}$$

where $\hat{\varepsilon}_t$ is the residual vector of (3). In the first step, the parameters in the univariate GARCH models are estimated for each residual series. Taking the estimates $\hat{\Xi}$ as given and using the transformed residuals $\hat{\zeta}_t = \hat{D}_t^{-1}\hat{\varepsilon}_t$, we estimate the parameters of the dynamic correlation in the second step. Given the estimates $\hat{H}_t$ obtained in the CCC GARCH and the DCC GARCH models, the optimal dynamic hedge ratios is estimated by:

$$\hat{\delta}_t^* = \hat{h}_{sf,t} / \hat{h}_{f,t}^2. \tag{10}$$

3. THE COPULA-BASED GARCH MODEL

3.1 Model Specification

All of the models mentioned in the previous section are estimated under the assumption of multivariate normality. By contrast, the use of a copula function allows us to consider the marginal distributions and the dependence structure both separately and simultaneously. Therefore, the joint distribution of the asset returns can be specified with full flexibility, which will thus be more realistic.

Following Glosten et al. (1993) and Hansen (1994), we specify the GJR-skewed-*t* models for shocks in the spot and futures returns. Under the same error correction model (3), the conditional variance for asset *i*, $i = s, f$, is given by:

$$h_{i,t}^2 = c_i + b_i h_{i,t-1}^2 + a_{i,1}\varepsilon_{i,t-1}^2 + a_{i,2}k_{i,t-1}\varepsilon_{i,t-1}^2,$$

$$\varepsilon_{i,t} \mid \Psi_{t-1} = h_{i,t} z_{i,t}, \quad z_{i,t} \sim skewed\text{-}t(z_i \mid \eta_i, \phi_i), \tag{11}$$

with $k_{i,t-1} = 1$ when $\varepsilon_{i,t-1}$ is negative; otherwise $k_{i,t-1} = 0$. The density function of the skewed-*t* distribution is:

$$skewed\text{-}t\left(z \mid \eta, \phi\right) = \begin{cases} bc\left(1 + \dfrac{1}{\eta - 2}\left(\dfrac{bz + a}{1 - \phi}\right)^2\right)^{-\frac{\eta+1}{2}}, z < -\dfrac{a}{b} \\ bc\left(1 + \dfrac{1}{\eta - 2}\left(\dfrac{bz + a}{1 + \phi}\right)^2\right)^{-\frac{\eta+1}{2}}, z \geq -\dfrac{a}{b} \end{cases}. \tag{12}$$

The values of *a*, *b*, and *c* are defined as:

$$a \equiv 4\phi c \frac{\eta - 2}{\eta - 1}, \quad b \equiv 1 + 3\phi^2 - a^2, \quad \text{and} \quad c \equiv \frac{\Gamma\left(\dfrac{\eta + 1}{2}\right)}{\sqrt{\pi\left(\eta - 2\right)}\Gamma\left(\dfrac{\eta}{2}\right)},$$

where η is the kurtosis parameter and ϕ is the asymmetry parameter. These are restricted to $4 < \eta < 30$ and $-1 < \phi < 1$.[8] Thus, the specified marginal distributions of spot and futures returns are asymmetric, fat-tailed, and non-Gaussian.

Assume that the conditional cumulative distribution functions of z_s and z_f are $G_{s,t}(z_{s,t} \mid \Psi_{t-1})$ and $G_{f,t}(z_{f,t} \mid \Psi_{t-1})$, respectively. The conditional copula function, denoted as $C_t(u_t, v_t \mid \Psi_{t-1})$, is defined by the two time-varying cumulative distribution functions of random variables $u_t = G_{s,t}(z_{s,t} \mid \Psi_{t-1})$ and $v_t = G_{f,t}(z_{f,t} \mid \Psi_{t-1})$. Let Φ_t be the bivariate conditional cumulative distribution functions of $z_{s,t}$ and $z_{f,t}$. Using the

[8] This distribution becomes a symmetrical Student *t*-distribution when the asymmetry parameter ϕ is equal to 0 and turns to the standard normal distribution when the kurtosis parameter η approaches ∞.

Sklar theorem, we have:

$$\Phi_t(z_{s,t}, z_{f,t} \middle| \Psi_{t-1}) = C_t(u_t, v_t \mid \Psi_{t-1}) = C_t(G_{s,t}(z_{s,t} \middle| \Psi_{t-1}), G_{f,t}(z_{f,t} \middle| \Psi_{t-1}) \middle| \Psi_{t-1}). \qquad (13)$$

The bivariate conditional density function of $z_{s,t}$ and $z_{f,t}$ can be constructed as:

$$\begin{aligned} &\varphi_t(z_{s,t}, z_{f,t} \mid \Psi_{t-1}) \\ &= c_t(G_{s,t}(z_{s,t} \mid \Psi_{t-1}), G_{f,t}(z_{f,t} \mid \Psi_{t-1}) \mid \Psi_{t-1}) \times g_{s,t}(z_{s,t} \mid \Psi_{t-1}) \times g_{f,t}(z_{f,t} \mid \Psi_{t-1}), \end{aligned} \qquad (14)$$

where $c_t(u_t, v_t \mid \Psi_{t-1}) = \dfrac{\partial^2 C_t(u_t, v_t \middle| \Psi_{t-1})}{\partial u_t \partial v_t}$, $g_{s,t}(z_{s,t} \mid \Psi_{t-1})$ is the conditional density of $z_{s,t}$, and $g_{f,t}(z_{f,t} \mid \Psi_{t-1})$ is the conditional density of $z_{f,t}$.

