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Abstract

We develop a theoretical analysis of the choice of firms between fixed-price offerings and uniform-price auctions for selling shares in IPOs and privatizations. We consider a setting in which a firm goes public by selling a fraction of its equity in an IPO market where insiders have private information about intrinsic firm value. Outsiders can, however, produce information at a cost about the firm before bidding for shares. Firm insiders care about the extent of information production by outsiders, since this information will be reflected in the secondary market price, giving a higher secondary market price for higher intrinsic-value firms. We show that auctions and fixed-price offerings have different properties in terms of inducing information production. Thus, in many situations, firms prefer to go public using fixed-price offerings rather than IPO auctions in equilibrium.

We relate the equilibrium choice between fixed-price offerings and IPO auctions to various characteristics of the firm going public. Unlike the existing literature, our model is able to explain not only the widely-documented empirical finding that underpricing is lower in IPO auctions than in fixed-price offerings (e.g., Derrien and Womack (2000)), but also the fact that, despite this, auctions are losing market share around the world. Our model thus suggests a resolution to the above “IPO auction puzzle,” and indicates how current IPO auction mechanisms can be reformed to become more competitive with fixed-price offerings. Our results also provide various other hypotheses for further empirical research.

JEL Classification Code: G30, G32, C72, D44, D82

1 Introduction

Most new issues (i.e., initial public offerings, or IPOs) of equity are sold using a fixed-price offering mechanism, where the firm going public sets a fixed-price for the equity, in consultation with the investment bank taking it public.1 However, many have argued recently, based on results from the economic theory of auctions, that the best way to sell stock in IPOs is to conduct an auction of the shares of the company going public.2 Indeed, an investment banking firm, W.R. Hambrecht & Co., has been founded with the explicit objective of selling IPO shares using a Dutch (or more precisely, Vickrey) auction. Unfortunately for W.R. Hambrecht, the post-IPO stock price performance of these IPO auctions has been less than stellar, in the sense that the stock of those companies that have followed the auction method have languished badly subsequent to the IPO, and only a few companies have chosen to auction shares in their IPOs in the U.S. Further, the auction method of selling IPOs, far from gaining in popularity and replacing fixed-price offerings worldwide, been losing market share worldwide, and is increasingly being replaced by the fixed-price mechanism even in the few countries where it was in place.3

The fact that IPO auctions, while theoretically optimal in terms of maximizing proceeds from the IPO (and empirically documented as involving a smaller amount of underpricing), have been losing market share to fixed-price offerings has been characterized by several authors as a puzzle.4 The objective of this paper is to develop a

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1 In the setting of our analysis, fixed-price offerings and “book-building” methods are equivalent. We will discuss why this is the case later on in this introduction.

2 The argument that is often made in favor of IPO auctions is often empirical as well as theoretical. See, for example, Ausubel and Cramton (1998): “...in the case of initial public offerings of corporate stock, the magnitude of underpricing under current American practice appears to be vastly larger than necessary. The substantial underpricing is indicative of a badly-performing mechanism for selling new issues...why are IPOs not done instead by an efficient auction?” We show in this paper that, in many situations, fixed-price offerings are better for issuers than IPO auctions, even though such offerings may be underpriced to a greater extent. However, we also characterize situations where auctioning shares in IPOs is indeed optimal.

3 At one time or another, IPO auctions have been used in Belgium, Brazil, Chile, France, Hong Kong, Israel, Japan, Korea, Portugal, Singapore, Switzerland, Taiwan, and the U.K. They have fallen out of use in many of these countries.

4 For example, Jenkinson and Ljungqvist (1996) comment: “Auction-like mechanisms such as tenders in the United Kingdom, the Netherlands, and Belgium, or offres publiques de vente in France, are generally associated with low levels of underpricing; most Chilean IPOs have also used auctions, and have been modestly underpriced , at least by emerging-markets standards. This is not surprising, given that, unlike fixed-price offers, tenders allow market demand to at least partially influence the issue price. What is curious, though, is that we do not observe a shift towards greater use of auctions.” Durrien and Womack (2000) make similar comments.
resolution to this “IPO auction puzzle,” based on a theoretical analysis rooted in the realities of the IPO market. We argue that there are two problems with the argument that auctions maximize the proceeds from IPOs and therefore are the optimal way of selling shares in IPOs. First, it is based on results from auction theory developed in the context of a monopolist auctioning off various goods in the product market. However, unlike in the case of a monopolist trying to maximize the proceeds from a one-time sale of various items, the objective of a firm in selling shares is not to maximize the proceeds from a one-time sale of stock. This is because companies care very much about the price of their stock in the secondary market (one reason why they care about the secondary market price of their stock is that many companies wish to issue more equity two or three years down the road from an IPO; also, if the stock price continues to languish, they can be subject to a takeover at bargain basement prices). Thus, in practice, companies face a dynamic choice: they want to obtain high proceeds from the sale of stock, but they also care about the secondary market price of their stock after the IPO.

The second problem with existing arguments about the optimality of auctions is that they take the information structure of the problem as given. In other words, in much of auction theory, the information that various bidders have about the value of the object being sold is taken as unalterable, and the focus is often on comparing auctions in terms of their ability to extract and aggregate the information available with outsiders into the selling price. However, in many auction situations, bidders can produce information about the true value of the object being sold at a cost. For example, when the government is auctioning off rights to drill for oil or other mineral rights, various participants can spend resources to learn more about the value of the mineral rights (by drilling a test hole in the case of oil rights). In particular, investors in the new issues market can devote time and other resources to learn more about the true value of the firm going public. This is important because different ways of selling various objects have different properties when it comes to inducing information production by outsiders. In particular, we show here that in many situations, a fixed-price offering can induce more investors to learn about the true value of a firm going public compared to an IPO share auction, with implications for the cash flows obtained by the firm and its insiders from these two mechanisms.

Combining the above two ingredients, we show that in many cases, a company that wishes to maximize a dynamic objective function (i.e., maximize the cash flow to the firm in the long run, rather than the proceeds from a one-time offering of stock) would in fact choose a fixed-price offering rather than an auction. We consider a
setting in which a firm goes public by selling a fraction of its equity in an IPO market characterized by asymmetric information between firm insiders and outsiders. Outsiders, can, however, produce information at a cost before bidding for shares in the IPO. Auctioning off shares in a setting where outsiders can learn more about the company at a cost will maximize the proceeds from a one-shot offering, but may be detrimental to the company’s long-run value, since not enough investors will choose to produce information about the company. Insiders care about getting a large number of outsiders to produce information about their firm, since this information will be reflected in the secondary market price (thereby leading to a larger secondary market price for truly higher intrinsic-value firms). Thus, in equilibrium, truly higher valued firms would prefer to sell their shares in a fixed-price offering (rather than auctioning them off) because the former is the mechanism that will maximize the long-term value of their firm. Since lower-intrinsic valued firms will also mimic higher intrinsic value firms by setting the same offer price, this price will be such that it induces the optimal extent of information production by outsiders.

There are two important differences between the initial offer price emerging from an IPO auction and the fixed-offer price set by a firm in an IPO. First, the price at which shares are sold in the IPO auction is determined as a result of competition from various informed bidders. This means that the initial offer price in the auction will aggregate, to a significant degree, the information produced by outsiders, unlike in the case of a fixed-price offering, where the offer price is set by the firm. Second, in common value auctions (such as IPO share auctions), bidders, whose information will be correlated with the true value of the firm (and therefore with that of each other), will compete away much of the surplus from each other. Since each bidder expects to be compensated for the cost of producing information, this means that the initial offer price emerging in an IPO auction will be able to support only a smaller number of informed entrants into the auction compared to the number of investors producing information in a fixed-price offering (where the firm can set the offering price to attract the optimal extent of information production by outsiders).

The above intuition is useful in understanding many of our results. First, if a firm is very well known or otherwise suffers from low levels of information asymmetry prior to the IPO (so that outsiders’ cost of information production is small), then our analysis implies that auctioning its equity is optimal, since the number of informa-

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5 Throughout this paper, the auction we consider is a (k+1)th price auction, where, if the firm going public is selling k shares, the uniform price paid by all investors is equal to the (k+1)th highest bid.
tion producers even in an auction is large enough that the disadvantage of lower information production is offset by the greater price received by higher intrinsic-value firms in the IPO. In contrast, if the firm is young, or small, or suffers from a greater extent of information asymmetry for any other reason (so that outsiders’ information production costs are significant), then fixed-price offerings will be the equilibrium choice of the firm, since, in this case, considerations of inducing information production and their impact on the secondary market price become important. Similarly, if the fraction of equity sold by the firm in the IPO is relatively large, then IPO auctions are the equilibrium choice, since, in this case, secondary market considerations are relatively unimportant to firm insiders at the time of the IPO. If, in contrast, the firm is selling only a small fraction of its equity in the IPO (as in the case of many firms going public in the U.S.), then fixed-price offerings are again the equilibrium choice, since, in this case, firm insiders place relatively less weight on maximizing the proceeds from a one-shot equity offering, and more weight on the impact of the IPO mechanism on the secondary market price of its equity.

There is by now a substantial empirical literature comparing the properties of IPOs sold by auction and by fixed-price offerings, in various countries (when both mechanisms are available in the same country) or across countries (see, e.g., Derrien and Womack (2000), Jacquillat (1986), MacDonald and Jacquillat (1974), Jenkinson and Mayer (1988), Kaneko and Pettway (2001), Aggarwal, Leal and Hernandez (1993), Celis and Maturana (1998), and Kandel, Sarig and Wohl (1999)). A prominent recent example of this literature is Derrien and Womack (2000), who document, using French data, that both the mean and the variance of underpricing is lower in fixed-price offerings compared in those sold through IPO auctions. Our model can explain this empirical finding of Derrien and Womack. In our setting, the offering price in an IPO auction aggregates the information produced by outsiders, so that this price is greater for higher-intrinsic-value firms (and lower for lower-intrinsic-value firms) in IPO auctions than in fixed-price offerings. At the same time, there is less information production in auctions compared to fixed-price offerings, which implies a lower amount of information is reflected in the opening price in the secondary market. Since the impact of increased information production is to increase the separation between higher and lower intrinsic-value firms in the secondary market, the price movement from the IPO to the secondary market is therefore smaller for IPO auctions than for fixed-price offerings, leading to both a lower mean and a lower variance in the underpricing of IPOs in auctions relative to fixed-price offerings.\footnote{All available evidence from other countries also indicate that underpricing is much lower in IPO auctions compared to shares sold in fixed-price offerings.} In addition to explaining...
these and other regularities documented by the empirical literature, our model also generates as yet untested predictions useful for further empirical research (see section 5 for a detailed discussion).

The approach we take here differs in two important respects from that in book-building literature. Following the seminal paper of Benveniste and Spindt (1989), a number of papers in this literature (see, e.g., Benveniste and Wilhelm (1990)) assume that outside institutional investors have information superior to the firm (and its underwriters), and demonstrate that underpricing is part of the optimal mechanism to induce truth-telling by institutional investors about their own valuation of the firm going public.\(^7\) In contrast to the above literature, our assumption here is that it is insiders who have information superior to outsiders about their firm’s true value.\(^8\)

Our view is that, while outsiders may indeed have private information about their own valuation of a certain firm going public (and therefore about their demand schedule for its equity), it is indeed firm insiders who are most likely to have superior information about the intrinsic (long-term) value of their own firm. We believe that these two different assumptions are complementary, in the sense that, in principle, both kinds of information problems may exist simultaneously in some settings, and may have relevance in pricing equity in the IPOs of some firms.

A second important difference between our approach and that in the book-building literature is that, in the latter, the objective of firm insiders is simply to maximize the proceeds from a one-shot equity offering. This means that, in the book-building literature, underpricing is a cost imposed on the firm because of the presence of informed outsiders so that an important measure of the success of the IPO equity sales mechanism in the above setting is its ability to minimize underpricing. This has important consequences for the ability of papers in this literature to explain the IPO auction puzzle. For example, papers which argue that the book-building mechanism is the optimal mechanism and therefore essentially equivalent (if not superior) to auction methods (see, e.g., Benveniste and Wilhelm (1990), or Bias and Faugeron-Crouzet (2002)) are clearly not in a position to explain the greater underpricing observed in the book-building method relative to that in IPO auctions (see, e.g.,

\(^7\) Some other papers in this literature are Sherman (2000), Sherman and Titman (2000), and Maksimovic and Pichler (2001). See also Spatt and Srivastava (1991), who show that a posted-price mechanism augmented by preplay communication and participation restrictions, can replicate any optimal auction.

