Option Implied Cost of Equity and Its Properties

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June 23, 2008

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We would like to thank the comments of Betty Simkins, Ali Nejadmalayeri, Chung-Ying Yeh, and the editor Bob Webb. We are particularly grateful for the suggestions and comments of an anonymous referee. The financial support of National Science Council of Taiwan and the research assistance of Der-Fong Chen, Hsuan-Li Su, Yun-Yi Wang, and Pei-Shih Weng are acknowledged.
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Abstract

The estimation of the cost of equity capital (COE) is one of the most important tasks in financial management. Existing approaches compute the COE using historical data, i.e. they are backward-looking methods. This paper derives a method to calculate forward-looking estimates of the COE using the current market prices of stocks and stock options. Our estimates of the COE reflect the expectation of the market investors about the COE during the life of the investment project. We test empirically our method and compare it with the Fama/French (1993) three-factor model for the S&P 100 firms. The empirical results indicate that our COE estimates (1) are plausible and stable over the years as required by appropriate discount rates for capital budgeting, (2) yield an equity risk premium close to the market equity risk premium reported by Fama and French (2002), (3) generate strong return-risk relationships, and (4) are significantly related with investor sentiment.

JEL codes: G12; G13; G31.

Keywords: Cost of capital; Capital budgeting; Fama/French three-factor model; Equilibrium option prices; Black-Scholes.
1. Introduction

The estimation of the cost of equity capital (COE) is an important issue for both practitioners and academics. The COE is widely used in applications such as the valuation of an investment project of a firm and the estimation of equity risk premiums. In particular, the COE often affects how the services of a firm in the public sector are regulated by its supervising commission. Therefore, the estimation precision of the COE has a significant impact on a firm’s value. According to the survey of Bruner, Eades, Harris, and Higgins (1998) and Graham and Harvey (2001), the most popular market-based methods for estimating the COE in practice are the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), the average historical returns, and a multibeta CAPM (with extra risk factors in addition to the market beta).\footnote{The risk factors include the fundamental factors (Fama and French, 1993), the momentum (Jegadeesh and Titman, 1993), and the macroeconomic factors (Chen, Roll, and Ross, 1986; Ferson and Harvey, 1993).} Although these methods are simple to apply, they all rely exclusively on historical data, i.e. they are backward-looking methods. Since the COE estimates are usually aimed to serve as the discount rate for future cash flows of an investment project, a backward-looking method may not perform well unless the patterns of COE are known and stable over the years in the future. As a result, these estimates of COE are usually imprecise, especially when they are applied to estimate the COE of an industry. For example, Fama and French (1997) pointed out that the standard errors of the COE estimates are typically above 3.0\% per year.

In contrast to the above backward-looking methods, this paper provides a forward-looking method to estimate the COE using the current market prices of equity and equity options.\footnote{Some accounting-based models also utilize forward information such as analyst forecasts. But we focus on the COE estimation using market-based models only.} The option market prices are widely used to estimate implied volatility, which is commonly found to be the best predictor for future volatility (see e.g. Poon and Granger (2003) for a detailed survey on this issue).\footnote{About the information content and forecasting performance of implied volatility based on option market prices see also Day and Lewis (1992), Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Christensen and Prabhala (1998), Blair, Poon, and Taylor (2001), Pong, Shackleton, Taylor, and Xu (2004),
contain incremental or superior information in addition to the information provided by historical data because they reflect market expectations. Inspired by the implied volatility literature, one can expect that the estimates of the COE based on option market prices may contain incremental or superior information in addition to the information contained in the traditional estimates of the COE obtained with historical data.

To obtain the COE implied by option market prices, we first develop an option pricing model in which the expected return of the underlying asset is a tractable parameter. To the best of our knowledge there are only two papers in the literature that discuss the estimation of the expected return of assets using option market prices. Heston (1993) presented an option pricing formula based on the log-gamma distribution under which the expected return of the stock is determined by both the location and the volatility parameters. Unfortunately, his pricing formula depends on the location parameter \( \mu \) but is independent of the volatility parameter, \( \sigma \). Hence this option pricing model alone can not be used to estimate the COE. McNulty, Yeh, Schulze, and Lubatkin (2002) also developed a forward-looking approach to calculate the COE based on option market prices. Although their approach is interesting, the method is ad hoc and lacks theoretical support. In contrast to Heston (1993) and McNulty et al. (2002), our option pricing formula not only depends on the expected return of the underlying asset or COE but is also derived in an equilibrium representative agent economy. Hence, our COE estimates are obtained in a general equilibrium model. Moreover, our option pricing formula is analytically tractable. Thus our option pricing model can be easily applied to estimate the COE of a firm or industry.

We compare our estimates of the COE for the market and industry portfolios composed by the component firms of the S&P 100 index with the estimates obtained with the Fama/French three-factor model from January 1996 to December 2005.\(^4\) There are at least four interesting findings from our empirical results. First, our option-implied COE estimates are more reasonable and stable over the years than those obtained with the Fama/French method,

\(^4\)Fama and French (1993, 1996) indicate that the three-factor model can describe the expected returns of financial assets more appropriately than the CAPM.
and thus are more reliable discount rates for capital budgeting. The mean and volatility of our estimates of averaged annual COE over the sample period is about 11% and 3%, respectively. In contrast, the mean level of the Fama/French’s estimates is too high (14%), and their values sometimes are extremely high (e.g. 63%) or even negative. Second, the equity risk premium of the market and industry portfolios calculated from our COE estimates is consistent with the existing literature on equity risk premium. For example, the equity premium of the market portfolio from our COE estimates is 6.96 percent which is close to the average equity premium reported by Fama and French (2002) of 7.43 percent. Third, the return-risk relationship for various industry portfolios is stronger using the option-implied COE estimates than using the Fama/French estimates. Finally, our COE estimates are significantly associated with investor sentiment proxied by the VIX index and the sentiment index of Baker and Wurgler (2006), while we find no obvious return-sentiment relationships for the Fama/French estimates.

With forward-looking information, option prices provide a reliable source for estimating the COEs for both the market and industry portfolios. Therefore, this study contributes to the literature not only by developing an option pricing model in which the expected return is tractable, but also by providing a plausible and reliable alternative for the COE estimation.

Ferguson and Shockley (2003) advance with a theoretical rationale for the three-factor model of Fama-French (1993). They show, when equity is a call written on a firm, that loadings on portfolios formed on relative leverage and relative distress subsume the powers of the Fama and French (1993) returns to small minus big market capitalization (SMB) portfolios and returns to high minus low book-to-market (HML) portfolios factors in explaining cross-sectional returns. Our paper also extends their results for our setting.

The theoretical set up of our paper is closer to Brennan (1979), Stapleton and Subrahmanyan (1984) and Camara (2003, 2005). In ours, like in these papers, there is a single-period economy. It is assumed that the stock price has a continuous distribution at the end of the period, and that dynamic trading does not exist. In such situation a riskless hedge

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5See also the important papers by Rubinstein (1979) and Schroder (2004).
is not possible to construct and to maintain, and markets are incomplete. In order to price options in this single-period economy, it is assumed that there is a nonsatiated, risk-averse representative agent who maximizes his expected end-of-period utility of wealth when he selects his optimal portfolio. While in that research the authors looked for utility functions and distributions of wealth that could be linked and produce preference-free option pricing formulas, we search for utility functions and distributions of wealth that can be linked and produce an option pricing formula dependent of the expected rate of return of the stock or cost of equity capital (COE).

The remainder of this paper is organized as follows. Section 2 derives an equilibrium option pricing model whose pricing formulae depends on the expected (mean) return or COE. Section 3 discusses the empirical implementation procedures and describes the data. Section 4 presents the empirical results for the component firms of the S&P 100 index. Section 5 extends the Ferguson and Shockley (2003) model to our setting. We provide some concluding remarks in Section 6.

2. The Option Valuation Model

This section starts by presenting our assumptions on the preferences of the representative agent and the stochastic behavior of aggregate wealth. Then we derive a pricing kernel that avoids arbitrage opportunities to arise in the economy. Assuming that stock prices under the actual probability measure are lognormally distributed, we obtain the equilibrium probability density function that is used to price all the assets in the economy. We derive closed-form solutions for call and put prices in this representative agent economy.

We assume that there is a representative agent with the following marginal utility function of aggregate wealth:

\[ U'(W_T) = W_T^\alpha + \beta, \]  

(1)

where aggregate wealth, \( W_T \), is positive, and \( \alpha < 0 \) and \( \beta \geq 0 \) are preference parameters.
The representative agent is nonsatiated and risk-averse since \( U'(W_T) > 0 \) and \( U''(W_T) < 0 \) respectively. It can easily be verified that the preferences of the investor are also characterized by decreasing absolute risk aversion (DARA) and decreasing proportional risk aversion (DPRA).\(^6\)

Elton and Gruber (1995, p. 218) argue that “while there is general agreement that most investors exhibit decreasing absolute risk aversion (DARA), there is much less agreement concerning relative risk aversion”. Later, Zhou (1998, p. 1730) provides empirical evidence that “justifies the assumption that consumer’s utility function exhibits DPRA”.