3.2 Parameter Estimation

At time t, the log-likelihood function can be derived by taking the logarithm of (14):

$$\log \varphi_t = \log c_t + \log g_{s,t} + \log g_{f,t}. \qquad (15)$$

Let the parameters in $g_{s,t}$ and $g_{f,t}$ be respectively denoted as θ_s and θ_f while the other parameters in c_t are denoted as θ_c . These parameters can be estimated by maximizing the following log-likelihood function:

$$L_{s,f}(\theta) = L_s(\theta_s) + L_f(\theta_f) + L_c(\theta_c), \qquad (16)$$

with $\theta = (\theta_s, \theta_f, \theta_c)$ and L_k representing the sum of the log-likelihood function values across observations of the variable k.

Since the dimensions of the estimated equation may be quite large, it is difficult in practice to achieve a simultaneous maximization of $L_{s,f}(\theta)$ for all of the parameters.

In order to effectively solve this problem, we follow the two-stage estimation procedure proposed by Joe (1997)[9] and then adopted by Patton (2006b) and Bartram et al. (2007).

In the first stage, the parameters of the marginal distribution are estimated from the univariate time series by:

$$\begin{aligned} \hat{\theta}_s &\equiv \text{argmax} \sum_{t=1}^{T} \log g_{s,t}(z_{s,t} \mid \Psi_{t-1}; \theta_s), \\ \hat{\theta}_f &\equiv \text{argmax} \sum_{t=1}^{T} \log g_{f,t}(z_{f,t} \mid \Psi_{t-1}; \theta_f). \end{aligned} \tag{17}$$

In the second stage, given the marginal estimates obtained above, the dependence parameters are estimated by:

$$\hat{\theta}_c \equiv \text{argmax} \sum_{t=1}^{T} \log c_t(\Psi_{t-1}; \hat{\theta}_s, \hat{\theta}_f, \theta_c). \tag{18}$$

3.3 The Copula Functions

Three types of copulas are employed to combine the marginal distributions into the joint distributions. First, let ψ be the cumulative distribution function of the standard normal, and the Gaussian copula density function can be written as:

$$c_t^N(u_t, v_t | \rho_t) = \frac{1}{\sqrt{1-\rho_t^2}} \exp\left\{ -\frac{\rho_t^2(\psi^{-1}(u_t)^2 + \psi^{-1}(v_t)^2) - 2\rho_t \psi^{-1}(u_t)\psi^{-1}(v_t)}{2(1-\rho_t^2)} \right\}, \tag{19}$$

where ρ_t is constrained within the interval $(-1, 1)$. The Gaussian copula is symmetric and implies zero dependence in the extreme tails. The second copula we will consider

[9] It is called the inference functions for margins method; see Joe (1997).

is the Gumbel copula. The density function for the Gumbel copula is:

$$c_t^G(u_t,v_t\,|\kappa_t)=\frac{C_t^G(u_t,v_t\,|\kappa_t)(\ln u_t \ln v_t)^{\kappa_t-1}[(-\ln u_t)^{\kappa_t}+(-\ln v_t)^{\kappa_t}]^{\kappa_t^{-1}}+\kappa_t-1}{u_t v_t[(-\ln u_t)^{\kappa_t}+(-\ln v_t)^{\kappa_t}]^{2-\kappa_t^{-1}}}, \qquad (20)$$

where

$$C_t^G(u_t,v_t\,|\kappa_t)=\exp\{-[(-\ln u_t)^{\kappa_t}+(-\ln v_t)^{\kappa_t}]^{1/\kappa_t}\},$$

the association parameter $\kappa_t=(1-\tau_t)^{-1}$, $\tau_t\in(-1,1)$, and τ_t is Kendall's tau measuring the co-movements of markets in the presence of non-linear relationships. The Gumbel copula implies a higher dependence at right tails of the marginal distributions. Finally, the Clayton copula density function is:

$$c_t^C(u_t,v_t\,|\kappa_t)=\frac{(1+\kappa_t)(u_t^{-\kappa_t}+v_t^{-\kappa_t}-1)^{-2-\kappa_t^{-1}}}{(u_t v_t)^{\kappa_t+1}}, \qquad (21)$$

where $\kappa_t=2\tau_t/(1-\tau_t)$. The Clayton copula implies a higher dependence at left tails.

It should be noted that we can also use the Gumbel survival (Clayton survival) copula to construct the joint distribution with left (right) tail dependence.[10] Although Gumbel (Clayton) and Clayton survival (Gumbel survival) copulas have a similar dependence structure, there is no evidence yet for selecting an exclusive copula in applications of dynamic hedging. For comparison, we simply consider three types of original copulas which have been widely used in economic and financial applications. Gumbel and Clayton copulas are the two asymmetric Archimedean copulas suggested

[10] Replacing u_t (v_t) with 1-u_t (1-v_t), the density of the survival copula is a mirror image of the density of the original copula; i.e., $c^*(u_t,v_t)=c(1-u_t,1-v_t)$.

by Cherubini et al (2004) for financial applications.[11]

Following Patton (2006a) and Bartram et al. (2007), we assume that the dependence parameters ρ_t or τ_t rely on the previous dependences and historical information, (u_{t-1}-0.5)(v_{t-1}-0.5).[12] If the number of the latter term is positive, thereby indicating that both u_{t-1} and v_{t-1} are either bigger or smaller than the expectation (0.5), then we infer that the correlation is higher than when the number is negative. The time-varying parameters ρ_t and τ_t are specified respectively as:

$$\begin{aligned}(1-\beta_1 L)(1-\beta_2 L)\rho_t &= \omega + \gamma\left(u_{t-1}-0.5\right)\left(v_{t-1}-0.5\right),\\ (1-\beta_1 L)(1-\beta_2 L)\tau_t &= \omega + \gamma\left(u_{t-1}-0.5\right)\left(v_{t-1}-0.5\right),\end{aligned} \tag{22}$$

where both β_1 and β_2 are positive and satisfy $0 \le \beta_2 \le \beta_1 \le 1$, and then the copula parameters are $\theta_c = (\beta_1, \beta_2, \omega, \gamma)'$.