\(^8\) This assumption of firm insiders with private information is consistent with the large literature on IPO underpricing which is not driven by book-building considerations (see, e.g., Allen and Faulhaber (1989), Welch (1989), and Chemmanur (1993)), as well as almost the entire non-IPO literature in corporate finance dealing with private information in various other settings (see, e.g., Myers and Majluf (1984) and Leland and Pyle (1977) on equity issues, Miller and Rock (1985) on dividend policy, or Ross (1977) on capital structure policy). In our setting, information production by outsiders does not give them an informational advantage over firm insiders; it simply brings the precision of their information closer to that of insiders, thereby reducing their informational disadvantage with respect to insiders.
Derrien and Womack (2000) for a study documenting this using French data; similar observations have been made by a number of other studies in various countries).9 On the other hand, papers which argue that uniform-price auctions are the optimal method for selling IPOs (e.g., Bias, Bossearts and Rochet (2002)) are unable to explain why uniform-price auctions are not only not gaining market share in various countries for selling IPOs, but in fact are losing market share.10

In contrast to the above literature, in our setting the insiders’ goal in pricing equity in the IPO is to maximize their dynamic objective function, and minimizing underpricing is not the objective (though, in same cases, the mechanism that maximizes the insiders’ dynamic objective may also happen to be the one that minimizes underpricing). This means that our model is able to explain much more of the empirical evidence comparing auctions with fixed-price offerings, since, in our setting, firms may adopt fixed price offerings even in some situations where they involve greater underpricing. Further, rather than arguing that one or the other method is the always the optimal mechanism, the focus of this paper is on characterizing the situations under which either fixed-price offerings or auctions are optimal for a given firm’s IPO or in the privatization of a particular government-owned firm.11 It should also be noted that, in our setting, since outsiders do not have information superior to firm insiders, insiders possess all value-relevant information required for pricing their firm’s IPO. Therefore, here fixed price offerings and book-building methods are essentially equivalent, and the comparison we make in this paper is between fixed price offerings (and book-building methods) on the one hand, and IPO auctions on the other.12

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9 The empirical literature so far has not provided a single market in the world where non-auction IPOs yielded lower initial returns (underpricing) than IPOs that were auctioned. This seems to lead to the conclusion that, if minimizing underpricing were the only objective of the firm, auctioning shares in IPOs would be the right thing to do.

10 Biais and Faugerson-Crouzet (2002) compare fixed-price offerings, market-clearing uniform-price auctions, and the Mise en Vente (an auction procedure used in France) in a setting where outsiders have private information about their demand for the firm’s shares and the objective of the firm is to maximize IPO proceeds. In an analysis along the lines of Wilson (1979) and Back and Zender (1993), they argue that uniform-price auctions may not be optimal for selling shares if auction participants are asked to submit their entire demand functions, since bidders can tacitly collude by submitting demand functions such that the clearing price is very low. In contrast, in the Mise en Vente (which has some similarities to book-building methods), the price underreacts to demand and thereby unravels tacit collusion on low prices. However, Biais, Bossearts and Rochet (2002) argue that uniform-price auctions may indeed be optimal if the underwriter has private information about the demand for IPO shares, institutional investors have private information about share value, and the underwriter and institutional investors are able to collude.

11 Such an analysis of the settings under which each mechanism is optimal has become particularly crucial in the light of the recently accelerating pace of innovations in information technology (e.g., the internet) in the U.S. and other countries, making it very easy and inexpensive to conduct on-line auctions of shares (as is evidenced, for instance, by the advent of W.R Hambrecht & Co). However, whether such IPO share auctions will indeed become prevalent will clearly depend upon how successful they are in meeting the requirements of firms going public. In section 5, we will also discuss the implications of our analysis for reforms of the IPO auction process.

12 Some authors have made a distinction between fixed-price offerings and book-building methods by characterizing fixed-price
While our primary goal here is to develop an analysis of the relative merits of auctions and fixed price offerings in selling equity in IPOs, this paper also makes a contribution to auction theory and the industrial organization literature dealing with optimal procurement mechanisms. First, this paper endogenizes the information structure in auctions, unlike much of the auction literature that takes the information available to various participants in an auction as given. Second, this is the first paper we are aware of which compares auctions with fixed price offerings in an environment of information production. We will give one example here of a setting outside finance where our analysis can be applied. Consider the case of the U.S Department of Defense (DOD) awarding contracts for weapons procurement (about $80 billion of the DOD budget went toward weapons procurement as of 1992). It is generally acknowledged that one of the most important features of the DOD’s weapons procurement activity is to induce R&D on the part of weapons contractors (since such R&D will lead to better weapons in the future). Thus, the task of awarding weapons procurement contracts is analogous to the IPO problem described above, since, while the DOD is concerned with containing costs, it is also concerned with promoting R&D. The point here is that, while conditioning the award of a procurement contract directly on the amount of R&D performed by a contracting firm is impractical, firms will perform R&D on their own to better compete in the bidding phase. Thus, we can re-interpret the acquisition of information about the value of the firm (in the IPO context) as a process of stochastic cost reduction in the production of a weapon in this case. Thus, the question in the weapons-procurement context is whether auctioning off these weapons contracts, or awarding these contracts based on a predetermined fixed payment (price) by the DOD will be better in the long run (in terms of balancing the twin objectives of cost containment and inducing maximum R&D).

The rest of this paper is structured as follows. Section 2 describes the basic model, while section 3 characterizes the equilibrium of the basic model and develops results. Section 4 develops two extensions of the basic model: section 4.1 allows the firm to choose between IPO auctions and fixed-price offerings in a setting where the fraction of fixed-price offerings as only those where information about outsiders’ demand is not incorporated into the offering price. In contrast, they argue that in IPOs with bookbuilding, underwriters can set the offer price after canvassing potential buyers, and allocate shares on a discretionary basis, thus rewarding outsiders for revealing their information. Such distinctions do not apply to our setting, since outsiders here do not have information superior to firm insiders. Thus, when we refer to fixed-price offerings, we mean offerings where the firm or its underwriters fix the offer price before the securities are offered to the public; our fixed-price offerings encompass all non-auction IPO mechanisms in the U.S as well as in other countries.

13 However, there is a small but growing literature that has endogenized this information structure in auctions to varying degrees. Examples of this literature include Milgrom (1981), Mathews (1977), Persico (2000), Gaier (1997), and Haush and Li (1993). None of this literature, however, has compared the information production properties of fixed-price mechanisms and auctions, focusing instead on comparing the properties of various auctions in terms of inducing information production.
Issuer announces an IPO, along with the IPO mechanism (fixed-price offering or auction)  

\[ t = 0 \]  

Investors decide whether to produce information about the issuer, and whether to bid in the IPO  

\[ t = 1 \]  

Issuer sells the remaining equity in a seasoned equity offering  

\[ t = 2 \]  

Cash flows are realized and distributed to shareholders

Figure 1: Time Line of the Model

of equity sold by the firm is endogenous; section 4.2 allows the issuing firm to make the same choice between a fixed-price offering and an IPO auction with an endogenous reservation price (no reservation price is set by the issuer in the IPO auction in the basic model). Section 5 describes the empirical and policy implications of our model. Section 6 concludes the paper. The proofs of all propositions and lemmas are in Appendix A. Appendix B summarizes the notation used in this paper.

2 The Basic Model

There are three dates in this model. At \( t = 0 \), a private firm goes public by selling a proportion \( \alpha \in (0, 1) \) of its equity in an IPO, using one of the following two mechanisms: a fixed-price offering or a \((k+1)\)th-price auction.\(^{14}\)

Outside investors then decide whether to produce information about the value of the issuing firm, and whether to bid in the firm’s IPO. At \( t = 1 \), the issuing firm’s stock is traded in the secondary market. The firm sells the remaining fraction \( 1 - \alpha \) of its equity to outsiders in a seasoned equity offering at the price prevailing in the secondary market.\(^{15}\) At \( t = 2 \), cash flows are realized and distributed to shareholders. The time line of the model is given in Figure 1.

\(^{14}\) We choose to compare the fixed-price offering with the \((k+1)\)th-price sealed-bid auction (uniform price auction) because it is by far the most widely used form of IPO auction in practice. For example, this kind of IPO auction is used in Israel, France, and also by an investment banking firm, W. R. Hambrecht, in the U.S. Searching for the optimal auction type is beyond the scope of this paper.

\(^{15}\) The assumption of a seasoned equity offering is made only for concreteness. Welch (1989) documents that in the 1977-1982 period, 288 out of 1028 IPO firms reissued a total of 395 seasoned offerings, and the average proceeds from the seasoned equity offerings are three times their average IPO proceeds. Even in the absence of a seasoned equity offering, our results go through qualitatively as long as firm insiders place some weight on the secondary market price in their objective function, which seems to be the case in practice. The assumption that insiders care about the secondary market price is also made in Allen and Faulhaber (1989), Chemmanur (1993), and Welch (1989).
2.1 The Issuing Firm’s Private Information and IPO Mechanisms

The issuing firm, which is risk-neutral, may be either good (type G) or bad (type B). The present value of cash flows of a good firm is \( v = v_G \), and that of a bad firm is \( v = v_B \), where \( v_G > v_B \). We assume both types of firms have positive NPV projects. For simplicity, we normalize \( v_G \) to 1, and \( v_B \) to 0.\(^{16}\) While the issuing firm knows its own type, outside investors observe only the prior probability \( \theta \) of a firm being of type G. The equity offered in the IPO is divided into \( k \) shares. We assume that each investor in the IPO is allowed to bid for a maximum of one share.\(^{17}\)

The issuing firm can choose one of the following two IPO mechanisms: a fixed-price offering or a \((k+1)\)th price auction. If the issuing firm chooses a fixed-price offering, it sets an offering price \( F \) per share, and all buyers pay this price. If the total demand is higher than \( k \) shares in the fixed-price offering, there will be rationing of shares, and the \( k \) shares will allocated to bidders randomly, with each bidder having an equal probability of being allocated one share.\(^{18}\) In the case when the total demand is less than \( k \) shares, the IPO fails.\(^{19}\)

In the case when the issuing firm chooses to auction its shares, the shares are allocated as follows. Investors simultaneously submit sealed bids for shares. The \( k \) highest bidders are each allocated one share, and pay a uniform price, which is the \((k+1)\)th highest bid (we will refer to this price as the clearing price). If there is a tie, so that there are more than \( k \) bidders above the clearing price, all investors bidding strictly above the clearing price are allocated one share with probability 1, with the remaining shares allocated with equal probability to those who bid at the clearing price. For example, suppose there are 2 shares for auction, and there are 4 bidders, who bid 0.9, 0.8, 0.8, 0.7, respectively. Then the offering price will be 0.8, with the bidder who bids 0.9 being allocated one share, and the two bidders who bid 0.8 having a 50% chance of being allocated one share.\(^{20}\)

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\(^{16}\) This normalization is a simplification which allows us to to keep the mathematical complexity to a minimum. It should be obvious that our results will remain qualitatively unaffected in the absence of this assumption.

\(^{17}\) Since investors are risk-neutral and not wealth-constrained, if they are allowed to buy at most one share, they will bid for either one share or nothing. So this assumption is equivalent to assuming that every participant is allowed to bid for either one share or nothing.

\(^{18}\) Our results are unchanged if we make the assumption that in the event that the demand for shares exceeds supply in a fixed-price offering, all investors will be allocated fractions of shares on a pro rata basis. However, we have chosen to make the assumption of rationing since this is the allocation rule for shares followed in practice.

\(^{19}\) We will see later that, in the basic model, the IPO of neither firm type fails in equilibrium when the firm chooses an IPO auction. Even when the firm chooses a fixed-price offering, the type G firm’s IPO never fails in equilibrium; only a type B firm’s IPO has a positive probability of IPO failure.

\(^{20}\) Our results will be exactly the same if we make the alternative assumption that in the case of a tie, those who bid at the clearing price will be allocated equal fractions of a share. In this case, those two who bid 0.8 will be allocated 0.5 share each in our numerical
The objective of the issuing firm is to maximize the combined revenue from the sale of equity in the IPO and in the seasoned equity offering. The issuing firm’s choice of IPO mechanism will affect the amount of information production about the firm, which will in turn determine the secondary market price (and therefore the price at which equity can be sold in the seasoned equity offering). In this sense, the IPO mechanism determines both the revenue from the IPO and the revenue from the seasoned equity offering. Hence the issuing firm will choose the IPO mechanism (and offering price in the case of a fixed-price offering) to maximize its combined revenue.

2.2 The Investors’ Information Production Technology

There are a large number of risk-neutral investors in the market, who do not know the true type of the firm, but have a prior belief that the firm is of type $G$ with probability $\theta$, and of type $B$ with the complementary probability, i.e.,

$$Pr(v = 1) = \theta, \quad Pr(v = 0) = 1 - \theta.$$ (1)

In addition to the equity of the issuing firm, there is a risk-free asset in the economy, the net return on which is normalized to 0.