We assume that aggregate wealth, \( W_T \), has a lognormal distribution:

\[
W_T \sim \Lambda \left( \ln W_0 + (\mu - \frac{1}{2}\sigma_w^2)T, \sigma_w^2T \right).
\]

This assumption precludes negative wealth, and allow us to obtain tractable results. We start by obtaining the pricing kernel of the economy.

**Lemma 1. (The pricing kernel)** Assume that the marginal utility function of the representative agent is given by equation (1) and that aggregate wealth has a lognormal distribution as in equation (2). Then the pricing kernel is given by:

\[
\phi(W_T) = \frac{\beta + W_0^\alpha}{\beta + W_0^\alpha \exp \left( \alpha \mu_w T + (\alpha^2 - \alpha) \frac{\sigma_w^2}{2} T \right)}.
\]

**Proof:** By definition (see e.g. Camara (2003)), the pricing kernel is given by:

\[
\phi(W_T) = \frac{U'(W_T)}{EP[U'(W_T)]}.
\]

Since \( U'(W_T) = \beta + W_0^\alpha \), we have \( EP[U'(W_T)] = \beta + EP[W_0^\alpha] \), where \( P \) is the actual probability measure. Also, since \( W_T \sim \Lambda(\ln W_0 + (\mu - \frac{1}{2}\sigma_w^2)T, \sigma_w^2T) \) we have \( W_0^\alpha \sim \Lambda(\ln W_0^\alpha + (\alpha \mu - \frac{1}{2}\alpha\sigma_w^2)T, \alpha^2\sigma_w^2T) \) by the properties of the standard lognormal distribution. Hence, using the formula of the expected value of a lognormal random variable we

\(^6\)The utility function \( U(W_T) = \frac{1}{\alpha+1}W_T^{\alpha+1} + \beta W_T \) is a monotonic transformation of the power utility function. See e.g. Varian (1992) on obtaining utility functions as monotonic transformations of existing utility functions.
write $E^P[W_T^\alpha] = W_0^\alpha \exp\left(\alpha \mu_w T + (\alpha^2 - \alpha)\frac{\sigma^2}{2} T\right)$. Making the appropriate substitutions in equation (4) yields equation (3). □

It is important to make some observations about the pricing kernel given by equation (3) since this is the stochastic discount factor that adjusts all assets for risk, and rules out arbitrage opportunities to arise in the economy. The pricing kernel is positive since both the numerator and the denominator are positive, it has a displaced lognormal distribution, and has expectation $E[\phi(W_T)] = 1$. The novelty here is that the pricing kernel has a displaced lognormal distribution. This contrasts with the pricing kernel implicit in the Black-Scholes model obtained by Rubinstein (1976), Brennan (1979), Schroder (2004), and others. These authors show that a necessary and sufficient condition for the Black-Scholes model to hold in a representative agent economy is that the pricing kernel $\phi(W_T)$ has a standard lognormal distribution. Hence, the Black-Scholes model does not hold in our economy even if the stock price has a standard lognormal distribution as in Black-Scholes (1973), unless $\beta = 0$ which is the special case studied by those authors.

In a representative agent economy, the price of the stock is given by the following standard valuation equation (see e.g. Cochrane (2001) and Camara (2003)):

$$S_0 = e^{-rT} E^P[\phi(W_T)S_T].$$

(5)

In our economy, the stock price has a lognormal distribution under the actual probability measure $P$, as in the Black-Scholes model. The next proposition derives the distribution of the stock price under the equivalent probability measure $R$. This distribution differs from the lognormal distribution under the risk-neutral probability measure $Q$ implicit in the Black-Scholes model.

**Proposition 2. (The $R$ measure)** Assume that the marginal utility function of the representative agent is given by equation (1) and that aggregate wealth has a lognormal distribution as in equation (2). Assume that the stock price has a lognormal distribution under the actual probability measure $P$, i.e. $S_T \sim \Lambda(\ln(S_0) + (\mu - \frac{1}{2}\sigma^2)T, \sigma^2T)$ under $P$. 

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Then:
\[ S_0 = e^{-rT} E^P [\phi(W_T)S_T] = e^{-rT} E^R [S_T], \]  
where the stock price has a mixture of lognormal distributions under the equivalent probability measure \( R \), i.e. \( S_T \sim x \cdot \Lambda(\ln(S_0) + (\mu - \frac{1}{2}\sigma^2)T, \sigma^2T) + (1 - x) \cdot \Lambda(\ln(S_0) + (\mu + \alpha\rho \sigma \omega \sigma - \frac{1}{2}\sigma^2)T, \sigma^2T) \) under \( R \), the weight \( x \), with \( 0 \leq x < 1 \), is a preference function (defined in the proof of the Proposition), and \( \rho \) is the correlation between aggregate wealth and the stock price.

**Proof:** See Appendix.

The stock price follows a standard lognormal distribution under \( P \) (as in the Black-Scholes model), but it follows a mixture of standard lognormal distributions under \( R \). The expected return of the asset under \( P \) is the cost of equity capital, \( \mu \). In our economy, asset prices are also given by the expectation of the asset payoffs under the equivalent measure \( R \), and then discounted at the riskless rate of return. Therefore, the expected rate of return of any asset under \( R \) is the riskless rate of return. There is only one difference between the measure \( R \) and the risk-neutral measure \( Q \) implicit in the Black-Scholes model. While the risk-neutral measure \( Q \) is independent of preference parameters the measure \( R \) depends on a preference parameter \( x \). The density function of the stock price at time \( T \) under the equivalent measure \( R \) is (as we show in the proof of Proposition 2) given by:
\[ f_R(S_T) = xf(S_T; \ln(S_0) + \mu T - \frac{1}{2}\sigma^2T, \sigma^2T) + (1 - x)f(S_T; \ln(S_0) + (\mu + \alpha\rho \sigma \omega \sigma - \frac{1}{2}\sigma^2T, \sigma^2T), \]
which is a mixture of two lognormal densities. This density depends on preference parameters. Option prices are uniquely determined by the evaluation of the expectation of their payoffs under \( R \), and then discounted at the riskless return.

**Proposition 3. (Asset prices)** The evaluation of the current prices of the stock, \( S_0 \), call, \( P_c \), and put, \( P_p \), yields the following equations:
\[ 1 = e^{-rT} \left[ xe^{\mu T} + (1 - x)e^{(\mu + \alpha \rho \sigma \omega \sigma)T} \right], \]  

8
\[ P_c = e^{-rT} x \left[ S_0 e^{\mu T} N(d_1) - KN(d_2) \right] \\
+ e^{-rT} (1 - x) \left[ S_0 e^{(\mu + \alpha \rho \sigma_w \sigma)T} N(d_3) - KN(d_4) \right] , \quad (9) \]

\[ P_p = e^{-rT} x \left[ KN(-d_2) - S_0 e^{\mu T} N(-d_1) \right] \\
+ e^{-rT} (1 - x) \left[ KN(-d_4) - S_0 e^{(\mu + \alpha \rho \sigma_w \sigma)T} N(-d_3) \right] , \quad (10) \]

where:

\[ d_1 = \frac{\ln \left( \frac{S_0}{K} \right) + (\mu + \frac{\sigma^2}{2})T}{\sigma \sqrt{T}} , \]
\[ d_2 = \frac{\ln \left( \frac{S_0}{K} \right) + (\mu - \frac{\sigma^2}{2})T}{\sigma \sqrt{T}} , \]
\[ d_3 = \frac{\ln \left( \frac{S_0}{K} \right) + (\mu + \alpha \rho \sigma_w \sigma + \frac{\sigma^2}{2})T}{\sigma \sqrt{T}} , \]
\[ d_4 = \frac{\ln \left( \frac{S_0}{K} \right) + (\mu + \alpha \rho \sigma_w \sigma - \frac{\sigma^2}{2})T}{\sigma \sqrt{T}} , \]

\( N(.) \) is the cumulative distribution function of the standard normal, \( K \) is the strike price, and \( T \) is the maturity date of the options.

**Proof:** Proposition 2 shows that, in our economy, the prices of the stock, call, and put are given by:

\[ S_0 = e^{-rT} E^P [\phi(W_T) S_T] = e^{-rT} E^R [S_T] , \]
\[ P_c = e^{-rT} E^P [\phi(W_T) (S_T - K)^+] = e^{-rT} E^R [(S_T - K)^+] , \]
\[ P_p = e^{-rT} E^P [\phi(W_T) (K - S_T)^+] = e^{-rT} E^R [(K - S_T)^+] , \]

where the stock price has a mixture of lognormal distributions under the equivalent probability measure \( R \), i.e. \( S_T \sim x \cdot \Lambda(\ln(S_0) + (\mu - \frac{1}{2} \sigma^2)T, \sigma^2T) + (1 - x) \cdot \Lambda(\ln(S_0) + (\mu + \alpha \rho \sigma_w \sigma - \frac{1}{2} \sigma^2)T, \sigma^2T) \) under \( R \). Hence, evaluating the expectations under \( R \), yields the desired results. \( \Box \)

We obtain the next result when we use the equilibrium relation given by equation (8) into options prices to eliminate a set of preference parameters from option prices.