After estimating the parameters in different copula-based GARCH models, the conditional variances $h_{s,t}^2$ and $h_{f,t}^2$ are obtained from Equation (11), and the conditional covariance are generated by numerical integration with respect to Equation (14); i.e.:

$$h_{sf,t} = h_{s,t} h_{f,t} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z_{s,t} z_{f,t} \varphi(z_{s,t}, z_{f,t} \mid \Psi_{t-1}) dr dw. \tag{23}$$

The dynamic hedge ratios for the copula-based GARCH models are then calculated from Equation (10). Note that the proposed hedge ratios consider asymmetric specifications for the joint and marginal distributions of assets.

[11] Alternatively, Hu (2006) proposes a mixed copula model which combines a copula with its survival copula to take account of possible structures of tail dependence.

[12] Like ρ_t in the Gaussian copula, Kendall's tau τ_t in the Clayton and Gumbel copulas is bounded between -1 and +1. Therefore, we specify the conditional dependence process for the parameter τ_t instead of κ_t in the Clayton and Gumbel copula-based models.

4. EMPIRICAL RESULTS

4.1 Data and Diagnostic Analysis

This section examines the performances of alternative hedging models for stock index and currency futures. Two of the most highly traded stock index futures for the S&P 500 and FTSE 100 index are studied herein. To stress the effectiveness of our copula-based GARCH models, it may be more interesting to consider an application for which the futures are less correlated with the underlying asset. Hence, we also compare the performances of alternative models for cross-hedging the currency exposure of holding the MSCI Switzerland index (MSCI-SWI) with Swiss Franc (USD/SWF) futures. All data are obtained from Datastream, running from 2 January 1995 to 31 October 2005. The asset returns are the changes in the logarithm of the daily closing prices. Table 1 reports the results of diagnostic analysis.

<Table 1 is inserted about here>

The stylized facts of the asset returns, such as skewness, leptokurtosis, and significant Bera-Jarque statistics, are present in our data, implying that the unconditional distributions of spot and futures returns are asymmetric, fat-tailed, and non-Gaussian. The Ljung-Box tests show that there is no serial correlation in any of the S&P 500 and USD/SWF returns, however, serial correlation is displayed in the FTSE 100 and MSCI-SWI returns. Finally, both the $Q^2(24)$ and LM statistics for the ARCH effects present strong autocorrelations in the squared returns for all assets.

Table 2 reports the results of the unit root and cointegration tests. The augmented Dickey-Fuller (ADF) tests show that the spot and futures prices have a unit root, but first-differencing leads to stationarity. The Johansen trace statistics show that the spot and futures prices for stock markets are cointegrated and that the error correction terms should be considered in the model specification. Since the estimates of cointegrating parameters $\hat{\lambda}$ are close to 1 for the S&P 100 and FTSE 100 data, it is reasonable to impose the restriction λ=1 into the error correction models - that is, the error correction term becomes $S_{t-1} - F_{t-1}$.

<Table 2 is inserted about here>

The trace statistic shows that the MSCI-SWI index and USD/SWF futures are not cointegrated, and thus the error-correction term is omitted in (3). Because researchers often specify the conditional mean to be a function of previous returns, we include the MA(1) term in the cross-hedging models (e.g., see Baillie and Myers, 1991).

4.2 Estimation of the Parameters

Tables 3, 4, and 5 present the estimation results of the different hedging models, with all of the parameters having been estimated by the maximum likelihood method. For the CCC GARCH model (Table 3), the constant correlations ρ between the spot and futures returns for the two stock markets are positive and close to 1 (around 0.97). However, the correlation between the MSCI-SWI spot and USD/SWF futures is much lower (only 0.27). All $a_i + b_i$ estimates are also close to 1, which implies that shocks in the stock and futures markets have high persistence in volatility. Hence, the long-run average variance $\text{var}(f_t)$ is not a good proxy for use in calculating the short-horizon hedge ratio.

<Table 3 is inserted about here>

For the DCC GARCH model (Table 4), the $\theta_1 + \theta_2$ estimates are close to (but less than) 1, which implies that the correlations between the futures and underlying assets are highly persistent. Such high persistence means that shocks can push the correlation away from its long-run average for some considerable time, although the correlation is eventually mean-reverting. Replacing the CCC GARCH model with the DCC GARCH model can capture the variation in correlation between the spot and futures markets. Note that another interpretation for the high persistence in correlations may be due to non-linear behaviors in the joint and marginal distributions of assets (e.g., see de Lima, 1998).

<Table 4 is inserted about here>

For the copula-based GARCH models, Panel A of Table 5 shows the estimates of parameters for conditional means, variances, and marginal distributions. The significant $a_{i,2}$ estimates for the stock indices indicate that negative shocks have greater impacts than positive shocks on the conditional variances, with the asymmetric effect on volatility in the S&P 500 returns being stronger than that for the FTSE 100 returns. However, this asymmetric effect is not significant for currency (USD/SWF) futures, which is consistent with previous studies.

<Table 5 is inserted about here>

Panels B, C, and D of Table 5 show the estimates of parameters for different copula functions. Since the autoregressive parameter β_1 is greater than 0.9 for all copulas and portfolios, the dynamic copula parameters ρ_t and τ_t are highly persistent. It implies that shocks to the dependence structure between the spot and futures returns can persist for some considerable time, in turn affecting the estimated hedge ratio. The parameter γ is significantly positive at the 10% level, which suggests that the latest information on returns is an appropriate measure for modeling the dynamic dependence structure.

In terms of model fitting, the log-likelihood functions in most of the copula-based GARCH models are higher than those for the CCC GARCH and DCC GARCH models. Comparing with the Gumbel and Clayton copulas, the Gaussian copula has the highest log-likelihood for the direct hedge. This finding is consistent with the results of Malevergne and Sornette (2003) and Bartram et al. (2007), who demonstrate that returns from most pairs of major stock indices are compatible with the Gaussian copula. In other words, allowing the dependence measure to be time-varying could be more crucial than permitting the dependence structure to be asymmetric, because spot and futures returns in the direct hedge are tied closely by their no-arbitrage condition. However, the Gumbel copula has the highest log-likelihood for the cross hedge, in which spot and futures returns are less linearly correlated and thus allowing asymmetric dependence in dynamic hedging may also be important.