After the issuing firm chooses its IPO mechanism (auction versus fixed-price offering, and the offering price in the latter case), outside investors make one of the following three choices based on their prior valuation of the firm and other parameters (e.g., information production cost C): engage in uninformed bidding, produce information about the firm and then decide how to bid, or ignore the IPO and invest in the riskless asset. In the case of an IPO auction, we can show that uninformed bidding is always dominated by informed bidding.\(^{21}\) In the case of a fixed-price offering, if sufficiently precise information is available to investors at a low enough cost, and the offering price is not too low, informed bidding always dominates uninformed bidding. In order to focus on issues related to information production, we assume that the model parameters are such that only informed bidding occurs in the fixed-price offering as well.\(^{22}\)

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\(^{21}\) We will show in the basic model that a bidder who produces information and receives a signal $M$ (which is equivalent to receiving no signal at all) has 0 expected payoff. The payoff to an investor who bids without producing information is always less than that to a bidder who produces information and receives a signal $M$. This means uninformed bidding has negative payoff.

\(^{22}\) This assumption translates into a parametric restriction (available to interested readers) on the fraction of equity sold in the IPO, $\alpha$, and the cost of information production to outsiders.
If an investor chooses to produce information about the issuing firm, he has to pay an information production cost $C$, and will receive a signal about the type of the issuing firm. We assume each information producer receives a signal, which can be high ($H$), medium ($M$), or low ($L$), with the following probabilities:

\[
\begin{align*}
\text{Prob}(S_i = H|v = 1) &= p = \text{Prob}(S_i = L|v = 0); \\
\text{Prob}(S_i = M|v = 1) &= 1 - p = \text{Prob}(S_i = M|v = 0); \\
\text{Prob}(S_i = H|v = 0) &= 0 = \text{Prob}(S_i = L|v = 1),
\end{align*}
\]  

(2)

where $p \in (0, 1)$ is the probability that a signal reveals the true value of the issuing firm. The signals received by different information producers are independent of each other. After receiving the above private signal, each information producer decides whether to bid for one share or not (in the case of a fixed-price offering) or how much to bid (in the case of an IPO auction), using Bayes’ rule where appropriate.

### 3 Market Equilibrium

*Definition of Equilibrium:* An equilibrium consists of (i) a choice of IPO mechanism by the issuing firm (conditional on its type) at time 0 (between fixed-price offering and IPO auction), and the offering price in the case of a fixed-price offering; (ii) a system of beliefs formed by investors about the type of the issuing firm after observing the issuer’s IPO choice; (iii) a choice made by each investor whether to produce information after seeing the choice of the issuing firm in the IPO; (iv) a decision of whether to bid for one share or not (in the case of a fixed price offering) or how much to bid (in the case of an IPO auction) made by each information producer after observing a private signal about the type of the firm; (v) a price at which the stock of the issuing firm is traded in the secondary market. The above set of prices, choices and beliefs must be such that (a) the choice of each party maximizes his objective, given the choices and beliefs of others and the expected secondary market price; (b) the beliefs of all parties are consistent with the equilibrium choices of others; further, along the equilibrium path, these beliefs are formed using Bayes’ rule; (c) the number of information producers is such that the payoff to each information producer equals the information production cost; (d) any deviation from his equilibrium strategy by any party is met by beliefs by other parties which yield the deviating party a lower payoff compared to that obtained in equilibrium.
Given our rich strategy space, there may be both separating and pooling equilibria in this model, depending on model parameter values. In principle, there can be two broad categories of equilibria: (1) Separating equilibria, in which type G and type B firms behave differently and reveal their true type; (2) Pooling equilibria, in which type B firms mimic the equilibrium choice made by type G firms. However, if an equilibrium is fully separating, there is actually no role for information production in that setting. Therefore, given that the focus of this paper is on the choice of firms between fixed-price offerings and IPO auctions in an environment of information production, we will focus on pooling equilibria here.23

To facilitate exposition, we discuss the model in reverse order. We discuss the equilibrium in the secondary market before going on to the equilibrium in the IPO market.

### 3.1 Equilibrium Price in the Secondary Market

We assume that there is no restriction on how many shares an investor can buy or short in the secondary market, and investors are not wealth-constrained. Note that the equilibrium secondary market price is dependent on the actual IPO mechanism (fixed-price offering versus auction) only to the extent that this affects the number of information producers about the firm. In other words, if the number of information producers is the same under the two IPO mechanisms, the expected secondary market price will be the same. At time 0, when the firm chooses between a fixed-price offering and an IPO auction, the actual realization of the signals obtained by outsiders is not known to insiders. Therefore, it is the expected secondary market price that enters the insiders’ objective function. Proposition 1 gives the expected equilibrium secondary market price for type G firm, type B firm, and firms across types, as a function of the number of information producers, n.

**Proposition 1 (Equilibrium Price in the Secondary Market)**

(i) The price in the secondary market aggregates all the information obtained by outsiders in the IPO.

(ii) The expected secondary market price of a type G firm, conditional on insider’s information at \( t = 0 \), is given by \( 1 - (1 - \theta)(1 - p)^n \). This price is increasing in the number of information producers, n.

(iii) The expected secondary market price of a type B firm, conditional on insider’s information at \( t = 0 \), is given by \( \theta(1 - p)^n \). This price is decreasing in the number of information producers, n.

(iv) The expected secondary market price of the issuing firm (across types) is \( \theta \), which is independent of the number of information producers, n.

23 However, conditions for the existence of other kinds of equilibria are available to interested readers upon request.
The secondary market price will reflect all the information obtained by participants in the IPO. The reason is as follows. Since there is no limit on how many shares an investor could buy or short in the secondary market, and investors are risk-neutral and not wealth-constrained, if an investor finds that the secondary market price is inconsistent with the signal he receives, he will keep trading until the private information is reflected in the price.

Given the information production technology of investors, the information (private signals) held by information producers could be one of the following three cases: (i) at least one signal is $H$; (ii) at least one signal is $L$; (iii) all signals are $M$. In case (i), the secondary market price must be 1. Since at least one information producer observes a signal $H$ in the IPO, he knows that the firm is of type $G$. If the secondary market price is less than 1, he has an incentive to demand more shares and drive the price up. Similarly, in case (ii) the secondary market price will be 0, since, otherwise, there is an incentive for the investor who observes $L$ to short shares. In case (iii), when all the signals are uninformative, nobody has any meaningful private information, the secondary market price will reflect this information and equal $\theta$.24

Since the price system here is fully revealing (i.e., the secondary market price incorporates the information produced by outsiders), outsiders do not have the incentive to engage in information production at time 1. This is because, while the costs of information production are privately incurred, the benefits no longer accrued to individual outsiders. To illustrate, consider the case where the secondary market price is $\theta$ (i.e., case (iii) discussed above).25 Suppose an investor incurs the information production cost $C$ at time 1, and obtain a signal $H$. In order to profit from this information, he has to buy equity at this time. However, no other investor would be willing to sell him any shares at a price $\theta$, since investors can infer his information from his demand function. A symmetric argument applies if the investor has a signal $L$. Thus no investor has the incentive to produce information in the secondary market.26

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24 The formal proof that the secondary market price is $\theta$ when all the signals are $M$ is given in the proof of Proposition 1 in the appendix.

25 If the secondary market price is 1 or 0, the true type of the firm is already revealed, then there is no need for information production.

26 In practice, the price system may be only partially revealing (perhaps due to additional uncertainty in the economy not modeled here). The equilibrium in the secondary market may then be a noisy rational expectations equilibrium. The intuition behind our model holds even in this case, since we merely require that outsiders’ incentives to produce information diminish after the start of trading in the equity in the secondary market. See Grossman (1976), Hellwig (1980), and Diamond and Verrecchia (1981) for a discussion of the reduction in investors’ incentives to produce information under alternative assumptions about the degree of noise in prices.

27 Consistent with this, there is considerable evidence that a large majority of small firms attract very little analyst coverage subsequent to their IPOs. Further, Rajan and Servaes (1997) document that the extent of analyst coverage following the IPO is
Part (ii) gives the expected secondary market price for the type G firm, and shows that it is increasing with the number of information producers in the IPO, \( n \). Part (iii) gives the expected secondary market price for the type B firm, and shows that it is decreasing with the number of information producers in the IPO. The intuition for the above results is this: when no investor observes the true value of the issuing firm, both types of firms will pool together and the market gives the average value to both types. Each signal has a small probability of revealing the true type of the issuing firm. Since the signals are independent, more signals means a higher probability that the true type is revealed. When the true value is revealed, the type G firm’s secondary market price will increase, and the type B firm’s will decrease. Thus, the expected secondary market price of the type G firm is increasing with the number of information producers, while that of the type B firm is decreasing. Part (iv) demonstrates that the average secondary market price of the firm across types is the prior average value of the issuing firm, and independent of the information production in the IPO. Since information production only separates the pool, it does not change the average value of the issuing firm.

Figure 2 illustrates the above intuition. In the underlying numerical example, we set \( p = 0.03 \) and \( \theta = 0.25 \). The solid curve is the expected secondary market price of the type G firm as a function of number of participants in the IPO, \( n \). We can see that it is increasing in \( n \). The dashed curve depicts the expected secondary market price for the type B firm, and we can see that it is decreasing in \( n \).
3.2 The IPO Market

We now discuss the equilibrium in the IPO market. We first discuss the case where the firm chooses to auction its shares in the IPO, and then discuss the case where the firm chooses to use a fixed-price offering.

3.2.1 The Case When the Issuing Firm Chooses an IPO Auction

In the case where the firm chooses to auction its shares in the IPO, the issuing firm clearly does not need to set a price: the offering price is determined by the bids submitted by investors. Each outsider decides whether or not to enter the auction (produce information) based on his prior probability \( \theta \) of the firm being of type G (and other IPO parameters). If he chooses to produce information, each investor observes a private signal and bids according to it. Below, we characterize the situation where the type G firm and the type B firm pool together by choosing to auction their shares in the IPO (we will show later that this is indeed what happens in equilibrium).

Each investor observes a private signal through the information production technology discussed before, and bids based on his updated value of the firm. The following proposition characterizes the equilibrium bidding strategies of investors in an IPO auction.

**Proposition 2 (Equilibrium Bidding Strategies of Investors in an IPO Auction)**

(i) When the issuing firm uses an IPO auction and there are \( n \geq k + 1 \) information producers, the equilibrium bidding strategy is as follows. Every bidder bids \( \frac{\alpha_k}{K} \) when he observes \( H \), 0 when he observes \( L \), and a random withdrawal \( b \) from the interval \((0, \frac{\alpha_k}{K}]\) with cdf \( M(b; n) \) when he observes \( M \), where \( M(b; n) \) is characterized by the following equation:

\[
\theta(\frac{\alpha_k}{K} - b)(p + (1 - p)(1 - M(b)))^{k-1}[1 - (1 - p)M(b)]^{n-k-1} = (1 - \theta)b[(1 - p)(1 - M(b))]^{k-1}[p + (1 - p)M(b)]^{n-k-1}.
\]

(ii) As long as \( p(1 - p)\theta(1 - \theta)\frac{n}{k}I(n_{\min}) \geq C \), where \( I(.) \) is defined in equation A.14 and \( n_{\min} \) is defined in the appendix, there will be \( n \geq k + 1 \) information producers in the IPO auction, so that the above equilibrium exists.

The true value of each share is either \( \frac{\alpha_k}{K} \) (type G firm) or 0 (type B firm). When an investor observes a signal \( H \), he knows that the firm is of type G and will bid the true value of the firm, which is \( \frac{\alpha_k}{K} \). Similarly, investors who observe \( L \) will bid 0 since only type B firm will have a signal \( L \). However, it is not an equilibrium for all investors who observe \( M \) to bid the expected value of the issuing firm conditional on signal \( M \), which is \( \theta \), since they will face a “winner’s curse”: they will have a greater probability of receiving shares when the firm is of type.
B. Therefore, it can be shown that all investors who observe $M$ will bid a random draw from the distribution $M(b; n)$ in equilibrium.

The following proposition gives the equilibrium number of information producers in an IPO auction.

**Proposition 3 (Equilibrium Number of Information Producers in an IPO Auction):**

(i) The equilibrium number of information producers is determined by:

$$\theta p \int_0^{\alpha/k} \left( \frac{\alpha}{k} - x \right) \binom{n-1}{1} (1-p)m(x) \binom{n-2}{k-1} [p + (1-p)(1-M(x))]^{k-1} [(1-p)M(x)]^{n-k-1} dx = C,$$

where $m(x)$ is the probability density function associated with $M(x)$, i.e., $m(x) = \frac{dM(x)}{dx}$, and $\binom{n-1}{1}, \binom{n-2}{k-1}$ are binomial probabilities;

(ii) The number of information producers is decreasing in the information production cost, $C$, and goes to infinity as the information production cost goes to 0.