**Proposition 4. (Call and put option prices)** The current prices of the call and put are
given by:

\[
P_c = e^{-rT} x \left[ S_0 e^{\mu T} N(d_1) - K N(d_2) \right] \\
+ e^{-rT} (1 - x) \left[ S_0 \left( \frac{e^{rT} - xe^{\mu T}}{1 - x} \right) N(d_3) - K N(d_4) \right],
\]

(11)

\[
P_p = e^{-rT} x \left[ K N(-d_2) - S_0 e^{\mu T} N(-d_1) \right] \\
+ e^{-rT} (1 - x) \left[ K N(-d_4) - S_0 \left( \frac{e^{rT} - xe^{\mu T}}{1 - x} \right) N(-d_3) \right],
\]

(12)

where:

\[
d_1 = \frac{\ln \left( \frac{S_0}{K} \right) + (\mu + \frac{\sigma^2}{2})T}{\sigma \sqrt{T}}, \\
d_2 = \frac{\ln \left( \frac{S_0}{K} \right) + (\mu - \frac{\sigma^2}{2})T}{\sigma \sqrt{T}}, \\
d_3 = \frac{\ln \left( \frac{S_0}{K} \left( \frac{e^{rT} - xe^{\mu T}}{1 - x} \right) \right) + \frac{\sigma^2}{2}T}{\sigma \sqrt{T}}, \\
d_4 = \frac{\ln \left( \frac{S_0}{K} \left( \frac{e^{rT} - xe^{\mu T}}{1 - x} \right) \right) - \frac{\sigma^2}{2}T}{\sigma \sqrt{T}},
\]

\(N(.)\) is the cumulative distribution function of the standard normal, \(K\) is the strike price, and \(T\) is the maturity date of the options.

**Proof:** Write equation (8) as \(\frac{1}{T} \ln \left( \frac{e^{rT} - xe^{\mu T}}{1 - x} \right) = \mu + \alpha \rho \sigma_w \sigma\). Then use this expression in equations (9) and (10) to eliminate the term \(\mu + \alpha \rho \sigma_w \sigma\). \(\square\)

**Corollary 5. (The Black-Scholes model) If \(\beta = 0\) then the Black-Scholes (1973) valuation equations obtain.**

**Proof:** If \(\beta = 0\) then, by equation (27) of the Appendix, we obtain that \(x = 0\). If \(x = 0\) in equations (11) and (12) then we have the Black-Scholes call and put prices. \(\square\)

Equations (11) and (12) show that, in our economy, option prices depend on the stock price \(S_0\), the strike price \(K\), the time to maturity \(T\), the interest rate \(r\), the stock volatility \(\sigma\), the preference function \(x\), and the rate of return required by stockholders or cost of equity capital (COE), \(\mu\). The parameters \(S_0\), \(K\), \(T\), and \(r\) are observable. Then equations (11) and
(12) can be solved for the three unknowns $x$, $\sigma$, and $\mu$ by minimizing the sum of squared differences between market prices and theoretical prices of options. This tell us what is the cost of equity capital (COE), $\mu$, implied by market prices including option prices.

3. Implementation Procedures and Data

3.1 Implementation Procedures

Assume that we are in an $n$-firm economy. As shown in equation (11) or (12), the unknown parameters include $x$, $\mu_i$, and $\sigma_i$ (for $i=1,2,\ldots,n$). Several loss functions can be considered when estimating the parameters with option prices. As is common in the literature, we minimize the sum of squared differences between the market and theoretical prices of options with the same time-to-maturity. In theory, for a time point the risk preference parameter $x$ is unique across $n$ assets, and all parameters ($x$, $\mu_i$, and $\sigma_i$) should be estimated simultaneously by minimizing the following loss function:

$$
\sum_{i=1}^{n} \sum_{j=1}^{m_i} (C_i(K_j) - c_i(K_j|x, \mu_i, \sigma_i))^2,
$$

(13)

where $C_i(.)$ and $c_i(.)$ denote the market and theoretical call prices, respectively, and $m_i$ is the number of option contracts with different strike prices $K_j$ for firm $i$. However, the problem of the dimension curse will occur for a multi-asset estimation. For example, we need to estimate 201 parameters in an optimization procedure when having 100 assets. Therefore, to make the estimation plausible, we adjust the above procedure to a two-step procedure.

In the first step, given a fixed $x$, $x_0$, we can easily estimate $\mu_i$ and $\sigma_i$ for all firms by minimizing their individual loss functions:

$$
L_i(x_0; \mu_i, \sigma_i) = \sum_{j=1}^{m_i} (C_i(K_j) - c_i(K_j|x, \mu_i, \sigma_i, x = x_0))^2,
$$

(14)
where $L_i(x_0; \mu_i, \sigma_i)$ is the loss function of firm $i$ given that $x = x_0$, where $i = 1, 2, \ldots, n$.

For each $x_0$, we obtain a set of estimates of $\hat{\mu}_i(x_0)$ and $\hat{\sigma}_i(x_0)$ for $n$ firms. By changing $x_0$ recursively from 0 to 0.99 with the interval of 0.01, we have 100 sets of estimates of $\hat{\mu}_i(x_0)$ and $\hat{\sigma}_i(x_0)$ ($i = 1, 2, \ldots, n$), respectively. We then choose $x_0$ with which we have the least sum of all individual loss functions and use the corresponding $\hat{\mu}_i(x_0)$ and $\hat{\sigma}_i(x_0)$ as the optimal estimates, i.e. choosing $x_0$ that minimizes the following function:

$$\sum_{i=1}^{n} L_i(x_0; \hat{\mu}_i(x_0), \hat{\sigma}_i(x_0)).$$

(15)

### 3.2 Data

An empirical implementation is conducted for the component firms (on December 31, 2005) of the S&P 100 index. Therefore, the primary data are the market prices of options written on the stocks of these firms. In order to estimate the cost of equity capital for a fixed horizon and make an appropriate comparison with the conventional estimates generated from an asset pricing model, we have to use the market prices of options with a fixed time-to-maturity at a regular frequency (e.g. monthly). Therefore, in this study we use the month-end market volatility surfaces of options on the stocks of the component firms of the S&P 100 index for the period from January 1996 to December 2005.

The volatility surfaces are collected from the database of OptionMetrics. For every month-end trading day, we have the Black-Scholes implied volatility surfaces made up of 13 strike prices reported as deltas for both call and put options with 2 different time-to-maturities (182 and 365 days). The calculation of volatility surfaces is based on a kernel smoothing

7 The delta ranges between 0.2 (-0.2) and 0.8 (-0.8) with the interval of 0.05 for call (put) options.
8 Although both option pricing theories and option trading experiences indicate that the marginal risk of an investment declines as a function of the square root of time and the falling marginal risk reduces the annual discount rate and so that the cost of equity capital that serves as the discount rate for capital budgeting depends on the investment duration, it is not common for a company to do capital budgeting for the projects with a very short duration such as 1 month or 3 months. Therefore, we use the options with 182 and 365 days to expire for our empirical implementation.
algorithm and an interpolation technique. The database also provides the month-end closing prices of the underlying stocks.

The risk-free interest rates are calculated from the OptionMetrics zero curves formed by a collection of continuously-compounded zero-coupon interest rates with various maturities. We use the linear interpolation method to generate the interest rates whose horizons exactly match the time-to-maturities of options.

As the underlying stocks pay discrete dividends, we use the OptionMetrics projected dividend amounts and ex-dividend dates that are based on the securities’ usual payments and frequencies in order to compute the present values of the projected dividend payments prior to the maturity dates. We then deduct the present values of projected dividend payments from the market prices of underlying stocks and then value the options as though the stocks pay no dividends.

With the adjusted prices of underlying assets and the matched risk-free rates, all volatility surfaces are converted to their Black-Scholes (European) option prices of non-dividend-paying stocks. As out-of-money options are usually traded more heavily than in-the-money ones, in-the-money options are excluded, and all put prices are converted to call prices using the put-call parity for the computation of the loss functions.9

To compare our estimates of the costs of equity capital with those estimated by a conventional method, the Fama/French three-factor model, we also collect the monthly time series of the three factors - the market portfolio return minus the risk-free interest rate (RmRf), a small-size portfolio return minus a big-size portfolio return (SMB), and a high-book-to-market-equity portfolio return minus a low-book-to-market-equity portfolio return (HML) - from the website of Kenneth R. French for the sample period from January 1993 to December 2005.10

9This procedure for data selection has been employed by many studies such as Bliss and Panigirtzoglou (2004) and Jiang and Tian (2005).