4.3 In-sample Comparison of Hedging Performance

This section evaluates the in-sample hedging performance of the different models. A hedge portfolio is composed of a spot asset and δ units of futures. For comparison, we calculate the variance of the returns to these portfolios over the sample:

$$\mathrm{var}(s_t - \delta_t^* f_t)\,, \tag{24}$$

where δ_t^* represents the estimated hedge ratios. Table 6 summarizes the results of the in-sample hedging performance for the different models.

<Table 6 is inserted about here>

Panel A of Table 6 considers the risk of hedged portfolios based on contemporaneous hedge ratios. It is shown that the copula-based GARCH models outperform the conventional and dynamic hedging models for stock markets, with the improvement over the conventional model by the Gaussian copula varying from 4.21% to 15.97% (the percentage change in variance reduction). As expected, the dynamic hedging models perform much better than the conventional hedging model for the FTSE 100 data, but the performance of the conventional hedging model is inferior to only the copula-based models for the S&P 500 index, demonstrating a smaller variance than both the CCC GARCH and DCC GARCH models.

The asymmetric copula-based models, such as Gumbel and Clayton copulas, perform more effectively for the cross hedge. The Gumbel copula has the largest variance reduction over other copula-based models, while the Gaussian copula is the worst. Empirical studies have found that stocks tend to crash together, but not boom together, which is not necessarily the case between stock and currency markets. The dependence could be asymmetric in either direction.

Panel A of Figure 1 plots the dynamics of Kendall's tau estimates generated from alternative copulas for the cross hedge.[13] This figure shows that the time-varying estimates of Kendall's tau are much lower than one and very volatile, which is consistent with Tong's (1996) argument that a continuous adjustment of the hedge portfolios is highly required in a cross hedge. Although the two paths of Kendall's tau generated from the Gaussian and Gumbel copulas are very close, it does not necessarily imply that these two models produce similar hedge ratios or hedging performance since the ratios and performance are determined not only by the dependence level, but also by the dependence structure of the two assets in the hedge portfolio.

<Figure 1 is inserted about here>

Following the evaluation of the contemporaneous variation in hedged portfolio returns in Panel A of Table 6, in Panels B and C we extend the respective holding period of the portfolios to one day and two days, as a check for robustness.[14] Again, as

[13] Those for direct hedge are available upon request.

[14] For example, we form a hedge portfolio at time *t* and evaluate the portfolio variance with the realized

compared to the other three hedging models, the copula-based GARCH models are most effective in reducing the variances of hedged portfolios. As expected, the amount of variance reduction becomes less significant when the portfolio holding period is prolonged. For the two-day holding period, the Gumbel and Clayton copula-based models are even inferior to the conventional hedge method for the S&P 500 and FTSE 100 data.

The in-sample comparison suggests overall that the proposed models have greater hedging effectiveness than either the conventional approach or the other two GARCH models. These findings support that when estimating the optimal hedge ratio, not only is it important to have time-varying variances, but it is of considerable value to employ suitable distribution specifications for the time series, as evidenced by the superior performance of the copula-based GARCH models.

4.4 Out-of-sample Comparison of Hedging Performance

When a model achieves good in-sample performance, it does not necessarily achieve good out-of-sample performance, because such 'over-fitting' could be quite penalizing. If we set out to use a different hedging model, our concern is certainly more about how well we can do in the future. It is therefore necessary to determine whether the proposed model still works well in terms of its out-of-sample hedging performance.

To compare such a performance, we adopt a method that involves rolling over a

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asset prices at time t+1 or t+2.

particular number of samples to determine the series of out-of-sample hedge ratios. More specifically, we take 2,000 observations from the sample (for example, from 2 January 1995 to 29 November 2002) and use these observations to estimate all of the hedging models. We then forecast the hedge ratio for the next day (30 November 2002) by computing the one-period-ahead covariance forecast divided by the one-period-ahead variance forecast. The calculation is repeated for the following day (1 December 2002), using the nearest available 2,000 observations before that day (from 3 January 1995 to 30 November 2002). By continually updating the model estimation through to the end of the dataset, we complete 733, 739, and 762 one-period forecasted hedge ratios for the S&P 500, FTSE 100, and MSCI-SWI indices, respectively.

Table 7 reports the one-period-ahead out-of-sample evaluation results of the different hedging models based on the forecasted hedge ratios. The hedging effectiveness of the copula-based GARCH model outperforms the other hedging methods for the FTSE 100 and MSCI-SWI indices, while the DCC GARCH model has better hedging performance for the S&P 500 index. In line with our in-sample findings, the Gaussian and Gumbel copulas have relative advantages for the out-of-sample direct and cross hedges, respectively. Hence, both allowing the dependence measure to be time-varying and permitting the dependence structure to be asymmetric are useful to improve hedging effectiveness. As shown in Panel B of Figure 1, the out-of-sample dynamics of Kendall’s tau estimates generated from alternative copulas for the cross hedge have the same properties as the in-sample dynamics mentioned earlier.

<Table 7 is inserted about here>

The copula-based GARCH models in general provide the best performance in both in-sample and out-of-sample hedges, with the DCC GARCH model in second place, followed by the CCC GARCH model and the conventional model in that order. Of the copula-based models, the Gaussian copula has the best performance for the direct hedge, while the Gumbel copula leads to the lowest portfolio variances for the cross hedge. Therefore, by specifying the joint distribution of spot and futures returns with full flexibility, we can use the copula-based GARCH models to effectively reduce the risk in hedged portfolios.