The left side of equation 4 is the payoff to one bidder, say, bidder $i$. To understand this formula, first note that the payoff to bidder $i$ is zero when he observes a signal of $M$ or $L$ (the proof is given in the appendix). The payoff is still 0 if he receives a signal $H$ and the clearing price is $\frac{\theta}{k}$ (this happens when at least $k+1$ investors observe the signal $H$). $\theta p$ is the probability that bidder $i$ observes a signal $H$. When the clearing price is $x \in (0, \frac{\theta}{k})$, his payoff is $(\frac{\theta}{k} - x)$, and $\binom{n-1}{1} (1-p)m(x) \binom{n-2}{k-1} [p + (1-p)(1-M(x))]^{k-1} [(1-p)M(x)]^{n-k-1}$ is the pdf of the clearing price being $x$ conditional on bidder $i$ observes $H$ and the clearing price is less than $\frac{\theta}{k}$. The payoff to each investor is a decreasing function of total number of information producers, $n$. So investors keep entering until the expected payoff to each information producer equals the cost of information production cost, $C$. When the information production cost goes to 0, there is no cost to produce information and enter the IPO auction, so that every investor enters the auction and the number of information producers goes to infinity.

**Proposition 4 (Revenue to the Issuing Firm in an IPO Auction):**

(i) When the issuing firm is of type $G$, its expected revenue from the IPO auction is

$$E[IR^G(n)] = [1 - \sum_{j=0}^{k} \binom{n}{j} p^j (1-p)^{n-j}] \frac{\alpha}{k}$$

$$+ k \sum_{j=0}^{k} \binom{n}{j} p^j (1-p)^{n-j} \int_0^{\alpha/k} x \binom{n-j}{1} m(x) \binom{n-j-1}{k-j} (1-M(x))^{k-j} M(x)^{n-k-1} dx; \quad (5)$$

and when the issuing firm is of type $B$, its expected revenue from the IPO auction is

$$E[IR^B(n)] = k \int_0^{\alpha/k} x \binom{n}{j} (1-p)m(x) \binom{n-1}{k-1} [(1-p)(1-M(x))]^k [p + (1-p)M(x)]^{n-k-1} dx. \quad (6)$$
(ii) In the IPO auction, if the issuing firm is of type G, the expected secondary market price is higher than the expected offering price; if the issuing firm is of type B, the expected secondary market price is lower than the expected offering price. In other words, the secondary market price is more informative than the IPO price when an IPO auction is used.

(iii) Define $E[R_G^C]$ as the expected total revenue (from the IPO and the seasoned equity offering) to the type G firm when the IPO auction is used. Then, for any $R < 1$, there exist a information production cost $C > 0$ such that $E[R_G^C] \geq R$ for any $C \leq \tilde{C}$.

When the issuing firm is of type G, with probability $1 - \sum_{j=0}^{k} \binom{n}{j} p^j (1-p)^{n-j}$, there will be more than $k$ information producers who observe $H$ and bid $\frac{2}{k}$. The offering price in this case would be $\frac{3}{k}$ per share. With probability $\binom{n}{j} p^j (1-p)^{n-j}$, there will be $j \in \{0, 1, ..., k\}$ information producers who observe $H$ and bid $\frac{2}{k}$. The other $n-j$ information producers will observe $M$ and bid a withdrawal from the distribution $M(b; n)$, so that the offering price per share would be the $(k+1-j)th$ highest signal from $n-j$ signals withdrawn from $M(b; n)$. Thus the expected revenue to the type G firm in IPO is as specified in equation 5.

In order to understand equation 6, note that if the issuing firm is of type B, the revenue from the IPO auction is 0 if at most $k$ bidders observe the signal $M$. The probability density function that the clearing price is $x \in (0, \frac{2}{k})$ is given by $\binom{n}{j} (1-p)m(x) \binom{n-1}{k-j} [(1-p)(1-M(x))]^k[p+(1-p)M(x)]^{n-k-1}$. The integration in equation 6 gives the expected clearing price conditional on the issuing firm being of type B. Since there are $k$ shares, the expected revenue from the IPO to the type B firm is given by the right hand side of equation 6.

Part (ii) of Proposition 4 demonstrates that the IPO auction price is less informative than the secondary market price. This is because in the IPO auction, each bidder can buy at most one share. They cannot use their private information to the full extent. However, in the secondary market, there is no such constraint, thus investors will make full use of their private information until all the private information is reflected in the price.

We say that an issue is underpriced if the average initial offering price (across types) is lower than the average secondary market price. We demonstrate in the following proposition that there is always underpricing in the IPO auction.

**Proposition 5 (Underpricing in an IPO Auction):** There is always underpricing if the issuing firm uses auction in its IPO. The average percentage underpricing is given by $\frac{\tilde{C}}{n - \tilde{n}}$. 

It is costly for bidders to produce information about the issuing firm, so in equilibrium their payoff from bidding in the IPO auction must exactly cover the information production cost. In equilibrium, the aggregate
information production cost of investors is $n_a C$, which is borne by the issuing firm through underpricing. The average expected secondary market value of fraction of equity sold in the IPO is $\theta_0$, so the percentage underpricing is \( \frac{n_a C}{\theta_0 - n_a C} \).

### 3.2.2 The Case When the Issuing Firm Chooses a Fixed-Price Offering

We now characterize the situation where both the type G and the type B firm pool by issuing equity in the IPO using a fixed-price offering. In this case, the type G firm will set the optimal offering price $F$ to maximize its total revenue from the IPO and the seasoned equity offering, taking into account the effect of the number of information producers in the IPO on the expected secondary market (seasoned equity offering) price. The type B firm will mimic the IPO choice made by the type G firm. We will show later that this is indeed what happens in equilibrium for certain parameter values. The type G firm faces a trade-off when setting the optimal offering price $F$. On the one hand, a higher offering price means more revenue from the IPO; on the other hand, a higher offering price means less information producers in the IPO, hence lower revenue from the seasoned equity offering.

The objective of the type G firm is to

\[
\begin{align*}
\max_F & \quad kF + (1 - \alpha)[1 - (1 - \theta)(1 - p)^n] \\
\text{s.t.} & \quad \pi(F, n) = C.
\end{align*}
\]

(7)

The following proposition gives the equilibrium bidding strategies of investors in the fixed-price offering.

**Proposition 6 (Equilibrium Bidding Strategies of Investors in a Fixed-Price Offering):**

(i) Suppose there are $n \geq k + 1$ information producers in the IPO, and the offering price satisfies $F \leq \frac{\theta_0 (1 - p)^n}{\theta_0 - \theta_0 (1 - p)^n}$, then the equilibrium bidding strategies for information producers are: bid for one share if the signal is $H$ or $M$, and do not if the signal is $L$.

(ii) There exists an $\pi$ such that for $\alpha \leq \pi$, the above equilibrium exists.

Part (i) demonstrates that when the offering price is not too high, every information producer will bid for one share when the signal is $H$ or $M$, and not bid if the signal is $L$. Part (ii) shows that as long as the fraction of equity sold in the IPO is not too large, the equilibrium described in part (i) exists. The intuition here is as follows. When the type G firm only sells a small fraction of its equity in the IPO, most of its revenue will be from the seasoned equity offering, so that the type G firm can benefit most from a high secondary market price.
Furthermore, the cost of underpricing is low since revenue from the IPO is only a small fraction of the total revenue. In this case it is optimal for the type G firm to set $F$ low and the equilibrium in part (i) exists.

In equilibrium, we can see that if the issuing firm is of type G, all $n$ information producers will bid, since everyone’s signal will be either $H$ or $M$, and both will lead the information producer to bid for one share. Therefore, the type G firm’s IPO never fails in equilibrium. However, if the issuing firm is of type B, it is possible that a large number of information producers receive the signal $L$ and not bid, so that less than $k$ investors bidding for the shares in the IPO. In this case, the possibility of IPO failure arises. We assume that if there are less than $k$ information producers bidding for the shares, the IPO auction fails and the type B firm will be liquidated in the secondary market at a price of $0$.\textsuperscript{28} Since all the choices are made by type G firms, type B firms only mimic, the possibility of type B firms’ IPO failure does not affect our model.\textsuperscript{29}

The payoff to each information producer in the above bidding equilibrium is as follows. When the issuing firm is of type G, all $n$ information producers will bid, as a result, each information producer will be allocated one share with probability $\frac{k}{n}$. When the issuing firm is of type B, the total number of bidders depends on how many information producers have received the signal $M$. Consider an information producer, call him bidder 1. When the issuing firm is of type G, he will be allocated one share with probability $\frac{k}{n}$, and the payoff is $(\frac{\alpha}{k} - F)$ if he is allocated one share. When the issuing firm is of type B, he can receive either the signal either $M$ or $L$. When he receives a signal $L$, he will not bid. When he receives $M$, his probability of being allocated one share depends on the number of information producers receiving signal $M$ among information producers $2, ..., n$. We denote that number by $j$. He will be allocated one share with probability $\frac{k}{j+1}$ when $j \geq k - 1$, otherwise the IPO fails. So the payoff to each information producer is:

$$\pi(F, n) = \theta(\frac{\alpha}{k} - F) \frac{k}{n} - (1 - \theta)F \sum_{j=k-1}^{n-1} (\frac{n-1}{j}) (1 - p)^j p^{n-1-j} \frac{k}{j+1}.$$  \hfill (8)

The following lemma gives equilibrium number of information producers, $n_f$, and its relationship with the offering price $F$ set by the issuing firm.

\textsuperscript{28} This assumption is appropriate, since only the type B firm’s IPO fails in equilibrium, so IPO failure will reveal its true type.

\textsuperscript{29} It should be noted that expected secondary market price across types will be still $\theta$ even after accounting for the possibility of IPO failure by a type B firm. The reason is as follows. If the issuing firm is of type B, its true type will be revealed in the secondary market if at least one participant observes signal $L$ in the IPO. The only chance that the type B firm will be traded at a price $\theta$ in the secondary market is when no participant observes $L$. In the case where there are less than $k$ investors bidding for shares in the IPO, there are at least $n - k + 1$ participants who observed $L$, so the issuing firm’s type will be revealed in the secondary market (hence traded at 0) anyway.
Proposition 7 (Equilibrium Number of Information Producers in Fixed-Price Offerings):

(i) The equilibrium number of information producers in a fixed-price offering, \( n_f \), is characterized by the following equation:

\[
\theta \left( \frac{\alpha}{k} - F \right) \frac{k}{n} - (1 - \theta)F \sum_{j=k-1}^{n-1} \left( \frac{\alpha}{\alpha - 1} \right)^j (1 - p)^{j+1} \frac{k}{j+1} = C. \tag{9}
\]

(ii) The equilibrium number of information producers in a fixed-price offering is decreasing in the offering price, i.e., \( \frac{\partial n_f}{\partial F} < 0 \).

The payoff to each bidder is decreasing in the total number of participants. Investors will keep entering until the payoff to producing information equals the cost of doing so. When the offering price is lower, the payoff to each bidder is higher if we hold the number of participants constant. Thus, more investors will choose to produce information when the offering price is lower.

Proposition 8 (Underpricing in Fixed-Price Offerings):

(i) \( F < \theta \frac{\alpha}{k} \) represents an underpricing equilibrium.

(ii) There is always an underpricing equilibrium for \( \alpha \leq \alpha \).

(iii) The smaller the fraction of shares the firm sells in the IPO, the greater the degree of underpricing, i.e., \( \frac{\theta \alpha}{k - F} \) is decreasing in \( \alpha \).

Note that the average secondary market price of equity (of the entire firm) is always \( \theta \), so that the average secondary market price of each share is \( \theta \frac{\alpha}{k} \). When the offering price is lower than the expected secondary market price, the issue is underpriced.\(^{30}\) Part (ii) of the above proposition demonstrates that, in the equilibrium defined in Proposition 6, the IPO is always underpriced. The reason is that the issuing firm needs to compensate the investors for information production through underpricing. Part (iii) demonstrates that when the firms sells a small fraction of its shares in the IPO, it is optimal to underprice more. When the firm sells only a small fraction of its equity, the revenue would come mainly from the seasoned equity offering, so that the cost of underpricing is small, while the benefit from underpricing (a higher secondary market price) would be high. So it is optimal for the firm to underprice more when \( \alpha \) is smaller.

3.3 The Choice between IPO Auctions and Fixed-Price Offerings for Going Public

We now discuss the overall equilibrium of the model, including the firm’s choice of IPO mechanism between fixed-price offerings and IPO auctions.

\(^{30}\) The motivation for underpricing here is similar to that in Chemmanur (1993). However, Chemmanur (1993) focuses only on establishing a rationale for IPO underpricing in the context of a fixed-price offering. In contrast, the focus here is on firms’ choice between IPO auctions and fixed-price offerings.
Proposition 9 (Overall Equilibrium):

(i) In equilibrium, the type G firm chooses the mechanism which maximizes its expected combined revenue from the IPO and the seasoned equity offering. In the case when it chooses a fixed-price offering, the details of the offering are as characterized in section 3.2.2. In the case the firm chooses to auction its shares, the details of the offering are as characterized in section 3.2.1.

(ii) In equilibrium, the type B firm pools with the type G firm by choosing the same offering mechanism and offering price (in the case of a fixed-price offering).