10As a long period of historical prices is necessary for the COE estimation using the Fama/French factor model, the sample period for the historical data is three year longer than that for the option data.
We also investigate the relationships between the COEs and investor sentiment. Two frequently used proxies of investor sentiment, the VIX index and the sentiment index of Baker and Wurgler (2006) are adopted. The details of the calculation of the two indices can be found on the website of CBOE (www.cboe.com/micro/vix/vixwhite.pdf) and in the paper of Baker and Wurgler, respectively.

4. Empirical Results

4.1 General Properties of Option-implied Estimates

We use the end-of-the-month prices of options written on the component stocks of the S&P 100 index with two different time-to-maturities - 182 and 365 days - and follow the two-step procedure detailed in Section 3.1 to estimate the parameters, $x$, $\mu$, and $\sigma$, for the period from January 1996 to December 2005 (120 months).

Table 1 and Figure 1 respectively show the summary statistics and processes of estimates of all parameters for various time-to-maturities. Both the processes and distributions of $x$ estimates are very similar across time-to-maturities. The estimates of $x$ range from 0.70 to 0.98 and their mean level is about 0.85. As mentioned in Section 2, our option pricing model converges to the Black-Scholes model when $x$ approaches 0. The estimates clearly indicate that the market prices of equity options are very different from the Black-Scholes prices. Moreover, the estimates of $x$ do not change very much with time and their volatility is only about 0.06. The first-order autocorrelation coefficients for the two horizons are 0.69 and 0.6, respectively.

For any time point, while the $x$ parameter is fixed across firms, the $\mu$ and $\sigma$ estimates are either value-weighted or equally-weighted across firms. As the results produced by both approaches are almost the same, only value-weighted results are reported and the following discussion applies to both. The mean levels of the $\mu$ estimates for the 182- and 365-day maturities are slightly different, about 13% and 11%, respectively. This difference
seems consistent with the general argument in option pricing theories and option trading experiences that the marginal risk of an investment declines as a function of the square root of time and the falling marginal risk reduces the annual discount rate.\textsuperscript{11} In addition, the estimates of $\mu$ are not volatile for both horizons as the volatility is only about 0.04 or 0.03. Also, the skewness and kurtosis of $\mu$ estimates do not exhibit obvious differences across time-to-maturities. Moreover, the estimates of $\mu$ have a high first-order autocorrelation that ranges from 0.84 to 0.87, which, along with the low volatility, is consistent with the general sense that the COE for a firm is relatively stable over time.

The distributions and processes of $\sigma$ estimates are almost the same across time-to-maturities. This could be driven by the stylized fact that equity prices follow a random walk, because the similar annualized volatilities for different horizons indicate that the sum of short-term volatilities equals the long-term volatility. The mean level of $\sigma$ estimates is about 0.25 and the estimates are spread between 0.14 and 0.39. Similar to the finding for $\mu$ estimates, the volatility of $\sigma$ estimates is also very small, at about 0.06. Moreover, $\sigma$ is highly persistent as the first-order autocorrelation is as high as 0.96, which is consistent with the stylized fact, volatility clustering, observed in the market prices of financial assets.

In summary, our empirical properties of the estimates for $x$, $\mu$, and $\sigma$ are in line with the theoretical assumptions and the estimates of these parameters are very stable across time. If it is necessary to estimate the COE that properly matches the required investment duration, even when the options with the time-to-maturity that perfectly matches a particular investment horizon are not traded in the market, we still can utilize an interpolation technique with the COEs estimated from other maturities of option prices to appropriately generate the COE with the desirable maturity. By contrast, the COEs estimated from many conventional methods with historical equity prices do not have such flexibility.

\textsuperscript{11}This argument has been emphasized by McNulty et al. (2002).
4.2 Estimating Costs of Equity Capital with Alternative Methods

The conventionally standard approach for estimating the COE is the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965). As an alternative, Fama and French (1993) propose another two pricing factors, SMB and HML.\textsuperscript{12} However, our COE estimates from option market prices are forward-looking, while the COEs estimated by all capital asset pricing models rely on historical data. As recent evidence (Fama and French, 1993 & 1996) suggests that the Fama/French three-factor model is better than the CAPM in describing expected returns, in this study we compare our COE estimates with those estimated from the Fama/French three-factor model for the most common investment duration of capital budgeting, which is one year.\textsuperscript{13} First of all, we look at the market portfolio COE by comparing the COE estimates averaged across all component firms of the S&P 100 index.

The first column of Table 2 and Figure 2 displays the summary statistics and processes of alternative COE estimates for the market portfolio, respectively. It clearly presents that our option-implied COE estimates are much more stable, while the Fama/French estimates are very volatile and roughly range from 0.4 to -0.15. Moreover, the level of our option-implied estimates for the market is much more reasonable for serving as the discount rate for capital budgeting. The mean value is about 11\% with a small volatility (0.03), which is similar to the previous evidence of Fama and French that the average return of the component stocks of the S&P 500 index is 9.62\% for 1951 to 2000. In contrast, the mean level of the Fama/French COE estimates is about 14\% and the estimates sometimes are extremely high or even negative. Basically, our findings are consistent with the argument of Fama and

\textsuperscript{12}Some studies, such as Blanchard (1993), Claus and Thomas (2001), Gebhardt, Lee, and Swaminathan (2001), and Fama and French (2002), use valuation models with fundamentals to estimate expected returns. In this study we focus on the comparisons with the market-based estimates only.

\textsuperscript{13}As it is necessary to use a long period of historical data to obtain a smooth time series of COE estimates for the Fama/French three-factor model, we use a three-year sample period for the estimation. We first estimate the factor loadings with the three-year historical data and then use the averages of the historical factor values with the estimated factor loadings to generate the COE estimate for the time point. By rolling over the sample period month by month, we form the time series of COE estimates for the period from January 1996 to December 2005.
French (1997) and Pástor and Stambaugh (1999) in that the COE estimates from both the CAPM and the three-factor models are surely imprecise due to the uncertainty about true factor risk premiums and imprecise estimates of the factor loadings.

One major difference between our option-implied COEs and Fama and French’s COEs is the stability of the estimates. One question that arises is how stable are the actual COEs. Because the actual COEs are not observable, an alternative way to validate our COE estimates is to compare them with the empirical expected returns generated by the following asset pricing model:

\[ \mu_{i,t} = r_t - \alpha \rho_{i,t} \sigma_{w,t} \sigma_{i,t}. \]  

We generate the time series of \( \mu_{i,t} \) by updating \( r_t \) (risk free rate), \( \sigma_{w,t} \) (the return volatility of wealth proxied by the S&P 500 index), \( \sigma_{i,t} \) (the return volatility of the \( i \)th stock), and \( \rho_{i,t} \) (the correlation coefficient of wealth and the \( i \)th stock returns) and using an appropriate relative risk aversion coefficient \( -\alpha = \text{relative risk aversion coefficient} \) suggested by Bliss and Panigirtzoglou (2004). Consistent with the estimation with the Fama/French model, \( \sigma_{w,t}, \sigma_{i,t}, \) and \( \rho_{i,t} \) are calculated from three-year historical returns. We plot the time series of the value-weighted COEs across the component firms of the S&P 100 index with alternative \( \alpha \) ranging from -0.98 to -6.46 against those of our option-implied and Fama-French COEs in Figure 2. It is evident that our option-implied COEs coincides with the movements of the empirically generated COEs. Moreover, the variations of our estimated COEs are also of similar magnitude to that of the generated COEs while the variations Fama and French’s COEs seem too large.

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14 We thank the referee for pointing out this issue to us.
15 This asset pricing model is the CAPM derived under the power utility function and \( -\alpha \) is the relative risk aversion (RRA) coefficient under our notation. Note that \( \alpha \) is negative in our model, and therefore a positive risk premium is added to the riskless return to yield the expected rate of return. A similar formula is shown in Cochrane (2001, equation (9.6)) under the exponential utility function.
16 Bliss and Panigirtzoglou (2004) used the CME S&P 500 futures option prices to estimate the relative risk aversion coefficient under the power utility function and obtained a mean estimate of 3.72 with a standard deviation of 1.37. Based on their estimates, we set mean minus two standard deviations, mean value, and mean plus two standard deviations as the relative risk aversion coefficients (i.e. \( \alpha \) = -0.98, -3.72, and -6.46) to generate \( \mu_{i,t} \).
To further investigate the differences between the option-implied and the historical-data-generating COE estimates, we follow the ICB industry classifications to construct six industrial portfolios from the component stocks of the S&P 100 index. Only those industries including at least 10 firms are selected to avoid forming an industrial portfolio with too few firms. Six industries are selected and they are Industrials, Consumer Services, Consumer Goods, Health Care, Financials, and Information Technology. Table 2 and Figure 3 offer the summary statistics and processes of the portfolio COEs for various industries from alternative approaches. The results show the same pattern that we found in the COE estimates of the market portfolio.