5. CONCLUSIONS

In this paper we have proposed a class of new copula-based GARCH models to estimate risk-minimizing hedge ratios and have compared the hedging effectiveness of the model with that of three other models: conventional, constant conditional correlation GARCH, and dynamic conditional correlation GARCH hedging models. Through different copula functions, including Gaussian, Gumbel, and Clayton copulas, the proposed models can specify the joint distribution of the spot and futures returns with full flexibility and hence the distribution is quite realistic. Since the marginal and joint distributions can be specified separately and simultaneously, we estimate the conditional variance and covariance to obtain the optimal hedge ratio without the restrictive assumption of multivariate normality.

With full flexibility in the distribution specifications, the hedging effectiveness based on the proposed models is substantially improved as compared to the alternative models. The in-sample evidence and out-of-sample evidence for the direct hedge and cross hedge both indicate that the copula-based GARCH model outperforms other hedging methods. Therefore, with more precise specification of the joint distribution of assets, we can effectively manage the risk exposure of portfolios. These findings have crucial implications for risk management.

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Table 1. Summary statistics

Statistics	Asset S&P 500		FTSE 100		MSCI-SWI	USD/SWF
	Stock	Futures	Stock	Futures	Spot	Futures
Mean	0.0004	0.0004	0.0002	0.0002	0.0004	5.84e-6
Std. Dev.	0.0112	0.0116	0.0110	0.0115	0.0115	0.0071
Skewness	−0.1095	−0.1688	−0.1819	−0.1275	−0.0286	0.1880
Kurtosis	6.2742	6.7109	5.8887	5.6690	6.4756	4.8929
J-B	1225.32*	1580.01*	966.71*	819.81*	1370.46*	422.40*
$Q(24)$	34.37	30.84	82.57*	77.24*	47.36*	27.29
$Q^2(24)$	1065.20*	961.30*	3214.60*	2838.10*	1513.72*	162.73*
ARCH(5)	259.00*	243.50*	508.60*	467.80*	340.61*	52.57*

Notes:

[a] The sample period for the daily spot and futures returns runs from 2 January 1995 to 31 October 2005 and excludes holidays.

[b] J-B is the Jarque-Bera test for normality; $Q(24)$ is the Ljung-Box statistic for up to the 24th order serial correlation in the returns; $Q^2(24)$ is the Ljung-Box statistic for the serial correlations in the squared returns; and ARCH(5) is the LM test for up to the 5th order ARCH effects.

[c] * indicates significance at the 1% level.

Table 2. Unit root and cointegration tests

Statistics	Asset S&P 500		FTSE 100		MSCI-SWI	USD/SWF
	Stock	Futures	Stock	Futures	Spot	Futures
ADF (price)	−2.68	−2.66	−2.15	−2.12	−1.71	−1.46
ADF (return)	−53.09*	−53.94*	−33.92*	−34.40*	−49.34*	−54.38*
Trace	46.89*	–	48.75*	–	6.13	–
$\hat{\lambda}$	0.9995	–	0.9997	–	0.3718	–

Notes:

[a] The sample period for the daily spot and futures returns runs from 2 January 1995 to 31 October 2005 and excludes holidays.

[b] The ADF tests are applied to test the null hypothesis of a unit root for the spot and futures prices and the returns; the number of lags in the ADF tests is determined by the Schwarz information criterion; Trace is the Johansen trace test, with the null hypothesis being that there is no cointegration; $\hat{\lambda}$ is the estimated cointegrating parameter.

[c] * indicates significance at the 1% level.

Table 3. Constant correlation GARCH model estimations

Parameters	Asset: S&P 500		FTSE 100		MSCI-SWI	USD/SWF
	$i=s$	$i=f$	$i=s$	$i=f$	$i=s$	$i=f$
α_{0i}	0.0002	0.0008	0.0004	0.0007	0.0006	–4.40e-5
	(0.4839)	(0.0008)	(0.0272)	(0.0002)	(0.0009)	(0.7258)
α_{1i}	−0.1536	0.0502	−0.0139	0.1605	0.0439	–0.0319
	(0.0006)	(0.0268)	(0.0277)	(0.0001)	(0.0305)	(0.1387)
c_i	7.36e-7	9.64e-7	9.16e-7	1.08e-6	3.63e-6	8.09e-7
	(0.0000)	(0.0000)	(0.0009)	(0.0008)	(0.0000)	(0.0735)
a_i	0.0691	0.0728	0.0835	0.0911	0.0985	0.0231
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0041)
b_i	0.9226	0.9266	0.9104	0.9031	0.8728	0.9606
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.000)	(0.000)
ρ	0.9705		0.9728		0.2738	
	(0.0000)		(0.0000)		(0.0000)	
Log-likelihood	21270		21705		18625	

Notes:

[a] The table presents the maximum likelihood estimates of the constant conditional correlation GARCH model.

[b] Figures in parentheses are *p*-values, where 0.0000 indicates that the value is less than 0.00005. The model is described as follows:

$$\begin{aligned} s_t &= \alpha_{0s} + \alpha_{1s}(S_{t-1} - F_{t-1}) + \varepsilon_{st} \\ f_t &= \alpha_{0f} + \alpha_{1f}(S_{t-1} - F_{t-1}) + \varepsilon_{ft}, \end{aligned} \quad \varepsilon_t \sim N(0, H_t)$$

$$H_t = \begin{bmatrix} h_{s,t}^2 & h_{sf,t} \\ h_{sf,t} & h_{f,t}^2 \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} = D_t R D_t,$$

$$\begin{aligned} h_{s,t}^2 &= c_s + a_s \varepsilon_{s,t-1}^2 + b_s h_{s,t-1}^2 \\ h_{f,t}^2 &= c_f + a_f \varepsilon_{f,t-1}^2 + b_f h_{f,t-1}^2 \end{aligned}.$$

[c] For the cross hedge between the dollar value of the MSCI Switzerland index and the USD/SWF futures, the coefficient estimates of the error correction terms are replaced by the estimates of the MA(1) terms due to the inexistence of co-integration.