(iii) The equilibrium beliefs of investors at $t = 0$ is such that they assign a probability $\theta$ to any issuing firm choosing an equilibrium action being of type G. The equilibrium bidding strategies of investors in response to a firm choosing a fixed-price offering or an auction are given by sections 3.2.2 and 3.2.1 respectively. If investors observe a firm choosing an out-of-equilibrium strategy, they assign a probability $\theta$ to that firm being of type G.

(iv) The equilibrium in the secondary market in this case is as characterized in Proposition 1.

At time 0, the type G firm has to choose an IPO mechanism which maximizes its own expected total revenue. The type B firm will choose to mimic the type G firm in equilibrium. This is because if it chooses a different IPO mechanism (including an offering price in the case of a fixed-price offering) than the type G firm, it will be revealed to be a type B firm, thereby obtaining a lower expected revenue than that obtained from mimicking the type G firm. Consistent with this equilibrium strategy of the two types of firms, outsiders assign a probability $\theta$ to any firm following the equilibrium strategy of being type G.

The following proposition gives the issuing firm’s choice between fixed-price offerings and IPO auctions for different values of information production cost, $C$.

Proposition 10 (Choice between Fixed-Price Offerings and IPO Auctions as a Function of C): If the information production cost $C$ is less than a certain upper bound $C_{\text{max}}$, then:

(i) When the information production cost is low ($C \leq C''$), the firm will choose an IPO auction.

(ii) When the information production cost is high ($C \geq C''$), the firm will choose a fixed-price offering in its IPO.\(^{31}\)

The intuition behind this proposition is the following. When the information production cost is low, there will be enough participation (hence information production) in the IPO no matter which mechanism is used. From Section 2 we know that a large amount of information production in the IPO means that the secondary market price would be close to the fully revealing price, so that the revenue to the type G firm will be close to its true value. Furthermore, in the case of an IPO auction the offering price is also very informative since there is a lot

\(^{31}\) $C'$, $C''$ and $C_{\text{max}}$ are defined in the appendix.
of information production, so that the type G firm will have a very high revenue in this case. In contrast, when the fixed-price offering is used, both types of firms will get the same revenue from the IPO, which is not in the best interest of the type G firm in this case. Thus, the total revenue to the type G firm would be higher by using an IPO auction when information production cost is low.

When the information production cost is high, the number of information production would be small if the issuing firm uses an IPO auction. This means that neither the offering price nor the secondary market price would be informative, and the type G firm’s revenue would be close to the average value of the issuing firm across types. However, by using a fixed-price offering, the type G firm can optimally underprice its shares sold in the IPO and induce the optimal amount of information production so that the secondary market price would be much more revealing compared to the situation where the firm uses an IPO auction. So when the information production cost is high, the type G firm can have much higher revenue from the secondary market by using a fixed-price offering rather than using an IPO auction, and have only a slightly lower revenue from the IPO. Thus, the total revenue to the type G firm would be higher by using a fixed-price offering when information production cost is high, making this the equilibrium choice.

Figure 3 illustrates the above insights. Notice that for low values of C, an IPO auction is the equilibrium choice of the firm; for higher values of C, a fixed-price offering will be the equilibrium choice.
The following proposition gives the choice between a fixed-price offering and an IPO auction as a function of the fraction of equity sold in the IPO, $\alpha$.

**Proposition 11 (Choice between Fixed-Price Offerings and IPO Auctions as a Function of $\alpha$):** If $\alpha < \alpha$, and information production cost is relatively low ($C \leq \overline{C}$), then:

(i) When the firm sells a small fraction of its equity in the IPO ($\alpha < \alpha$), it will choose a fixed-price offering.

(ii) When the firm sells a large fraction of its equity in the IPO ($\alpha > \alpha'$), it will use an IPO auction.

The fraction of equity sold in the IPO affects two things. First, the number of information producers is monotonically increasing with the fraction of equity sold in IPO, $\alpha$, in the case of an IPO auction. Second, the larger the fraction of equity sold in the IPO, the larger the weight of the revenue from the seasoned equity offering in the issuing firm’s total revenue, so that the type G firm cares more about the secondary market price. Thus, when the firm sells a small fraction of its equity in the IPO, the number of information producers will be small in the case of an IPO auction, and neither the IPO price nor the secondary market price will be very revealing. However, if the issuing firm uses a fixed-price offering, it can underprice its equity in the IPO and induce a large amount of information production. The secondary market price will then be very revealing, so that the secondary market revenue to the type G firm will be quite high. Since the total revenue is dominated by the revenue from the secondary market, the total revenue to the type G firm in this situation will be higher when the issuing firm uses a fixed-price offering rather than an IPO auction.
Table 1: Comparison of Underpricing in Fixed-Price Offerings and IPO Auctions

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Optimal IPO Mechanism</th>
<th>Underpricing (in Percentage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>fixed-price offering</td>
<td>39.14%</td>
</tr>
<tr>
<td>0.08</td>
<td>fixed-price offering</td>
<td>26.58%</td>
</tr>
<tr>
<td>0.1</td>
<td>fixed-price offering</td>
<td>20.05%</td>
</tr>
<tr>
<td>0.12</td>
<td>fixed-price offering</td>
<td>16.08%</td>
</tr>
<tr>
<td>0.14</td>
<td>fixed-price offering</td>
<td>13.39%</td>
</tr>
<tr>
<td>0.16</td>
<td>fixed-price offering</td>
<td>11.46%</td>
</tr>
<tr>
<td>0.18</td>
<td>fixed-price offering</td>
<td>10%</td>
</tr>
<tr>
<td>0.2</td>
<td>fixed-price offering</td>
<td>8.85%</td>
</tr>
<tr>
<td>0.22</td>
<td>fixed-price offering</td>
<td>7.93%</td>
</tr>
<tr>
<td>0.24</td>
<td>IPO auction</td>
<td>6.18%</td>
</tr>
<tr>
<td>0.26</td>
<td>IPO auction</td>
<td>6.97%</td>
</tr>
<tr>
<td>0.28</td>
<td>IPO auction</td>
<td>7.41%</td>
</tr>
<tr>
<td>0.3</td>
<td>IPO auction</td>
<td>7.65%</td>
</tr>
<tr>
<td>0.32</td>
<td>IPO auction</td>
<td>7.75%</td>
</tr>
<tr>
<td>0.34</td>
<td>IPO auction</td>
<td>7.78%</td>
</tr>
<tr>
<td>0.36</td>
<td>IPO auction</td>
<td>7.76%</td>
</tr>
<tr>
<td>0.38</td>
<td>IPO auction</td>
<td>7.69%</td>
</tr>
<tr>
<td>0.4</td>
<td>IPO auction</td>
<td>7.61%</td>
</tr>
</tbody>
</table>

In contrast, when the issuing firm sells a large fraction of its equity in the IPO, there will be a large number of information producers even when the firm uses an IPO auction. This means that both the IPO price and the secondary market price will be high for the type G firm. In this case, the IPO auction dominates, since the fixed-price offering does not confer any additional advantage to the type G firm over an IPO auction (but with the disadvantage of a lower IPO price).

Figure 4 illustrates the above intuition. We can see that when the issuing firm sells a small fraction of its equity in the IPO ($\alpha < 0.32$), it will choose a fixed-price offering; when it sells a large fraction of its equity in the IPO ($\alpha > 0.32$), it will choose an IPO auction.

### 3.4 Comparison of Underpricing in Fixed-Price Offerings and in IPO Auctions: A Numerical Simulation

In this section, we use a numerical simulation to compare the degree of underpricing in fixed-price offerings and in IPO auctions. We assume that the equity in the IPO is divided into $k = 13$ shares. Investors believe with probability $\theta = 0.75$ that the issuing firm is of type G. If an investor chooses to produce information, he has a probability $p = 0.005$ of observing the true value of the firm. The information production cost is $C = 0.0000144164$. The fraction of equity sold in the IPO ranges from 6% to 40%.
Table 1 gives the optimal IPO mechanism and the percentage underpricing as a function of the fraction of equity sold in the IPO, \( \alpha \). We can see that when the firm sells a small fraction of equity in the IPO (i.e., \( \alpha \in [6\%, 22\%] \)), it is optimal to use fixed-price offerings, and when it sells a large fraction of equity in the IPO (i.e., \( \alpha \in [24\%, 40\%] \)), it is optimal to use IPO auctions. This confirms our results in Proposition 11. We can see that when the firm uses fixed-price offerings, the degree of underpricing is decreasing with \( \alpha \). For example, when \( \alpha = 6\% \), the underpricing is 39.14\%; when \( \alpha = 10\% \), underpricing decreases to 20.05\%; when \( \alpha \) increases to 20\%, the underpricing decreases to 8.85\%. This example confirms our results in Proposition 8.

From the example, we find that the degree of underpricing is not monotonic in \( \alpha \) when IPO auctions are used. When \( \alpha \) increases from 24\% to 34\%, underpricing increases monotonically from 6.18\% to 7.78\%. However, when \( \alpha \) increases from 34\% to 40\%, underpricing decreases monotonically from 7.78\% to 7.61\%. We know that the total number of information producers is increasing in \( \alpha \), hence the absolute value of underpricing is also increasing with \( \alpha \) (since the underpricing is to compensate information production costs of outsiders, more information production means a high absolute value of underpricing). However, the average value of shares sold in the IPO (which is always \( \theta \alpha \) from Proposition 1) also increases as \( \alpha \) increases. So the impact of an increase of \( \alpha \) on percentage underpricing is ambiguous. This is because when \( \alpha \) is relatively small, the number of information producers depends crucially on the fraction of equity sold in the IPO and increases very fast; for large values of \( \alpha \), the number of information producers increases at a lower rate. This is why the degree of underpricing is first increasing in \( \alpha \) and then decreasing in \( \alpha \) in the case of IPO auctions.

We find that, in Table 1, the average underpricing conditional on the firm choosing fixed-price offerings (for \( \alpha \in [6\%, 22\%] \)) is 17.05\%, while that conditional on the firm choosing IPO auctions (for \( \alpha \in [24\%, 40\%] \)) is only 7.42\%. This example is consistent with the empirical findings of Derrien and Womack (2000), who find that IPOs conducted by uniform price auctions have less underpricing than IPOs conducted by fixed-price offering (recall that in our model, fixed-price offerings include offerings conducted by book-building as well).

4 Extensions to the Basic Model

In this section, we extend the basic model in two different directions, by relaxing two of the assumptions in the basic model (one at a time). In the first subsection, we relax the assumption that the fraction of equity sold in
the IPO, $\alpha$, is exogenous. In the second subsection, we relax the assumption that the issuing firm does not set any reservation price in IPO auctions.

### 4.1 Fixed-Price Offerings versus IPO Auctions When the Fraction of Equity Offered is Endogenous

In the basic model, we assumed that the fraction of equity sold in the IPO, $\alpha$, is exogenous. In reality, the issuing firm may have some degree of freedom in choosing how much equity to sell in the IPO. In this subsection, we explore this possibility by assuming that the issuing firm can endogenously choose the fraction of equity sold in the IPO, subject to the constraint that at least a fraction $\alpha_{\text{min}}$ has to be sold (all other assumptions remain the same as in the basic model). In this case, the problem facing the issuing firm is two-dimensional: (a) an IPO mechanism: either a fixed-price offering (along with an optimal offering price) or an IPO auction; and (b) the optimal fraction of equity to offer in the IPO. Formally, the problem of the issuing firm is to

$$\max_{\alpha, q \in \{a, f\}} E[R^G_q(\alpha, n)]$$

s.t. $\pi^q(\alpha, n) = C$

where $E[R^G_q(\alpha, n)]$ and $\pi^q(\alpha, n)$ are the expected total revenue to the type G firm and the expected payoff to each information producer, respectively, when mechanism $q \in \{a, f\}$ is used ($f$ stands for fixed-price offering and $a$ stands for auction), a fraction $\alpha$ of the equity is sold in the IPO, and there are $n$ information producers. The following proposition characterizes the equilibrium choice of IPO mechanism with endogenous $\alpha$ as a function of $C$.

**Proposition 12 (Choice of IPO Mechanism with Endogenous $\alpha$ as a Function of $C$):**

(i) When the information production cost is low ($C < \hat{C}$), the firm chooses to auction its shares in the IPO;

(ii) When the information production cost is high ($C > C_{\text{max}}$), the firm will use a fixed-price offering to sell shares in the IPO.

The intuition behind the firm’s choice of mechanism when the fraction of equity sold is endogenous is similar to that in the basic model. When the information production cost is low, there will be a large number of information producers in an IPO auction even when $\alpha$ is small. Fixed-price offering dominates auction only for very low values of $\alpha$. Since the firm has to sell a minimum fraction of $\alpha_{\text{min}}$, auction dominates in this case. When the information
Figure 5: Endogenous Fraction of Equity Sold, $\alpha$, When $C$ Is Low

Figure 6: Endogenous Fraction of Equity Sold, $\alpha$, When $C$ Is Moderate
production cost is high, the amount of information production is low in auction even when the firm sells a large fraction of its equity in the IPO. As a result, the revenue to the type G firm is low for any $\alpha$. However, when a fixed-price offering is used, type G firm can choose to sell a small fraction but optimally underprice its equity in the IPO and induce a large number of information producers. As a result, the type G firm will choose fixed-price offering rather than in this case.