In terms of the equity premium (defined as the difference between the portfolio expected return and the risk-free interest rate) as shown in Table 3, the average equity premium for the market portfolio calculated from our option-implied COEs is 6.96 percent, which is close to the estimate of Fama and French (2002) for 1951 to 2000, 7.43 percent. In contrast, the average equity premium of Fama/French model for the market portfolio is 10.71 percent, which is about 3 percent more than its actual value. Moreover, although the premiums estimated from both approaches indicate that the COEs for Financials and Information Technology are higher than the market averages, the levels of equity premiums estimated by the Fama/French model are too high.

Using the implied volatilities of various industrial portfolios as their total risk proxies and assuming that the systematic risk is proportional to the total risk with the same ranking across industries, we find that the rank correlation coefficient between option-implied COEs and risk (0.89) is much higher than that between Fama/French’s COEs and risk (0.43). This means that the return-risk relationship is stronger using the option-implied estimates although both approaches show that the Information Technology industry is the most risky industry with the highest COEs among all six industries.

In summary, with forward-looking information, option prices provide a reliable source for estimating the COEs for both the market and industrial portfolios. Compared with the COEs estimated from historical data with a conventional asset pricing model, the option-implied
estimates are much more stable and reasonable. Therefore, in terms of both plausibility and reasonability, our option pricing model provides a reliable alternative for estimating COEs.

4.3 Costs of Equity Capital and Investor Sentiment

Although investor sentiment plays no role in classical finance theories, many empirical studies (Neal and Wheatley, 1998; Shiller (2000); Baker and Wurgler (2000, 2006); Campbell and Cochrane (2000); Menzly et al. (2004)) indicate that investor sentiment affects the determination of stock prices. Therefore, we would like to further investigate whether our COEs also show some particular relationships with investor sentiment across time.

Since there is no perfect proxies for investor sentiment, we use two frequently adopted measures, the VIX index compiled by CBOE and the sentiment index of Baker and Wurgler (2006) (hereafter B/W index) for our investigation. While the former is usually regarded as a fearness gauge, the latter is a bull-bear sentiment index based on the first principal component of a number of proxies suggested by previous studies. Essentially they represent two different types of investor sentiment with a low correlation coefficient of about 0.16.

As shown in Figure 4 consisting of the scatter plots of the COEs estimated from alternative sources against the VIX and B/W indices, our COEs are significantly and positively related to both indices. These empirical findings indicate not only that investors require a higher (lower) return when facing higher (lower) uncertainty, but also that investors expect a higher (lower) return when being bullish (bearish). By contrast, we find no significant relationships between the Fama/French COEs and the two sentiment indices. In general, the association of our option-implied COEs and investor sentiment is in line with the literature supporting the effect of investor sentiment on determining stock prices.
5. An extension of the Ferguson and Shockley model

Ferguson and Shockley (2003) show that firms’ leverage might account for error measurements in stock betas in a world where the CAPM holds. They show that a proxy stock beta, calculated with respect to the economy’s aggregate equity, depends on both the true stock beta and an error that depends on firm’s leverage. Note that aggregate wealth, \( W \), is equal to the market portfolio, \( M \), and that the market portfolio equals the market value of equity, \( E \), plus the market value of debt, \( D \), in the work of Ferguson and Shockley (2003), i.e. \( M = E + D \).

We start this section by showing that the CAPM holds in our economy for the special case of a preference parameter \( \beta = 0 \) which implies \( x = 0 \). From equation (8), when \( x = 0 \), we can obtain the CAPM:

\[
\mu_{si} = r + \beta_{si}(\mu_w - r),
\]

where \( \beta_{si} = \frac{\rho_{si,w}\sigma_w\sigma_{si}}{\sigma_w^2} \) is the beta of stock \( i \) calculated with respect to the true market portfolio or aggregate wealth, and where we added the indices \( si \) and \( s, iw \) to align our notation with Ferguson and Shockley (2003). This shows that the CAPM holds exactly in a single period economy where investors have a power utility function that displays CPRA and aggregate wealth (i.e. the market portfolio), and the stock price are bivariate lognormal distributed.\(^{17}\)

In our economy, where investors have DPRA, and aggregate wealth and the stock price are bivariate lognormal the CAPM does not hold as an equality. From equation (8), for an arbitrary \( x \), we can obtain:

\[
\mu_{si}T = \ln \left( \frac{e^{rT} - xe^{\mu_{si}T}}{1 - x} \right) + \beta_{si} \left[ \mu_wT - \ln \left( \frac{e^{rT} - xe^{\mu_wT}}{1 - x} \right) \right]
\]

and we note that the cost of equity \( \mu_{si} \) is an implicit function since it appears on both sides of the equation. This equation is not the traditional CAPM. However, using the first order

\(^{17}\)Brennan (1979) and Camara (2003) derive variations of the CAPM in a single period economy with CPRA and bivariate lognormal.
approximation of the exponential function, it is possible to show that in general the CAPM in our economy with DPRA and bivariate lognormal only holds as an approximation.

Ferguson and Shockley (2003) price the stock and the bond issued by the firm as contingent claims written on the assets of the firm. Following this approach, in our economy where the representative agent has a marginal utility function (1) and the market portfolio and the firm value are lognormal, the initial equity value $S_i$ and the initial debt value $B_i$ of firm $i$ are given by:

$$S_i = V_i \left[ x e^{(\mu_i - r)T} N(d_1)_i + \left( 1 - x e^{(\mu_i - r)T} \right) N(d_3)_i \right]$$

$$- F_i e^{-rT} \left[ x N(d_2)_i + (1 - x) N(d_4)_i \right],$$

$$B_i = V_i \left\{ N(-d_3)_i + x e^{(\mu_i - r)T} \left[ N(d_3)_i - N(d_1)_i \right] \right\}$$

$$+ F_i e^{-rT} \left[ x N(d_2)_i + (1 - x) N(d_4)_i \right],$$

where:

$$(d_1)_i = \frac{\ln \left( \frac{V_i}{F_i} \right) + (\mu_i + \frac{\sigma_i^2}{2})T}{\sigma_i \sqrt{T}},$$

$$(d_2)_i = \frac{\ln \left( \frac{V_i}{F_i} \right) + (\mu_i - \frac{\sigma_i^2}{2})T}{\sigma_i \sqrt{T}} = (d_1)_i - \sigma_i \sqrt{T},$$

$$(d_3)_i = \frac{\ln \left( \frac{V_i}{F_i} \left( \frac{e^{rT} - xe^{\mu_i T}}{1 - x} \right) \right) + \frac{\sigma_i^2}{2}T}{\sigma_i \sqrt{T}},$$

$$(d_4)_i = \frac{\ln \left( \frac{V_i}{F_i} \left( \frac{e^{rT} - xe^{\mu_i T}}{1 - x} \right) \right) - \frac{\sigma_i^2}{2}T}{\sigma_i \sqrt{T}} = (d_3)_i - \sigma_i \sqrt{T},$$

$V_i$ is the initial value of firm $i$ which has a zero coupon bond with face value $F_i$ as its debt, $\mu_i$ is the weighted average cost of capital (WACC) of firm $i$, and $\sigma_i$ is the volatility of the assets of the firm.

It should be noted that the probability implicit in option prices that shareholders will pay their debt at time $T$ and will buy back the firm from bondholders is given by:

$$N(y)_i = x N(d_2)_i + (1 - x) N(d_4)_i,$$

---

$^{18}$From now on, we use the notation of Ferguson and Shockley (2003). We use $\mu_i$ as the weighted average cost of capital (WACC), $\mu_{ei}$ the cost of equity (COE), and $\mu_{Bi}$ the debt yield.
which is also the probability implicit in option prices that the call will be exercised at time \( T \). Here \( N(\cdot) \) is the cumulative probability distribution function for a standard normal random variable with probability density function (pdf) \( Z(\cdot) \). Hence:

\[
Z(y)_i = xZ(d_2)_i + (1 - x)Z(d_4)_i,
\]

is the probability density function of the event that shareholders will buy back the firm from bondholders at the maturity. Then:

\[
MR_i = \frac{N(y)_i}{Z(y)_i}
\]

is the Mills Ratio implicit in option prices for the event that stockholders will exercise their call option and will buy back the firm from bondholders at the maturity of the zero coupon bond.