Table 4. DCC GARCH model estimations

Parameters	Asset: S&P 500		FTSE 100		MSCI-SWI	USD/SWF
	$i=s$	$i=f$	$i=s$	$i=f$	$i=s$	$i=f$
θ_1	0.0306		0.0279		0.0191	
	(0.0011)		(0.0244)		(0.0000)	
θ_2	0.9631		0.9675		0.9768	
	(0.0000)		(0.0000)		(0.0000)	
Log-likelihood	21392		21821		18684	

Notes:

[a] The table presents the maximum likelihood estimates of the dynamic conditional correlation GARCH model. Because the marginal processes are identical to those in the CCC GARCH model presented in Table 3, only the estimates of correlation parameters are reported.

[b] Figures in parentheses are *p*-values, where 0.0000 indicates that the value is less than 0.00005. The correlation process is specified as follows:

$$H_t = D_t R_t D_t = D_t J_t Q_t J_t D_t,$$

$$Q_t = (q_{ij,t})_{2\times 2} \qquad J_t = \mathrm{diag}\{q_{s,t}^{-1/2}, q_{f,t}^{-1/2}\}$$

$$Q_t = (1-\theta_1-\theta_2)\bar{Q} + \theta_1 \zeta_{t-1}\zeta_{t-1}^{'} + \theta_2 Q_{t-1}.$$

[c] For the cross hedge between the dollar value of the MSCI Switzerland index and the USD/SWF futures, the coefficient estimates of the error correction terms are replaced by the estimates of the MA(1) terms due to the inexistence of co-integration.

Table 5. Copula-based GARCH model estimations

Parameters	Asset					
	S&P 500		FTSE 100		MSCI-SWI	USD/SWF
	$i=s$	$i=f$	$i=s$	$i=f$	$i=s$	$i=f$
Panel A: Estimates of marginal processes						
α_{0i}	0.0001	0.0003	0.0001	0.0003	0.0003	–3.40e-5
	(0.5271)	(0.0699)	(0.5716)	(0.0608)	(0.0595)	(0.8030)
α_{1i}	−0.0313	0.0110	−0.0482	0.0730	0.0336	–0.0648
	(0.1453)	(0.1040)	(0.0371)	(0.0040)	(0.0200)	(0.0004)
c_i	1.32e-6	1.51e-6	8.08e-7	8.45e-7	3.44e-6	7.68e-7
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0213)
$a_{i,1}$	2.50e-14	4.07e-13	1.33e-12	0.0106	0.0232	0.0204
	(1.0000)	(1.0000)	(1.0000)	(0.0215)	(0.0331)	(0.0000)
$a_{i,2}$	0.1402	0.1413	0.1058	0.1040	0.0999	9.72e-13
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.9999)
b_i	0.9212	0.9183	0.9384	0.9309	0.8973	0.9646
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
η_i	11.8024	9.4632	19.9499	15.9410	11.8813	6.2239
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
ϕ_i	–0.0973	–0.1322	–0.1248	-0.1164	–0.0679	0.0639
	(0.0001)	(0.0000)	(0.0000)	(0.0000)	(0.0200)	(0.0055)
Panel B: Estimates of Gaussian dependence processes						
ω	0.0024		−0.0017		0.0015	
	(0.0000)		(0.0000)		(0.2584)	
β_1	0.9973		0.9317		0.9790	
	(0.0000)		(0.0000)		(0.0000)	
β_2	0.0002		0.0003		0.0003	
	(0.9767)		(0.8141)		(0.9996)	
γ	0.0034		0.0039		0.2054	
	(0.0011)		(0.0000)		(0.0163)	
Log-likelihood	21859		21978		18795	

Table 5. Copula-based GARCH model estimations (cont'd)

Panel C: Estimates of Gumbel dependence processes			
ω	0.0010	0.0022	0.0015
	(0.1030)	(0.1199)	(0.9985)
β_1	0.9978	0.9964	0.9693
	(0.0000)	(0.0000)	(0.0049)
β_2	1.40e-6	1.22e-6	0.0013
	(0.9999)	(0.9999)	(0.9999)
γ	0.0107	0.0101	0.1801
	(0.0007)	(0.0064)	(0.0980)
Log-likelihood	21772	21857	18797
Panel D: Estimates of Clayton dependence processes			
ω	0.0085	0.0252	0.0002
	(0.0142)	(0.0850)	(0.9988)
β_1	0.9857	0.9656	0.9875
	(0.0000)	(0.0000)	(0.0000)
β_2	0.0005	0.0002	0.0092
	(0.9910)	(0.9955)	(0.9995)
γ	0.0298	0.0201	0.0719
	(0.0004)	(0.0918)	(0.0927)
Log-likelihood	21627	21752	18749

Notes:

[a] The table presents the maximum likelihood estimates of the copula-based GARCH model.

[b] Figures in parentheses are *p*-values, where 0.0000 indicates that the value is less than 0.00005. The copula-based GARCH models are described as follows:

$$s_t = \alpha_{0s} + \alpha_{1s}(S_{t-1} - F_{t-1}) + \varepsilon_{st} \qquad \varepsilon_{i,t} \mid \Psi_{t-1} = h_{i,t} z_{i,t}$$

$$f_t = \alpha_{0f} + \alpha_{1f}(S_{t-1} - F_{t-1}) + \varepsilon_{ft}, \qquad z_{i,t} \sim skewed - t(z_i \mid \eta_i, \phi_i)$$

$$h_{s,t}^2 = c_s + b_s h_{s,t-1}^2 + a_{s,1}\varepsilon_{s,t-1}^2 + a_{s,2} k_{s,t-1}\varepsilon_{s,t-1}^2$$

$$h_{f,t}^2 = c_f + b_f h_{f,t-1}^2 + a_{f,1}\varepsilon_{f,t-1}^2 + a_{f,2} k_{f,t-1}\varepsilon_{f,t-1}^2,$$