Figures 5, 6 and 7 illustrate the firm’s equilibrium choice of offering mechanism for various values of $C$, as well as its equilibrium choice of $\alpha$. From Figure 5 we can see that when $C$ is low the type G firm will choose the IPO auction and sell the smallest possible fraction, $\alpha_{\text{min}}$. This is because there will be enough information production even when the firm sells the minimal fraction $\alpha_{\text{min}}$. Since the IPO price is not as informative as the secondary market price (for a type G firm, the expected offering price is always lower than the expected secondary market price), the type G firm tries to sell as little as possible in the IPO in this case.

However, when the information production cost is moderate, the type G firm will choose to auction its shares and sell a fraction more than $\alpha_{\text{min}}$, as illustrated in Figure 6. Since, in this case, the number of information producers in the IPO crucially depends on the fraction of equity sold, the type G firm faces a trade-off when choosing the optimal fraction of equity to offer. On the one hand, a large fraction sold means more information producers in the IPO, hence both the IPO price and the secondary market price would be higher. On the other hand, since the expected IPO price is always lower than the expected secondary market price for the type G firm,
a larger fraction of equity sold in the IPO means lower expected revenue to the type G firm. In this case, the type G firm will therefore choose an optimal \( \alpha \) larger than \( \alpha_{\min} \). In the figure, we assume \( p = 0.005, \theta = 0.1, k = 3 \) and \( C = 7.4993 \times 10^{-6} \). The optimal fraction of equity sold in the IPO is \( \alpha = 0.26 \), which is higher than the minimum required fraction, \( \alpha_{\min} = 0.1 \).

When the information production cost is high, the type G firm will choose a fixed-price offering, and again sell the minimum fraction \( \alpha_{\min} \), as illustrated by Figure 7. The reason why the type G firm chooses a fixed-price offering for large values of \( C \) has already been discussed in the basic model. The type G firm faces a trade-off when choosing the fraction of equity to sell: it can sell a large fraction in the IPO and underprice less, or sell a small fraction and underprice more in order to induce the same amount of information production. However, for a wide variety of parameter values it is always optimal for the type G firm to sell a small fraction and underprice more.

### 4.2 IPO Auction with an Endogenous Reservation Price

In the basic model, we assume that the issuing firm does not set any reservation price in the IPO auction. In this subsection, we relax this assumption by allowing the issuing firm to choose a reservation price \( r \) optimally in the IPO auction (all other assumptions remain the same as in the basic model).

The introduction of a reservation price will affect three things. First, by setting a reservation price, the issuing firm can protect itself from selling at a very low price in the IPO. It makes sure that the offering price is higher than the reservation price when the IPO succeeds. Second, there is a possibility of IPO failure when the issuing firm sets a high reservation price. When most bidders would like to make a low bid, there may be less than \( k+1 \) bidders bidding above \( r \), so that the IPO auction fails. We assume that if the IPO auction fails, the issuing firm is able to obtain financing from an alternative source (say, a private placement of equity). We denote the cash flow to the issuing firm from this alternative source by \( R_{\text{fail}} \), which is common to type G and type B firms.\(^{32} \) Third, when there is a reservation price in the IPO auction, the information producers’ payoff will be less, so that there will be less information producers in the IPO, which is not in the best interest of the type G firm. Formally, the

\(^{32} \)In the basic model we assume that type B firm will get 0 when IPO fails. Here we assume that both types get \( R_{\text{fail}} \). The reason is that in the basic model, only the IPO of the type B firm may fail, so that IPO failure will reveal its true type. Here both types may fail, so that IPO failure does not reveal firm type. Our results remain the same if we assume \( R_{\text{fail}} \) is different for the two types of firms.
objective of the type G firm in the IPO auction is now:

\[
\begin{align*}
\text{Max}_r & \quad E[R^G_n(n, r)] \\
\text{s.t.} & \quad \pi^a(n, r) = C,
\end{align*}
\]

where \(E[R^G_n(n, r)]\) and \(\pi^a(n, r)\) are the expected revenue to the type G firm and the payoff to each information producer, respectively, when the reservation price is \(r\) and there are \(n\) information producers. The benefit to the issuing firm of having a reservation price is that it can extract some of the surplus obtained by the bidders in the firm’s IPO. The cost of having a reservation price is that it may reduce the number of information producers about the firm and increase the probability of IPO failure. The equilibrium reservation price set by the issuing firm emerges from the above trade-off.

As in the basic model, the type B firm mimics the equilibrium strategy of the type G firm, so that the two types set the same reservation price in equilibrium. If the type G firm sets a reservation price \(r \in (0, \frac{\alpha}{k})\), this will affect the information production decisions and bidding strategies of the investors. The following proposition characterizes the equilibrium bidding strategies of information producers in an IPO auction with an endogenous reservation price.

**Proposition 13 (Equilibrium Bidding Strategies of Information Producers in an IPO Auction with an Endogenous Reservation Price):** If the issuing firm sets a reservation price \(r \in (0, \frac{\alpha}{k})\) in auction, then the equilibrium bidding strategy is as follows:

(i) Participants who observe \(H\) bid \(\frac{\alpha}{k}\);

(ii) Participants who observe \(L\) do not bid;

(iii) If a participant observes \(M\), he withdraws a random number \(b\) from the distribution \(M(b)\), which is defined in equation 3. He will bid \(b\) if \(b \geq r\), and not bid otherwise.

The bidding strategy in an IPO auction with an endogenous reservation price is very similar to that without reservation price in the IPO auction. The difference here is that if the amount the bidder would like to bid is lower than the reservation price, he does not place the bid in equilibrium. The following lemma gives the equilibrium number of information producers in the IPO auction with endogenous reservation price.

**Lemma 1 (Equilibrium Number of Information Producers in the IPO auction with an Endogenous Reservation Price):**

(i) The equilibrium number of participants is determined by the following equation

\[
\theta p \int_0^{\frac{\alpha}{k}} \left(\frac{x}{k}\right)^{n-2} \left(1 - p\right)m(x) \left(\frac{n-2}{k-1}\right) [p + (1 - p)(1 - M(x))]^{k-1}[(1 - p)M(x)]^{n-k-1}dx = C;
\]

\[12\]

30
(ii) The equilibrium number of information producers is decreasing in r.

The expected payoff to each bidder is given in the left side of Equation 12. It is similar to the case when there is no reservation price, as in the basic model. However, in this case, when there are not enough investors bidding above the reservation price r, the IPO auction fails, which is not in the best interest of the information producers since they will be allocated shares only when the offering price is high, and they can not be allocated shares when the offering price is low. Therefore the payoff to all participants in the IPO auction is decreasing in r, since in equilibrium, investors will enter until the payoff equals the cost of information production, higher reservation price means less information production about the issuing firm.

**Proposition 14 (Choice between a Fixed-Price Offering and an IPO Auction with an Endogenous Reservation Price as a function of \( \alpha \))**

If \( \alpha < \bar{\alpha} \), and information production cost is relatively low \((C \leq \bar{C})\), then:

(i) When the firm sells a small fraction of its equity in the IPO \((\alpha < \alpha^a)\), the firm will choose a fixed-price offering;

(ii) When the firm sells a large fraction of its equity in the IPO \((\alpha > \alpha^f)\), the firm will use an IPO auction with an endogenous reservation price.

The above proposition demonstrates that even when the issuing firm sets a reservation price in the IPO auction, we obtain a result similar to Proposition 11. The intuition here is as follows. The benefit to the issuing firm of having a reservation price in an IPO auction is that it enables it to extract some of their surplus from bidders. However, by the same token, the cost of having a reservation price is that it reduces the amount of information production among investors associated with the IPO auction, since the payoff to bidders is decreasing with the reservation price. Therefore, when the firm sells a small fraction of its equity, the information production is low if the IPO auction is used. In this case, a fixed-price offering dominates the IPO auction because, as we discussed before, the fixed-price offering does a good job of inducing information production. If, however, the issuing firm sells a large fraction of its equity in the IPO, an IPO auction dominates a fixed-price offering for the same reasons as discussed in the basic model.\(^{33}\) In this case, setting a reservation price yields a greater revenue to the issuer compared to an IPO auction with no reservation price.

\(^{33}\) Recall that, since the issuer has the freedom to set the reservation price optimally (including setting \( r=0 \), which is equivalent to setting no reservation price as in the basic model), an auction with an endogenous reservation price will also dominate a fixed-price offering in all settings where an IPO auction without a reservation price dominates the fixed-price offering.
Figure 8: Comparison of Fixed-Price Offering with Auctions with and without Reservation Price

Figure 8 presents the relationship between $\alpha$, the fraction of equity sold in the IPO and the choice between fixed-price offerings and auctions with an endogenous reservation price. As in the basic model, the firm chooses a fixed-price offering in the IPO for low values of $\alpha$, while for high values of $\alpha$, it chooses an auction (the solid line gives the expected revenue to the issuer in an IPO auction with endogenous reservation price while the dotted line gives the revenue in an IPO auction without a reservation price).

**Proposition 15 (Choice between a Fixed-Price Offering and an IPO Auction with an Endogenous Reservation Price as a Function of $C$):** If the information production cost $C$ is less than a certain upper bound $C_{\text{max}}$, then:

(i) When the information production cost is low ($C \leq C'$), the firm chooses an IPO auction with an endogenous reservation price.

(ii) When the information production cost is high ($C \geq C''$), the firm chooses a fixed-price offering in the IPO.

When the information production cost is high, we already know from the basic model that a fixed-price offering dominates an IPO auction without a reservation price. When the firm has the freedom to set a reservation price, it will not help in this case, since a reservation price will only restrict information production. When the information production cost is low, the IPO auction dominates a fixed-price offering. In this case, the introduction of a reservation price will not hurt the IPO auction since the issuing firm always has the freedom of setting $r=0$, which is equivalent to setting no reservation price. So an IPO auction with an endogenous reservation price dominates a fixed-price offering in this case.
Figure 9: Comparison of the Three Mechanisms as A Function of C

Figure 10: Endogenous Reservation Price as A Function of $R^{\text{fail}}$
The optimal reservation price is determined by the following trade-off. The higher the reservation price, the higher the payoff to a type G firm when the IPO succeeds. However, the higher the reservation price, the higher the probability of IPO failure. The optimal reservation price emerges from this trade-off. The optimal reservation price depends on the value of \( R_{\text{fail}} \). Intuitively, the higher the value the issuing firm get from IPO failure, the lower the cost of IPO failure, the higher the reservation price. Therefore we expect the optimal reservation price is increasing with the value from IPO failure. This is illustrated in Figure 10. From the figure we can see that for \( R_{\text{fail}} \leq 0.4 \), it is optimal for the issuing firm to set \( r = 0 \). For \( R_{\text{fail}} > 0.4 \), we have \( r > 0 \), and we can see that \( r \) is strictly increasing in \( R_{\text{fail}} \) in this region.

5 Empirical and Policy Implications

We highlight some of the empirical and policy implications of our model below.

*(i) The relationship between firm and IPO characteristics and the optimal mechanism for going public:* First, our model predicts that if a firm is young, or small, or suffers from a greater extent of information asymmetry for some other reason (so that outsiders’ information production costs are significant), then fixed-price offerings will be the equilibrium choice of the firm, since, in this case, considerations of inducing information production and their impact on the secondary market price become important. In contrast, if a firm is older, or larger, or has a well-known (reputable) product, or suffers from low levels of information asymmetry for some other reason (so that outsiders’ cost of evaluating the firm is smaller), then our analysis implies that it will choose an IPO auction. Second, our model predicts that, *ceteris paribus*, firms selling smaller fractions of equity in the IPO will choose fixed-price offerings, while those selling larger fractions of their equity will choose IPO auctions. Consistent with the former prediction, Derrien and Womack (2000) document that foreign firms going public in France (which can be expected to be lesser-known) make use of non-auction IPO mechanisms.

*(ii) The relationship between offering mechanism and IPO underpricing:* Our model predicts that IPO auctions will exhibit a significantly lower mean and variance of underpricing compared to fixed-price offerings. This is due to the fact that the offering price in an IPO auction aggregates the information produced by outsiders to a significant degree, so that this offering price is greater for higher intrinsic-value firms (and lower for lower-intrinsic-value firms) in IPO auctions than in fixed price offerings. At the same time, there is less information
production in IPO auctions compared to fixed-price offerings (where the offering price set by insiders to induce the optimal degree of information production), so that a lower amount of information is reflected in the opening price in the secondary market in this case. Since the impact of increased information production is to increase the separation between higher and lower intrinsic-value firms in the secondary market, the price jump (either upward or downward) from the IPO to the secondary market is therefore smaller for IPO auctions than for fixed price offerings, leading to both a lower mean (see section 3.4) and a lower variance of underpricing in IPO auctions. Evidence consistent with this prediction is provided by Derrien and Womack (2000), who document, using French data, that both the mean and the variance of underpricing is lower in IPO auctions compared to those sold through fixed-price offerings. The British privatization study of Jenkinson and Mayer (1988) also indicates that, in the U.K., the extent of underpricing was much lower in the auction sample than in the non-auction sample. A comparison of IPO underpricing from Japanese IPO auctions (Pettway and Kaneko, 1996) with underpricing in non-auction Japanese IPOs around the same period (Jenkinson, 1990) also indicates that underpricing in IPO auctions is significantly lower (see also Loughran, Ritter and Rydqvist (1994) and Kaneko and Pettway (2001)).