If the preference parameter \( \beta = 0 \) then \( x = 0 \), the value of the equity and the value of the debt of firm \( i \) are given by the Black-Scholes formulas, and the Mills ratio reduces to \( MR_i = N(d_4)_i/Z(d_4)_i \), where \( (d_4)_i = (\ln(V_i/F_i) + (r - 0.5\sigma^2_i)T)/(\sigma_i\sqrt{T}) \) as implicit in the Black-Scholes option prices. This Mills ratio \( MR_i = N(d_4)_i/Z(d_4)_i \) (with \( x = 0 \)) plays an important role in the analysis of Ferguson and Shockley (2003). They show that the proxy beta of firm’s assets \( \beta^E_i = \sigma_{i,E}/\sigma^2_E = \sigma_i\rho_{i,E}/\sigma_E \) declines with firm’s leverage \( F_i \) if their Mills ratio \( MR_i = N(d_4)_i/Z(d_4)_i \) is an upper bound for a quantity that uses in a fundamental way the correlation of the stock of the firm with the equity market.\(^{19}\) They also show, when this Mills ratio condition is satisfied, that the true beta of the stock \( \beta_{si} \) increases more than its proxy \( \beta^E_{si} = \sigma_{si,E}/\sigma^2_E = \sigma_{si}\rho_{si,E}/\sigma_E \) when the leverage of the firm \( F_i \) increases. We now generalize these results by showing that they hold in our economy when their Mills ratio \( MR_i = N(d_4)_i/Z(d_4)_i \) is replaced by our generalized Mills ratio (17).

**Proposition 6. (The impact of leverage on the proxy beta of firm’s assets)**

Assume that the marginal utility function of the representative agent is given by (1), that aggregate wealth or market portfolio is lognormal as in (2), and that the value of the firm

\(^{19}\)The volatility of the equity market is \( \sigma_E \) and the correlation between firm’s assets and the equity market is \( \rho_{i,E} \).
is lognormal under $P$, i.e. $V_{i,T} \sim \Lambda(\ln(V_i) + (\mu_i - 0.5\sigma_i^2)T, \sigma_i^2T)$. Then the proxy beta of firm’s assets $\beta^E_i$ declines with firm’s leverage $F_i$, i.e. $\frac{\partial \beta^E_i}{\partial F_i} < 0$ if:

$$\frac{2\rho^2_{si,E} - 1}{\sigma_E \rho_{si,E} \sqrt{T}} < MR_i.$$ (18)

**Proof:** See Appendix.

We can see from (18) that the Mills ratio (17) of the event that the debt will be paid is the upper bound for the quantity $\frac{2\rho^2_{si,E} - 1}{\sigma_E \rho_{si,E} \sqrt{T}}$, which depends on $\rho_{si,E}$ in a crucial way. When this quantity is bounded above by the Mills ratio of the event that the debt will be paid then the proxy beta of firm’s assets $\beta^E_i$ declines with firm’s leverage $F_i$. This result is relevant since the true beta of firm’s assets $\beta_i$ does not change when $F_i$ changes. Therefore measuring systematic risk with respect to the market equity $E$ instead of measuring systematic risk with respect to the market portfolio $M$ or aggregate wealth $W$ does not only leads to measurement errors, but more importantly to measurement errors that change with firm’s leverage. If the preference parameter $\beta$ in (1) is zero then $x = 0$, and Lemma 1 of Ferguson and Shockley (2003) obtains as a special case of our Proposition 6.

**Proposition 7. (The ratio of the true beta to the proxy beta)** Under the assumptions of Proposition 6 suppose that $\frac{\partial \beta^E_i}{\partial F_i} < 0$, that the proxy asset beta is positive i.e. $\beta^E_i > 0$, and that the true asset beta is positive i.e. $\beta_i = \sigma_i \rho_i, w/\sigma_w > 0$. Then the true beta of the stock $\beta_{si}$ increases more than its proxy $\beta^E_{si}$ when the leverage of the firm $F_i$ increases.

**Proof:** The result follows according to the proof of Proposition 1 of Ferguson and Shockley (2003) with their Mills ratio substituted by our Mills ratio (17).

This result extends Proposition 1 of Ferguson and Shockley (2003) for when $x \neq 0$. Their Proposition 2 uses the CAPM which only holds as an an approximation in our economy. Hence their Proposition 2 only holds (when $x \neq 0$) as an approximation in our set up.
6. Conclusions

The expected return of the stock or cost of equity capital (COE) does not affect most existing modern option pricing models. Then it is in general impossible to estimate the COE using such models. This paper contributes to the literature by deriving equilibrium option pricing formulae which explicitly depend on the expected return of the stock or COE. Thus, we are able to provide a forward-looking equilibrium estimate of the COE using our option pricing model and current market prices. Our empirical tests for the component firms of the S&P 100 index for the period 1996-2005 indicate that our COE estimates are superior to those obtained with the Fama/French (1993) three-factor model. For example, our COE estimates are reasonable and stable over the years, and thus can be used as discount rates for capital budgeting. They also generate an equity risk premium close to the average equity premium reported by Fama and French (2002). Moreover, our option-implied COEs show that not only a strong return-risk relationship, but also a significant return-sentiment relationship is observed with our estimates.

Future research can apply our estimates of expected returns to test the validity of asset pricing models such as the CAPM. Many issues can be reinvestigated with our method. For example, one can ask if the expected returns of the assets are linearly related to their betas? In contrast to all existing literature which uses historical returns to test CAPM, empirical tests based on our forward-looking estimates of expected returns and betas\textsuperscript{20} would be ex-ante tests. Therefore it should be possible to derive new results and insights using our option-implied expected return. Furthermore, it is important to further investigate the issues discussed in this paper using our generalization of the leverage-based theory of Ferguson and Shockley (2003) as well as their original model.

\textsuperscript{20}Forward-looking betas estimated from option prices are now possible, see Christoffersen, Jacobs, and Vainberg (2006)
Appendix

Proof of Proposition 2: The proof is in two steps. In the first step, we derive the asset-specific pricing kernel, \( \psi(S_T) \), by conditioning the pricing kernel, \( \phi(W_T) \), on the asset payoff \( S_T \). The asset specific pricing kernel is a positive random variable with \( E^P[\psi(S_T)] = 1 \). The asset-specific pricing kernel precludes arbitrage opportunities to arise between a specific underlying asset and derivatives written on that asset. In the second step, we derive the density function of the asset payoff \( S_T \) under the equivalent measure \( R \), i.e. \( f^R(S_T) \), by multiplying the asset-specific pricing kernel, \( \psi(S_T) \), and the actual density of the stock, \( f^P(S_T) = f(S_T; \ln(S_0) + (\mu - \frac{1}{2}\sigma^2)T, \sigma^2T) \), under the actual measure \( P \). We verify that \( f^R(S_T) \) is a true density since \( f^R(S_T) > 0 \) and \( \int_{-\infty}^{+\infty} f^R(S_T)dS_T = 1 \). Then we conclude that current prices (including option prices) in this economy are given by the expectation of asset payoffs with respect to the density \( f^R(S_T) \), and then discounted at the riskless rate of return.

First step: The asset-specific pricing kernel is (see Camara (2003)) given by:

\[
\psi(S_T) = E^P[\phi(W_T) \mid S_T] = \frac{E^P[U'(W_T) \mid S_T]}{\beta + W_0^\alpha \exp\left(\alpha\mu_w T + (\alpha^2 - \alpha)^\sigma_w^2 T\right)} = \frac{\beta + E^P[W_0^\alpha \mid S_T]}{\beta + W_0^\alpha \exp\left(\alpha\mu_w T + (\alpha^2 - \alpha)^\sigma_w^2 T\right)},
\]

where we have used the pricing kernel given by equation (3).

Since \( \ln(S_T) \sim N(\ln(S_0) + \mu T - \frac{1}{2}\sigma^2 T, \sigma^2 T) \) and \( \alpha \ln(W_T) \sim N(\ln(W_0^\alpha) + \alpha(\mu_w - \frac{1}{2}\sigma_w^2)T, \alpha^2\sigma_w^2 T) \), we have:

\[
\alpha \ln(W_T) \mid \ln(S_T) \sim N\left(\ln(W_0^\alpha) + \alpha(\mu_w - \frac{1}{2}\sigma_w^2)T + \rho \frac{\alpha \sigma_w}{\sigma} \left(\ln(S_T) - \left(\ln(S_0) + \mu T - \frac{1}{2}\sigma^2 T\right)\right), \alpha^2\sigma_w^2 T(1 - \rho^2)\right)
\]
due to the properties of the bivariate and conditional normal distributions. Then:

\[
W_T^\alpha \mid S_T \sim \Lambda\left(\ln(W_0^\alpha) + \alpha(\mu_w - \frac{1}{2}\sigma_w^2)T + \rho \frac{\alpha \sigma_w}{\sigma} \left(\ln(S_T) - \left(\ln(S_0) + \mu T - \frac{1}{2}\sigma^2 T\right)\right), \alpha^2\sigma_w^2 T(1 - \rho^2)\right) .
\]
Using the definition of expectation of a lognormal random variable, the asset-specific pricing kernel given by equation (19) can be written as:

$$
\psi(S_T) = \frac{\beta + W_0^\alpha \exp \left( \alpha (\mu_w - \frac{1}{2} \sigma_w^2)T + \rho \frac{\alpha \sigma_w}{\sigma} \left( \ln(S_T) - \left( \ln(S_0) + \mu T - \frac{1}{2} \sigma^2 T \right) \right) \right)}{\beta + W_0^\alpha \exp \left( \alpha \mu_w T + \left( \alpha^2 - \alpha \right) \frac{\sigma_w^2}{2} T \right)}.
$$