$$c_t^N(u_t, v_t | \rho_t) = \frac{1}{\sqrt{1-\rho_t^2}} \exp\left\{ -\frac{\rho_t^2(\psi^{-1}(u_t)^2 + \psi^{-1}(v_t)^2) - 2\rho_t \psi^{-1}(u_t)\psi^{-1}(v_t)}{2(1-\rho_t^2)} \right\},$$

$$c_t^G(u_t, v_t | \kappa_t) = \frac{C_t^G(u_t, v_t | \kappa_t)(\ln u_t \ln v_t)^{\kappa_t - 1}[(-\ln u_t)^{\kappa_t} + (-\ln v_t)^{\kappa_t}]^{\kappa_t^{-1}} + \kappa_t - 1}{u_t v_t[(-\ln u_t)^{\kappa_t} + (-\ln v_t)^{\kappa_t}]^{2-\kappa_t^{-1}}},$$

$$c_t^C(u_t, v_t | \kappa_t) = \frac{(1+\kappa_t)(u_t^{-\kappa_t} + v_t^{-\kappa_t} - 1)^{-2-\kappa_t^{-1}}}{(u_t v_t)^{\kappa_t + 1}},$$

$$(1 - \beta_1 L)(1 - \beta_2 L)\rho_t = \omega + \gamma(u_{t-1} - 0.5)(v_{t-1} - 0.5), \text{ or}$$

$$(1 - \beta_1 L)(1 - \beta_2 L)\tau_t = \omega + \gamma\left(u_{t-1} - 0.5\right)\left(v_{t-1} - 0.5\right).$$

[c] For the cross hedge between the dollar value of the MSCI Switzerland index and the USD/SWF futures, the coefficient estimates of the error correction terms are replaced by the estimates of the MA(1) terms due to the inexistence of co-integration.

Table 6. Comparison of the effectiveness of in-sample hedging

Models	Portfolio Variance			Variance Reduction over the Conventional Method (%)		
	S&P 500	FTSE 100	MSCI-SWI	S&P 500	FTSE 100	MSCI-SWI
Panel A: $\text{var}(s_t - \delta_t^* f_t)$						
Conventional	0.0712	0.0626	1.2480			
CCC GARCH	0.0725	0.0593	1.3069	–0.0013 (−1.83)	0.0033 (5.27)	–0.0589 (−4.72)
DCC GARCH	0.0719	0.0587	1.2383	–0.0007 (−0.98)	0.0039 (6.23)	0.0097 (0.78)
Gaussian Copula	0.0682	0.0526	1.2395	0.0030 (4.21)	0.0100 (15.97)	0.0085 (0.68)
Gumbel Copula	0.0690	0.0529	1.2361	0.0022 (3.09)	0.0097 (15.50)	0.0119 (0.95)
Clayton Copula	0.0695	0.0589	1.2379	0.0017 (2.39)	0.0037 (5.91)	0.0101 (0.81)
Panel B: $\text{var}(s_{t+1} - \delta_t^* f_{t+1})$						
Conventional	0.0713	0.0626	1.2481			
CCC GARCH	0.0722	0.0601	1.3049	–0.0009 (−1.26)	0.0025 (3.99)	–0.0568 (−4.55)
DCC GARCH	0.0718	0.0596	1.2382	–0.0005 (−0.70)	0.0030 (4.79)	0.0099 (0.79)
Gaussian Copula	0.0671	0.0524	1.2397	0.0042 (5.89)	0.102 (16.29)	0.0084 (0.67)
Gumbel Copula	0.0677	0.0530	1.2366	0.0036 (5.05)	0.0096 (15.34)	0.0115 (0.92)
Clayton Copula	0.0683	0.0597	1.2372	0.0030 (4.21)	0.0029 (4.63)	0.0109 (0.87)
Panel C: $\text{var}(s_{t,t+2} - \delta_t^* f_{t,t+2})$						
Conventional	0.0707	0.0677	1.2483			
CCC GARCH	0.0732	0.0681	1.3045	–0.0025 (−3.54)	–0.0004 (−0.59)	–0.0562 (−4.50)
DCC GARCH	0.0722	0.0667	1.2400	–0.0015 (−2.12)	0.0010 (1.48)	0.0083 (0.66)
Gaussian Copula	0.0704	0.0674	1.2424	0.0003 (0.42)	0.0003 (0.44)	0.0059 (0.47)
Gumbel Copula	0.0708	0.0676	1.2392	–0.0001 (−0.14)	0.0001 (0.15)	0.0091 (0.73)
Clayton Copula	0.0711	0.0683	1.2406	–0.0004 (−0.57)	–0.0006 (−0.89)	0.0077 (0.62)

Notes:

[a] We evaluate the effectiveness of the hedging models with the contemporaneous, 1-day-later, and 2-day-later prices, for which the respective results are reported in Panels A, B, and C.

[b] Computation of the variance in portfolio returns is carried out by Equation (24) and all reported values of variance have been multiplied by 10^4.

[c] The variance reduction over the conventional method is given as $-(\sigma_i^2 - \sigma_{conventional}^2)$. The numbers in the parentheses, in percentage terms, are given as:

$$-100(\sigma_i^2 - \sigma_{conventional}^2)/\sigma_{conventional}^2 .$$

Table 7. Comparison of the effectiveness of out-of-sample hedging

Models	Portfolio Variance			Variance Reduction over the Conventional Method		
	S&P 500	FTSE 100	MSCI-SWI	S&P 500	FTSE 100	MSCI-SWI
$\mathrm{var}(s_{t+1} - \delta^*_{t+1\|t} f_{t+1})$						
Conventional	0.0308	0.0280	0.8914			
CCC GARCH	0.0288	0.0268	0.9179	0.0020 (6.49)	0.0012 (4.29)	–0.0265 (–2.97)
DCC GARCH	0.0285	0.0266	0.8442	0.0023 (7.47)	0.0014 (5.00)	0.0472 (5.30)
Gaussian Copula	0.0288	0.0261	0.8428	0.0020 (6.49)	0.0019 (6.79)	0.0486 (5.45)
Gumbel Copula	0.0290	0.0263	0.8375	0.0018 (5.84)	0.0017 (6.07)	0.0539 (6.05)
Clayton Copula	0.0295	0.0278	0.8533	0.0013 (4.22)	0.0002 (0.71)	0.0381 (4.27)

Notes:

[a] The method adopted to determine the series of out-of-sample hedge ratios involves rolling over a particular number of samples (2000 observations). The variation in portfolio returns is evaluated using the spot and futures returns which are observed at the forecasted date.