IPOs where shares are auctioned seem to have lower underpricing compared to those using fixed-price offerings in Taiwan as well (see Lin and Sheu (1997), Liaw, Liu and Wei (2000), and Ritter (2002)). A comparison of the initial returns from shares bought in Chilean IPO auctions (Aggarwal et al, 1993) with the developing-market average for non-auction IPOs around the same period (Jenkinson and Ljungqvist, 1996) also leads to similar conclusions in the Chilean and developing-market setting.

(iii) The relationship between offering mechanism and the average number of bidders in the IPO: Our model predicts that the number of bidders in fixed-price offerings will be significantly larger than that in IPO auctions. Recall that, in our setting, firms set the offering price in a fixed-price offering so as to induce a greater extent of information production in the IPO, leading to a larger number of bidders in fixed-price offerings as well.

(iv) A Resolution of the IPO Auction Puzzle: Unlike the existing literature, our model is able to explain why auctions are losing market share to fixed-price offerings, while simultaneously predicting that fixed-price offerings will exhibit a greater extent of underpricing compared to IPO auctions. If, in practice, firm insiders’ objective is

34 However, the fact that Japanese IPOs auctions were discriminatory rather than uniform-price auctions calls for some caution in interpreting this evidence.
not to maximize the proceeds from a one-shot equity offering, it is indeed optimal for younger and smaller firms, and those selling smaller fractions of equity to go public using fixed-price offerings.\textsuperscript{35} Since a large majority of firms going public in the U.S as well as most other countries fall into this category, it is hardly surprising that IPO auctions are not gaining market-share in these countries. Our analysis indicates that one setting where IPO auctions may indeed be optimal is in the privatization of large, well-run government firms (where the government may be selling off its entire equity in the IPO) or in the IPOs of older, large firms (for instance, firms going public again after being taken-private in an LBO).\textsuperscript{36}

\textit{(v) The benefit from selling equity in tranches and the optimal fraction of equity to sell in the IPO:} First, our analysis indicates the benefits of selling equity in tranches, since the firm may be able to obtain a significantly higher share price in later sales of equity. Second, our results from section 4.1 indicate that, when the firm needs to raise only a small amount of capital from the IPO (so that the minimum amount of equity the firm needs to sell in the IPO is small), the fraction of equity to be sold in the IPO may have to be decided jointly with the choice of offering mechanism. Thus, when the outsiders’ cost of information production is large, the firm may choose to sell the smallest fraction of equity possible, using a fixed price offering. At the other extreme, when the outsiders’ cost of information production is very small, the firm may also choose to sell the smallest fraction of equity possible, but using an IPO auction. At moderate levels of the cost of information production, however, the firm may choose to sell a fraction of equity larger the minimum it needs to sell (from the point of view of raising capital) using either an IPO auction (for smaller C values in this range) or a fixed price offering (for larger C values in this range). This latter result obtains because the number of information producers tends to increase with the fraction of equity sold in IPOs.\textsuperscript{37}

\textsuperscript{35} The fact that, on average, firms sell only about one third of their equity in IPOs in the U.S. seems to indicate that insiders may indeed not be focused on maximizing proceeds from a one-shot equity offering here.

\textsuperscript{36} The privatization of large, well-known government-owned firms seems to be one setting where IPO auctions seem to have been used extensively to date (e.g., in privatizations in Britain and several other countries). In terms of firms going public in the U.S., our analysis indicates that IPO auctions have been tried by precisely the wrong kind of firms, namely, small, lesser known firms (possibly because IPO auctions appealed to these firms purely from the point of view of providing savings in investment banking fees); older, larger, or better-known firms might have made use of IPO auctions with considerably more success (though most firms going public clearly do not belong to this category).

\textsuperscript{37} The determination of optimal tranche size was an often-debated problem in the privatizations of government firms in many countries. Our analysis provides two pieces of guidance in this case. First, it indicates that, even when the government has no reason to hold on to large fractions of equity after the IPO, it is optimal from the point of revenue maximization to sell the equity in tranches. Second, even when there is no minimum capital to be raised from an IPO or privatization, considerations of inducing information production dictate that the government (or firm going public) sell a certain minimum fraction of equity.
(vi) Determination of the reservation price in IPO auctions: Our analysis of section 4.2 indicates that firms which have little or no immediate capital requirements, or have a low opportunity cost of postponing their IPOs (either because they have alternative sources of capital, or because the stock issue was not meant to raise significant capital, as in the case of many privatizations) will set a higher reservation price compared to those which do have significant capital requirements or a significant opportunity cost of postponing their IPO.

(vii) Reforming the procedures used in existing IPO auctions: Our analysis indicates that existing IPO auction procedures may be reformed in several directions to make IPO auctions more competitive with fixed-price offerings. First, our analysis indicates that firms going public may benefit from offering the IPO at a discount to the clearing-price in the IPO auction. Second, this discount may be adjusted to account for the characteristics of firms going public: for instance, a greater discount may be offered in the IPOs of younger, smaller, or lesser-known firms. The idea here is to encourage greater information production by outsiders, over and above that “naturally occurring” in auctions.38

6 Conclusion

We have developed a theoretical analysis of the choice of firms between fixed-price offerings and uniform price auctions for selling shares in IPOs and privatizations. We considered a setting in which a firm goes public by selling a fraction of its equity in an IPO market where insiders have private information about intrinsic firm value, but where outsiders could produce information at a cost about true firm value before bidding for shares. We showed that, while auctioning off shares in such a setting may maximize the proceeds from a one-shot offering, it does not maximize long-run firm value, since not enough investors will choose to produce information about the firm in equilibrium. Insiders care about inducing the optimal degree of information production by outsiders, since this information will be reflected in the secondary market price, giving a higher secondary market price for truly higher intrinsic-value firms. Thus, we demonstrated that, in many situations, firms will prefer to go public using fixed-price offerings rather than IPO auctions in equilibrium, since such offerings allow the firm to induce the optimal extent of information production. We related the equilibrium choice of firms between fixed-price

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38 The IPO auctions in France do seem to set the offer price 2 to 5% below the auction-clearing price (see MacDonald and Jacquillat (1970) or Biais and Faugeron-Crouzet (2000)). However, the French auction officials do not seem to pre-commit to such a policy. It may be optimal not to pre-commit to a discount to the clearing price since investors may bid more aggressively in response to the pre-committed discount, thus offsetting any advantage provided by the discount in terms of inducing information production.
offerings and auctions to various characteristics of the firm going public and that of its IPO. Unlike the existing literature, our model is able to explain not only the widely-documented empirical finding that both the mean and variance of underpricing is lower in IPO auctions than in fixed price offerings (e.g., Derrien and Womack (2000)), but also the fact that, despite this, auctions are losing market share around the world. Our model thus provides a resolution of the "IPO auction puzzle," and suggests how current IPO auction mechanisms may be reformed to become more competitive with fixed-price offerings. Our results also provide various other hypotheses for further empirical research.

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Appendices

A Proofs of Propositions:

Before the proofs of the propositions in the paper, we define some functions and point out some of their properties, which will be used later in the formal proofs.

\( n_{\text{min}} \): The function \( I(n) \) obtains its maximum value at this point, and \( \frac{\partial I(n)}{\partial n} < 0 \) for all \( n \geq n_{\text{min}} \). Note that the function \( I(n) \) is independent of both \( \alpha \) and \( \theta \).

\( \alpha_f^l \): The minimum fraction of equity the firm must sell to make sure that there will be at least \( k + 1 \) investors produce information in the IPO when fixed-price offering is used.

\( \alpha_f^l \): The minimum fraction of equity the firm must sell to make sure that there will be at least \( k + 1 \) investors produce information in the IPO when auction is used. Note we always have \( \alpha_f^l > \alpha_f^f \), since when using fixed-price offering, the issuing firm can always set the offering price low to make sure that there will be enough entry.

\( C_{\text{max}} \): The maximum information production cost which makes sure that auction is feasible when \( \alpha = 1 \), i.e.,

\[
C_{\text{max}} = p(1-p)\theta(1-\theta)\frac{1}{k}I(n_{\text{min}}). \tag{A.1}
\]

\( \tilde{\alpha} \): The maximum \( \alpha \) makes sure that the equilibrium used in fixed-price offering exists for all \( C \in (0, C_{\text{max}}) \), i.e.,

\[
\tilde{\alpha} \equiv \min\{\overline{\alpha}(C) : C \in (0, C_{\text{max}})\}, \tag{A.2}
\]

which makes sure that for any \( C \leq C_{\text{max}} \), and \( \alpha \leq \tilde{\alpha} \), the NE exists.

**Proof of Proposition 1:** We first prove that the secondary market price will be \( \theta \) if all \( n \) signals are \( M \). We use \( M^n \) to denote the event that all \( n \) signals are \( M \), then we have

\[
SP(M^n) = E[v|M^n] = 1 \times Pr(v = 1|M^n) + 0 \times Pr(v = 0|M^n) = Pr(v = 1|M^n)
\]

\[
= \frac{Pr(v = 1)Pr(M^n|v = 1)}{Pr(v = 1)Pr(M^n|v = 1) + Pr(v = 0)Pr(M^n|v = 0)} = \frac{\theta(1-p)^n}{\theta(1-p)^n + (1-\theta)(1-p)^n} = \theta, \tag{A.3}
\]

which completes the proof.

Suppose there are \( n \) participants in the firm’s IPO. If the firm is of good type \( G \), with probability \( (1-p)^n \) all participants will receive the signal \( M \), and the secondary market price is \( \theta \); with probability \( 1 - (1-p)^n \) at least one participant will receive a signal \( H \), and the secondary market price is 1. So the expected secondary market price for a high value firm when there are \( n \) participants in the IPO market is:

\[
E[SP^G(n)] = \theta(1-p)^n + (1 - (1-p)^n) \times 1 = 1 - (1-\theta)(1-p)^n, \tag{A.4}
\]

which is increasing in \( n \) since \( \partial E[SP^G(n)]/\partial n = -(1-\theta)(1-p)^n \ln(1-p) > 0 \). Similarly, the expected secondary market price for a low value firm when there are \( n \) participants in the firm’s IPO is:

\[
E[SP^B(n)] = \theta(1-p)^n + (1 - (1-p)^n) \times 0 = \theta(1-p)^n, \tag{A.5}
\]

which is decreasing in \( n \) since \( \partial E[SP^B(n)]/\partial n = \theta(1-p)^n \ln(1-p) < 0 \). The expected secondary market price for the firm across types is

\[
E[SP(n)] = \theta E[P_2^H] + (1-\theta)E[P_2^f]. \tag{A.6}
\]

Plug equations A.4 and A.5 in, we have

\[
E[SP(n)] = \theta, \tag{A.7}
\]
which is independent of $n$.

**Proof of Proposition 2**: Suppose the other n-1 bidders, 2, 3, ... n-1, use strategy defined in the proposition, if we can prove that it is also optimal for bidder 1 to use the strategy, then the strategy is a symmetric NE. When bidder 1 observes H, he knows that the true value of each share is $\frac{\theta}{k}$. Suppose $i$ bidders receive signal H other than bidder 1, where i could range from 0 to n-1.

Case 1: $i < k$. The k-th highest bid other than 1 will be a bid from a participant with signal M, hence the clearing price is less than $\frac{\theta}{k}$, let’s call it $m_k$. If bidder 1 bids $\frac{\theta}{k}$, he will win one unit and pay a price of $m_k$, the payoff is positive; if he bids a value in $[0, m_k)$, he can never win the object and payoff is zero; if he bids a value in $(m_k, 1)$, the payoff is the same as bidding $\frac{\theta}{k}$. We can see that bidding $\frac{\theta}{k}$ is weakly optimal in this case.

Case 2: $i \geq k$. The k-th highest bid among investors 2, 3, ..., n is $\frac{\theta}{k}$. If investor 1 bids $\frac{\theta}{k}$, he will win one unit with probability $\frac{\theta}{k^2}$, and pay a price of $\frac{\theta}{k}$, the payoff is zero; if he bids a value in $[0, \frac{\theta}{k})$, he can never win the object and payoff is zero. We can see that bidding $\frac{\theta}{k}$ is also weakly optimal in this case.