(20)

which is positive since both the numerator and denominator are positive. We also conclude that:

$$
E^P[\psi(S_T)] = \frac{\beta + W_0^\alpha \exp \left( \alpha \mu_w T + \left( \alpha^2 - \alpha \right) \frac{\sigma_w^2}{2} T \right)}{\beta + W_0^\alpha \exp \left( \alpha \mu_w T + \left( \alpha^2 - \alpha \right) \frac{\sigma_w^2}{2} T \right)} = 1.
$$

(21)

Equation (21) can be obtained by noting that we have implicitly in equation (20) the following relation:

$$
E^P[W_T^\alpha | S_T] = W_0^\alpha \exp \left( \alpha (\mu_w - \frac{1}{2} \sigma_w^2)T + \rho \frac{\alpha \sigma_w}{\sigma} \left( \ln(S_T) - \left( \ln(S_0) + \mu T - \frac{1}{2} \sigma^2 T \right) \right) \right) + \frac{1}{2} \alpha^2 \sigma_w^2 T (1 - \rho^2).
$$

(22)

which is a lognormal random variable. Hence:

$$
\ln \left\{ E^P[W_T^\alpha | S_T] \right\} = \ln(W_0^\alpha) + \alpha (\mu_w - \frac{1}{2} \sigma_w^2)T + \rho \frac{\alpha \sigma_w}{\sigma} \left( \ln(S_T) - \left( \ln(S_0) + \mu T - \frac{1}{2} \sigma^2 T \right) \right) + \frac{1}{2} \alpha^2 \sigma_w^2 T (1 - \rho^2)
$$

is a normal variate. Evaluating the mean and variance of this normal random variable yields:

$$
E^P \left[ \ln \left\{ E^P[W_T^\alpha | S_T] \right\} \right] = \ln(W_0^\alpha) + \alpha (\mu_w - \frac{1}{2} \sigma_w^2)T + \frac{1}{2} \alpha^2 \sigma_w^2 T (1 - \rho^2),
$$

(23)

$$
Var^P \left[ \ln \left\{ E^P[W_T^\alpha | S_T] \right\} \right] = \rho^2 \alpha^2 \sigma_w^2 T.
$$

(24)

Using the relation between the normal and the lognormal random variables yields:

$$
E^P[\{E^P[W_T^\alpha | S_T]\}] = W_0^\alpha \exp \left( \alpha \mu_w T + \left( \alpha^2 - \alpha \right) \frac{\sigma_w^2}{2} T \right).
$$

(25)

Summing up these previous remarks yields equation (21).

Second step: We set $f^R(S_T) = \psi(S_T) \cdot f^P(S_T)$. Then, following Camara (2003), $E^P[\phi(W_T)S_T] = E^P[E^P[\phi(W_T)S_T | S_T]] = E^P[\psi(S_T)S_T] = E^R[S_T]$. We write equation (20) as:

$$
\psi(S_T) = x + (1 - x) \exp \left( \rho \frac{\alpha \sigma_w}{\sigma} \left( \ln(S_T) - \left( \ln(S_0) + \mu T - \frac{1}{2} \sigma^2 T \right) \right) - \frac{1}{2} \alpha^2 \sigma_w^2 T \rho^2 \right).
$$

(26)
Note that \( \beta \) the actual density of the stock, \( f \) with 0

We obtain as intermediary results the following:

\[
f^R(S_T) = xf(S_T; \ln(S_0) + \mu T - \frac{1}{2} \sigma^2 T, \sigma^2 T) + (1 - x)f(S_T; \ln(S_0) + (\mu + \alpha \rho \sigma\sigma)T - \frac{1}{2} \sigma^2 T, \sigma^2 T).
\]

(28)

Since \( f(S_T; \ln(S_0) + \mu T - \frac{1}{2} \sigma^2 T, \sigma^2 T) \) and \( f(S_T; \ln(S_0) + (\mu + \alpha \rho \sigma\sigma)T - \frac{1}{2} \sigma^2 T, \sigma^2 T) \) are two lognormal densities, we see that \( \int_{-\infty}^{\infty} f^R(S_T)dS_T = 1 \). Also, since that \( f^R(S_T) \) is a mixture of lognormal densities and \( f^P(S_T) \) is a lognormal density then \( P \) and \( R \) are equivalent measures since both give the same probability to the set \( (0, +\infty) \) and therefore to its complement \( (-\infty, 0] \) This completes the proof of proposition 2.

\( \Box \)

Proof of Proposition 6: We start with some preliminary results that are important for the proof of the Proposition. Let \( dS_i = \frac{\partial S_i}{\partial V_i}dV_i \) (see Chiang (1984, p. 194) for this notation). Then \( r_{si} = \frac{\partial S_i}{\partial V_i}r_i \) where \( r_{si} = \frac{\partial S_i}{S_i} \) and \( r_i = \frac{dV_i}{V_i} \). Also \( \sigma^2_{si} = E[(r_{si} - \bar{r}_{si})^2] = \left( \frac{\partial S_i}{\partial V_i} \right)^2 \sigma^2_i \) since \( \sigma^2_i = E[(r_i - \bar{r}_i)^2] \). Similarly, \( \sigma^2_{Bi} = \left( \frac{\partial B_i}{\partial V_i} \right)^2 \sigma^2_i \). We also note that

\[
\begin{align*}
\frac{\partial (d_1)}{\partial V_i} &= \frac{\partial (d_2)}{\partial V_i}, \\
\frac{\partial (d_3)}{V_i} &= \frac{\partial (d_4)}{\partial V_i},
\end{align*}
\]

are

\[
Z(d_3)_i = Z(d_3)(\frac{V_i}{F_i}(e^{\mu T - xe^{\mu T}})).
\]

Then:

\[
\begin{align*}
\frac{\partial S_i}{\partial V_i} &= xe^{(\mu_i - r_i)T}N(d_1)_i + \left(1 - xe^{(\mu_i - r_i)T}\right)N(d_3)_i, \\
\frac{\partial B_i}{\partial V_i} &= N(-d_3)_i + xe^{(\mu_i - r_i)T}[N(d_3)_i - N(d_1)_i].
\end{align*}
\]

Note that \( \beta^E = \frac{\sigma_i}{\sigma_E} = \frac{1}{\sigma_E} \sum_j S_j \sigma_{i,j} = \frac{1}{\sigma_E} \sum_j \frac{\partial S_i}{\partial V_j}V_j \sigma_{i,j} \) since \( \sum_j S_j = E, \sigma_{i,j} = \sigma_{i,j} \rho_{i,j}, \sigma_{sj} = \frac{\partial S_i}{\partial V_j}S_j \sigma_j, \) and \( \rho_{i,j} = \rho_{i,j} \). Then:

\[
\frac{\partial \beta^E}{\partial F_i} = \frac{E \sigma^2_j \left[ \partial \left( \sum_j \frac{\partial S_i}{\partial V_j}V_j \sigma_{i,j} \right) / \partial F_i \right] - \sum_j \frac{\partial S_i}{\partial V_j}V_j \sigma_{i,j} \left[ E \frac{\partial \sigma^2_f}{\partial F_i} + \sigma^2_E \frac{\partial \beta^E}{\partial F_i} \right]}{(E \sigma^2_E)^2}.
\]

We obtain as intermediary results the following:

\[
\partial \left( \sum_j \frac{\partial S_j}{\partial V_j}V_j \sigma_{i,j} \right) / \partial F_i = -V_i \sigma_i^2 xe^{(\mu_i - r_i)T}Z(d_1)_i + \left(1 - xe^{(\mu_i - r_i)T}\right)Z(d_3)_i.
\]
\[
\frac{\partial E}{\partial F_i} = \frac{\partial S_i}{\partial F_i} = -e^{-rT} [xN(d_2)_i + (1 - x)N(d_4)_i],
\]

\[
\frac{\partial \sigma_E^2}{\partial F_i} = \frac{1}{E} \frac{2V_i}{\partial F_i} \frac{\partial}{\partial F_i} \left( \frac{\alpha S}{\partial V_i} \right) \sigma_{i,E} - \sigma_E^2 \frac{1}{E} \left( 2 \frac{\partial E}{\partial F_i} \right),
\]

\[
\frac{\partial}{\partial F_i} \left( \frac{\partial S}{\partial V_i} \right) = -\frac{xe^{(\mu_i - r)T} Z(d_1)_i + (1 - xe^{(\mu_i - r)T}) Z(d_3)_i}{F_i \sigma_i \sqrt{T}},
\]

\[
E \frac{\partial \sigma_E^2}{\partial F_i} + \sigma_E^2 \frac{\partial E}{\partial F_i} = 2V_i \frac{\partial}{\partial F_i} \left( \frac{\alpha S}{\partial V_i} \right) \sigma_{i,E} - \sigma_E^2 \frac{\partial E}{\partial F_i},
\]

which we use in the comparative statics \( \frac{\partial \beta}{\partial F_i} \). The remaining part of the proof is identical to Ferguson and Shockley (2003, Lemma 1). □
References


Lintner, J., 1965, The Valuation of Risk Assets and the Selection of Risky Investments in


Table 1: Summary Statistics of Parameter Estimates

This table presents the summary statistics of the estimates of $x$ (the risk preference parameter), $\mu$ (the expected-return parameter), and $\sigma$ (the volatility parameter) in the option pricing formulae. The estimates are generated from the month-end prices of options written on the component stocks of the S&P 100 index with alternative time-to-maturities. The $x$ parameter is fixed across firms, while $\mu$ and $\sigma$ are averages either value-weighted or equally-weighted across firms. Only value-weighted results are reported as both produce similar results. The sample period is from 1996 to 2005.