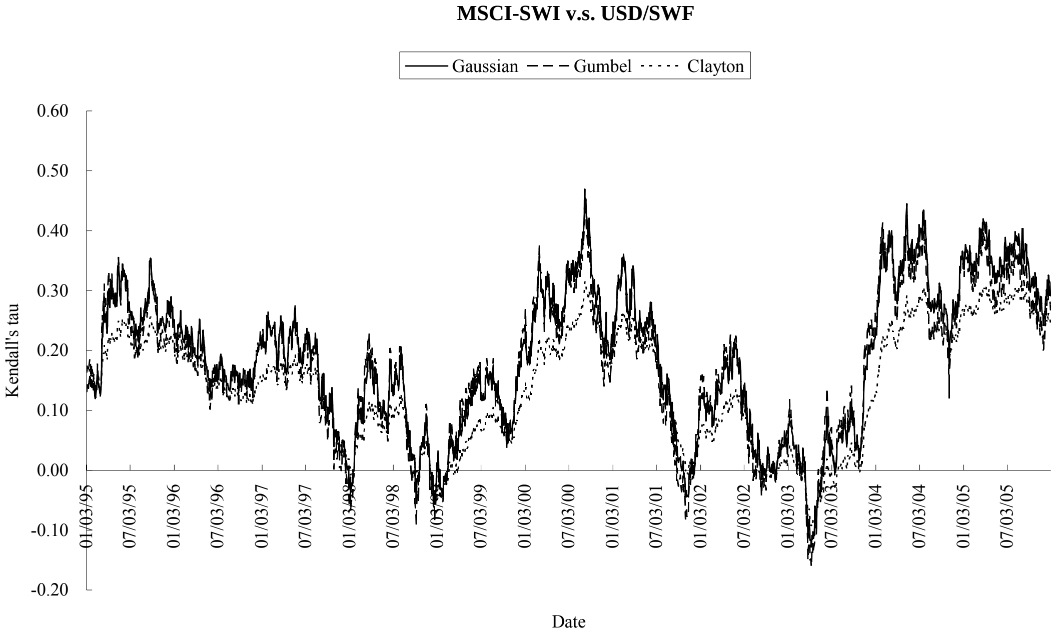
[b] Computation of the variance in portfolio returns is carried out by Equation (24) and all reported values of variance have been multiplied by 10^4.

[c] The variance reduction over the conventional method is given as $-(\sigma_i^2 - \sigma_{conventional}^2)$. The numbers in the parentheses, in percentage terms, are given as:

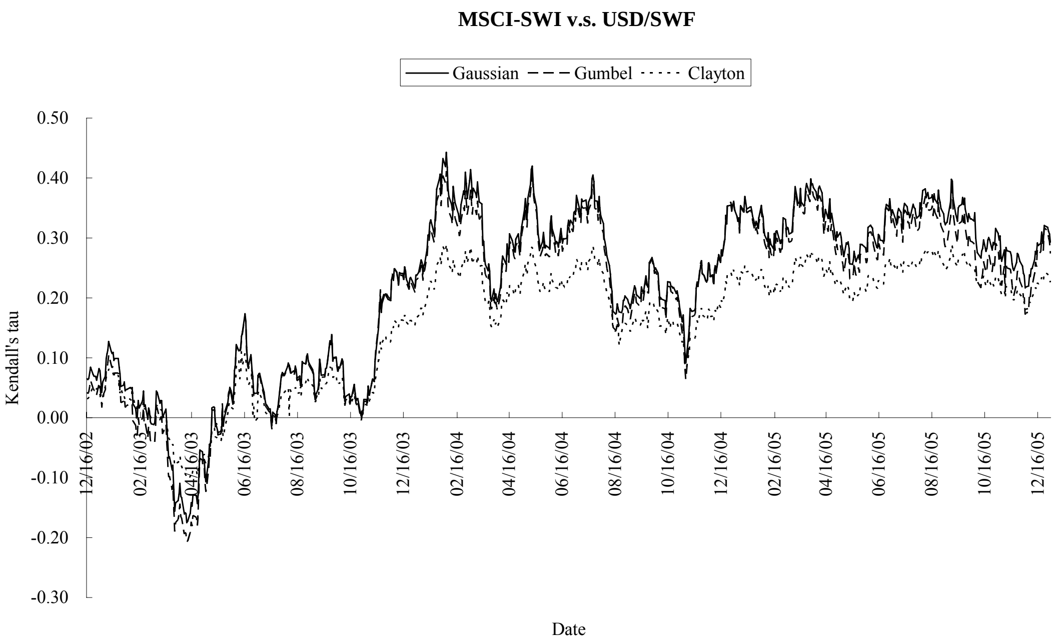
$$-100(\sigma_i^2 - \sigma_{conventional}^2)/\sigma_{conventional}^2 .$$

Figure 1. The Time-varying Dependence from Alternative Copula Models

Panel A: In-sample



Panel B: Out-of-sample



Notes:

[a] This figure shows the in-sample and out-of-sample estimates of time-varying Kendall's tau between the dollar value of the MSCI Switzerland index and USD/SWF futures