<table>
<thead>
<tr>
<th>Time-to-maturity (days)</th>
<th>$x$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>182</td>
<td>365</td>
<td>182</td>
</tr>
<tr>
<td>Mean</td>
<td>0.8486</td>
<td>0.8500</td>
<td>0.1398</td>
</tr>
<tr>
<td>Median</td>
<td>0.8500</td>
<td>0.8500</td>
<td>0.1292</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.9600</td>
<td>0.9800</td>
<td>0.2511</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.7000</td>
<td>0.7100</td>
<td>0.0697</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0527</td>
<td>0.0624</td>
<td>0.0409</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.0423</td>
<td>0.0697</td>
<td>0.5486</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.4913</td>
<td>2.3425</td>
<td>2.4287</td>
</tr>
<tr>
<td>Jarque-Bera Prob.</td>
<td>0.5143</td>
<td>0.3221</td>
<td>0.0218</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.6900</td>
<td>0.6000</td>
<td>0.8700</td>
</tr>
</tbody>
</table>
Table 2: Summary Statistics of Costs of Equity Capital for Various Industries

This table presents the summary statistics of the costs of equity capital (COE) for various industrial portfolios composed by the component stocks of the S&P 100 index. Only those industries including at least 10 firms are selected. The COEs are estimated with either our option pricing formula or the Fama/French 3-factor model. The option-implied COEs are estimated from the prices of options with one year to expire. The Fama/French COEs are estimated with three-year historical stock prices. The averages are value-weighted. The sample period is from 1996 to 2005.

**Panel 1: Option-implied**

<table>
<thead>
<tr>
<th>Sector</th>
<th>Market</th>
<th>Industrials</th>
<th>Consumer Services</th>
<th>Consumer Goods</th>
<th>Health Care</th>
<th>Financials</th>
<th>Information Technology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.1055</td>
<td>0.1060</td>
<td>0.1080</td>
<td>0.1003</td>
<td>0.1012</td>
<td>0.1079</td>
<td>0.1189</td>
</tr>
<tr>
<td>Median</td>
<td>0.0955</td>
<td>0.0958</td>
<td>0.1026</td>
<td>0.0947</td>
<td>0.0952</td>
<td>0.0984</td>
<td>0.1077</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.1979</td>
<td>0.2064</td>
<td>0.2109</td>
<td>0.1871</td>
<td>0.1717</td>
<td>0.2098</td>
<td>0.2290</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.0606</td>
<td>0.0557</td>
<td>0.0433</td>
<td>0.0475</td>
<td>0.0411</td>
<td>0.0486</td>
<td>0.0563</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0292</td>
<td>0.0313</td>
<td>0.0315</td>
<td>0.0264</td>
<td>0.0281</td>
<td>0.0316</td>
<td>0.0367</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.7034</td>
<td>0.7194</td>
<td>0.5825</td>
<td>0.7015</td>
<td>0.3715</td>
<td>0.6705</td>
<td>0.7515</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.7340</td>
<td>2.8223</td>
<td>3.0777</td>
<td>3.0054</td>
<td>2.4767</td>
<td>2.8649</td>
<td>2.7416</td>
</tr>
</tbody>
</table>

**Panel 2: Fama/French Model**

<table>
<thead>
<tr>
<th>Sector</th>
<th>Market</th>
<th>Industrials</th>
<th>Consumer Services</th>
<th>Consumer Goods</th>
<th>Health Care</th>
<th>Financials</th>
<th>Information Technology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.1430</td>
<td>0.1274</td>
<td>0.1159</td>
<td>0.1195</td>
<td>0.1355</td>
<td>0.1503</td>
<td>0.2027</td>
</tr>
<tr>
<td>Median</td>
<td>0.1732</td>
<td>0.1787</td>
<td>0.1107</td>
<td>0.1084</td>
<td>0.1984</td>
<td>0.1536</td>
<td>0.2916</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.3991</td>
<td>0.3197</td>
<td>0.5713</td>
<td>0.3017</td>
<td>0.3750</td>
<td>0.3989</td>
<td>0.6302</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.1367</td>
<td>-0.1627</td>
<td>-0.2685</td>
<td>-0.0508</td>
<td>-0.1181</td>
<td>-0.0942</td>
<td>-0.3623</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.1636</td>
<td>0.1498</td>
<td>0.2032</td>
<td>0.1033</td>
<td>0.1660</td>
<td>0.1361</td>
<td>0.2834</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.1896</td>
<td>-0.5089</td>
<td>0.2426</td>
<td>0.2113</td>
<td>-0.0868</td>
<td>0.0498</td>
<td>-0.3264</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.5752</td>
<td>1.8647</td>
<td>2.2762</td>
<td>1.5434</td>
<td>1.3702</td>
<td>1.7859</td>
<td>1.7290</td>
</tr>
</tbody>
</table>
Table 3: Equity Premiums
This table presents the equity premiums of the market and various industrial portfolios composed by the component stocks of the S&P 100 index. Only those sectors including at least 10 firms are selected. The risk premium is defined as the difference between the portfolio’s expected return and the risk-free interest rate. The expected returns are estimated with either our option pricing formula or the Fama/French 3-factor model. The option-implied expected returns are estimated from the prices of options with one year to expire. The Fama/French expected returns are estimated with three-year historical stock prices. The averages are value-weighted. The sample period is from 1996 to 2005.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Market</th>
<th>Industrials</th>
<th>Consumer Services</th>
<th>Consumer Goods</th>
<th>Health Care</th>
<th>Financials</th>
<th>Information Technology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option-implied</td>
<td>0.0696</td>
<td>0.0702</td>
<td>0.0722</td>
<td>0.0644</td>
<td>0.0654</td>
<td>0.0721</td>
<td>0.0831</td>
</tr>
<tr>
<td>Fama/French</td>
<td>0.1071</td>
<td>0.0915</td>
<td>0.0801</td>
<td>0.0837</td>
<td>0.0997</td>
<td>0.1144</td>
<td>0.1669</td>
</tr>
</tbody>
</table>
Figure 1: Processes of Parameter Estimates

This figure presents the processes of the estimates of $x$ (the risk preference parameter), $\mu$ (the expected-return parameter), and $\sigma$ (the volatility parameter) in the option pricing formulae. The estimates are generated from the month-end prices of options written on the component stocks of the S&P 100 index with alternative time-to-maturities. The $x$ parameter is fixed across firms, while $\mu$ and $\sigma$ are averages either value-weighted or equally-weighted across firms. Only value-weighted results are reported as both produce similar results. The sample period is from 1996 to 2005.

Panel A: $x$ estimates

Panel B: $\mu$ estimates

Panel C: $\sigma$ estimates
Figure 2: Processes of Costs of Equity Capital

This figure presents the processes of alternative costs of equity capital (COE) for the market portfolio composed by the component firms of the S&P 100 index. The COEs estimated with either our option pricing formula or the Fama/French three-factor model are compared with the empirical COE generated by using equation (16) and assuming an appropriate risk aversion parameter (-\(\alpha\)) suggested by Bliss and Panigirtzoglou (2004). The variables used to generate the empirical COE are computed from three-year historical returns. The option-implied COEs are estimated from the prices of options with one year to expire. The Fama/French COEs are estimated with three-year historical stock prices. The averages are value-weighted. The sample period is from 1996 to 2005.
Figure 3: Processes of Costs of Equity Capital for Various Industries

This figure presents the processes of the costs of equity capital (COE) for various industrial portfolios composed by the component stocks of the S&P 100 index. Only the industries including at least 10 firms are selected. The COEs are estimated with either our option pricing formula or the Fama/French three-factor model. The option-implied COEs are estimated from the prices of options with one year to expire. The Fama/French COEs are estimated with three-year historical stock prices. The averages are value-weighted. The sample period is from 1996 to 2005.
Figure 4: Relationships between Option-implied COE and Investor Sentiment Indices

This figure presents the scatter plots of the COEs estimated from alternative sources against the VIX index compiled by CBOE and the sentiment index of Baker and Wurgler (2006) (B/W index). Totally, there are 120 pairs of estimates for the sample period from 1996 to 2005.

Panel A: COE v.s. VIX

Panel B: COE v.s. B/W index