When a participant observes signal L, he knows that $\nu = 0$. Suppose that investors 2, 3, ..., n use the equilibrium bidding strategy. If investor 1 bids 0, the payoff to him is always zero. If he bids above 0, and more than k other investors receives the signal M, there is some chance that he will receive one unit at a price higher than 0. If he bids less than 0, his bid will never be filled hence the payoff is 0. So it is optimal to bid 0 on L when all other investors use the equilibrium strategy.

We now characterize the bidding strategy of the investors who receive signal M. Define $M(b)$ as the cdf of a random variable with support on the interval $(b, \theta)$, and $m(b)$ is the corresponding pdf. Suppose bidders 2, 3, ..., n will use the following strategy: withdraw a bid from the distribution $M(b)$ after receiving a signal M. If investor 1 observes M, and if he submits a bid $b \in (b, \theta)$, the expected payoff to him is

$$
\pi(b) = \theta \int_{\theta}^{b} (\frac{\theta}{k} - x) \binom{n-1}{k-1} (1-p)m(x) (\frac{n-2}{k-1}) [p + (1-p)(1-M(x))]^{k-1}[(1-p)M(x)]^{n-k-1} dx

- (1-\theta) \int_{\theta}^{b} x \binom{n-1}{k-1} (1-p)m(x) (\frac{n-2}{k-1}) [(1-p)(1-M(x))]^{k-1}[p + (1-p)M(x)]^{n-k-1} dx.
$$

To understand that above formula, note that if we define $x$ as the k-th highest bid among investors 2, 3, ..., n, investor 1 will win one unit at price $x$ if $b > x$. When the firm is of type $G$, $\binom{n-1}{k-1} (1-p)m(x) (\frac{n-2}{k-1}) [(1-p)(1-M(x))]^{k-1}[p + (1-p)M(x)]^{n-k-1}$ is the pdf that the k-th highest bid among investors 2, 3, ..., n is $x$, and bidder 1’s payoff is $\frac{\theta}{k} - x$ if $x < b$ and 0 otherwise. When $\nu = 0$, $\binom{n-1}{k-1} (1-p)m(x) (\frac{n-2}{k-1}) [(1-p)(1-M(x))]^{k-1}[p + (1-p)M(x)]^{n-k-1}$ is the pdf that the k-th highest bid among investors 2, 3, ..., n is $x$, and bidder 1’s payoff is $-x$ if $x < b$ and 0 otherwise.

If investor 1 is willing to randomize in the range $(b, \theta)$, the payoff must be the same to him, otherwise he will choose to bid whichever gives him the highest payoff. This means $\partial \pi(b)/\partial b = 0$, which leads directly to Equation 3. Since $M(b) \to 0$ when $b \to 0$ and $M(b) = 1$ when $b = \frac{\theta}{k}$, we have

$$
b = 0 \text{ and } \theta = \frac{\alpha}{k}.
$$

(A.8)

Further note that $M(b)$ is strictly positive and strictly increasing over the interval $(0, \frac{\theta}{k})$, which qualifies $M(b)$ as a cdf.

As for part (ii), we will prove in Proposition 3 that the payoff to each information producer is at most $p(1-p)\theta(1-\theta)\frac{\alpha}{k}I(\theta_{\min})$, so as long as the information production cost $C$ is less than that, there will be at least $k+1$ participants in the IPO auction, so the equilibrium described in part (i) exists.

**Proof of Proposition 3**: Each participant’s expected payoff is

$$
E[\pi(n)] = Pr(s_i = H)E[\pi(n)|H] + Pr(s_i = M)E[\pi(n)|M] + Pr(s_i = L)E[\pi(n)|L].
$$

(A.9)

It is obvious that $E[\pi(n)|L] = 0$. The expected profit for a participant when he observes M is also 0. To see this, plug equation 3 into $\pi(b)$, we have $\pi(b) = 0$ for any $b$, which implies $E[\pi(n)|M] = 0$. The expected profit when bidder 1 observes H is

$$
E[\pi(n)|H] = \int_{0}^{\theta} \frac{\alpha}{k} - x \binom{n-1}{k-1} (1-p)m(x) (\frac{n-2}{k-1}) [p + (1-p)(1-M(x))]^{k-1}[1-p)M(x)]^{n-k-1} dx,
$$

(A.10)
where $x$ is the $k$th highest bid among the bids from bidder 2, 3, ..., and \((1^{n-1}) (1-p)m(x) (k-1)_{n-1} [p + (1-p)(1-M(x))]^{k-1}[(1-p)(1-M(x))]^{n-k-1}\) is the pdf of $x$ conditional on the firm is of type $G$ and $x < \frac{p}{k}$, $(\frac{p}{k} - x)$ is the payoff to bidder 1 in this case. So the expected profit of each participant is

\[
E[\pi(n)] = Pr(H)E[\pi(n)|H] + 0
\]

\[
= \theta p \int_0^1 \left( \frac{\alpha}{k} - x \right) (1^{-1}) (1-p)m(x) (k-1)_{n-1} [p + (1-p)(1-M(x))]^{k-1}[(1-p)(1-M(x))]^{n-k-1}dx.
\] (A.11)

From equation 3, we can express $x$ as a function of $M$,

\[
x = \frac{\theta[p + (1-p)(1-M)]^{k-1}[(1-p)(1-M)]^{n-k-1} + (1-\theta)[(1-p)(1-M)]^{k-1}[p + (1-p)M]^{n-k-1}}{\theta[p + (1-p)(1-M)]^{k-1}[(1-p)(1-M)]^{n-k-1} + (1-\theta)[(1-p)(1-M)]^{k-1}[p + (1-p)M]^{n-k-1}} \frac{\alpha}{k}.
\] (A.12)

If we change the integration on $x$ to integration on $M$, and rearrange, the equation A.11 becomes

\[
\pi(n) = p(1-p)\theta(1-\theta)\frac{\alpha}{k} I(n),
\] (A.13)

where

\[
I(n) = \int_0^1 \left( 1^{-1} \right) (n-2)_{k-1} [p + (1-p)(1-M)]^{k-1}[(1-p)(1-M)]^{n-k-1} + (1-\theta)[(1-p)(1-M)]^{k-1}[p + (1-p)M]^{n-k-1} dM.
\] (A.14)

We first prove one intermediate result: $I(n)$ is decreasing in $n$ when $n$ is large, and $I(n) \to 0$ as $n \to \infty$. The proof is as follows. We can rewrite $I(n)$ as

\[
I(n) = \int_0^1 J(M,n) \left( 1^{-1} \right) (n-2)_{k-1} (1-M)^{k-1}M^{n-k-1} dM,
\] (A.15)

where

\[
J(M,n) = \frac{[p + (1-p)(1-M)]^{k-1}(1-p)^{n-2}}{[\theta[p + (1-p)(1-M)]^{k-1}[(1-p)(1-M)]^{n-k-1} + (1-\theta)[(1-p)(1-M)]^{k-1}[p + (1-p)M]^{n-k-1}]},
\]

when $n \to \infty$, $[\frac{(1-p)M}{\theta[p + (1-p)(1-M)M]}^{n-k-1} \to 0$, and $(1-p)^{n-2} \to 0$, hence the numerator of $J(M,n)$ goes to 0 and the denominator goes to a constant $(1-\theta)[(1-p)(1-M)]^{k-1}$, so we have $J(M,n) \to 0$ as $n \to \infty$. Another thing we should note is that $\int_0^1 \left( 1^{-1} \right) (n-2)_{k-1} (1-M)^{k-1}M^{n-k-1} dM = 1$ for any $n$, so we can treat it as a density function, and will not explode when $n \to \infty$, so we have $I(n) \to 0$ hence $\pi(n) \to 0$ as $n \to \infty$.

When $n$ is large, $[\frac{(1-p)M}{\theta[p + (1-p)(1-M)M]}^{n-k-1}$ is very small, hence the value of the denominator of $J(M,n)$ is dominated by $(1-\theta)[(1-p)(1-M)]^{k-1}$. So if $n$ is large, the numerator of $J(M,n)$ is decreasing in $n$, while the denominator is almost constant, so $J(M,n)$ is decreasing in $n$. Further note that $\int_0^1 \left( 1^{-1} \right) (n-2)_{k-1} (1-M)^{k-1}M^{n-k-1} dM = 1$ for any $n$, when $n$ increases, it remains as a density function, only its shape change slightly, so we have $I(n)$ is decreasing in $n$ when $n$ is large.

We define $n_{min}$ such that for all $n \geq n_{min}$, $\pi(n)$ is decreasing in $n$, and $\pi(n)$ achieves maximum at $n_{min}$.

The payoff to each participant is decreasing in the total number of participants, so the number of participants is determined by the following equation

\[
\pi(n) = p(1-p)\theta(1-\theta)\frac{\alpha}{k} I(n) = C.
\] (A.16)

By implicit function theorem, we know that $\frac{\partial n}{\partial C} < 0$. When $C \to 0$, $\pi(n) \to 0$, hence $n \to \infty$. 

3
Proof of Proposition 4: When the issuing firm is of type G, with probability \(1 - \sum_{j=0}^{k} \binom{n}{j} p^{j}(1-p)^{n-j}\), there will be more than \(k\) participants observe \(H\) and bid \(\frac{n}{k}\). The clearing price would be \(\frac{n}{k}\). With probability \(\binom{n}{j} p^{j}(1-p)^{n-j}\), there will be \(j \in \{0,1,\ldots,k\}\) participants observe \(H\) and bid \(\frac{n}{k}\). The other \(n-j\) participants will observe \(M\) and will bid a withdrawal from the distribution \(\alpha g \{k+1\}\), so the offering price per share would be the \((k+1)\)th highest signal from \(n-j\) signals withdrawn from \(\alpha g \{k+1\}\). So the expected revenue to the type G firm in IPO market is given by equation 5. If we change the integration into an integration over \(\alpha\), we have:

\[
E[IR^{G}(n)] = \alpha - \alpha \int_{0}^{1} \frac{\sum_{j=0}^{k} \binom{n}{j} p^{j}(1-p)^{n-j}(1-\theta)[(1-p)(1-M)]^{k-1}[p+(1-p)M]^{n-k-1} \binom{n-j-1}{k-j} (1-M)^{k-j}M^{n-k-1} dM. 
\]

Use the fact that \(\int_{0}^{1} \binom{n}{j} \binom{n-j-1}{k-j} = \binom{k}{j} \binom{n}{k}\), we have

\[
E[IR^{G}(n)] = \alpha - \alpha \int_{0}^{1} \frac{\binom{n}{j} \binom{n-j-1}{k-j}}{\binom{k}{j} \binom{n}{k}} (1-\theta)[(1-p)(1-M)]^{k-1}[p+(1-p)M]^{n-k-1} + (1-\theta)[(1-p)(1-M)]^{k-1}[p+(1-p)M]^{n-k-1} dM.
\]

(A.17)

Similar to the proof that \(I(n)\) is decreasing in \(n\) and goes to 0 as \(n \to \infty\), we can easily show that \(E[IR^{G}(n)]\) is increasing in \(n\) for large \(n\), and goes to \(\alpha\) as \(n \to \infty\).

When the firm is of type B, if the \((k+1)\)th highest bid is 0, then the revenue is just 0. Suppose the \((k+1)\)th highest bid is \(x \in (0, \frac{n}{k})\), the pdf of \(x\) is \(\binom{k}{j} \binom{n-j-1}{k-j} [(1-p)(1-M)]^{k} [(1-p)(1-M)]^{n-k-1}\), and the expected revenue to the issuing firm is given by equation 6. This completes the proof of part (i).

From equation A.17, we can see

\[
E[IR^{G}(n)] < \alpha - \alpha \int_{0}^{1} \binom{n}{j} \binom{n-j-1}{k-j} (1-p)^{n} (1-M)^{k} M^{n-k-1} dM.
\]

(A.18)

Use the fact that \(\int_{0}^{1} \binom{n}{j} \binom{n-j-1}{k-j} (1-M)^{k} M^{n-k-1} dM = 1\), we have \(E[IR^{G}(n)] < \alpha [1 - (1-\theta)(1-p)^{n}]\). Note that is only the revenue from selling a fraction \(\alpha\) of the firm, so the value of the whole firm should be less than \([1 - (1-\theta)(1-p)^{n}]\), which is the expected secondary market price of the firm (in terms of a whole firm) when it is of type G.

Similarly, if we express equation 6 as an integration of \(M\), we have

\[
R^{B}_{\alpha}(n) = \alpha \int_{0}^{1} \frac{\theta(p+(1-p)(1-M)^{k-1}[1-p)(1-M)]^{n-k-1} \binom{n-j-1}{k-j} (1-p)^{k+1} (1-M)^{k} (p+(1-p)M)^{n-k-1} dM}{\theta(p+(1-p)(1-M)^{k-1}[1-p)(1-M)]^{n-k-1} + (1-\theta)(1-p)(1-M)^{k-1}[(1-p)(1-M)]^{n-k-1} dM}
\]

hence the value of the whole firm is higher than \(\theta(1-p)^{n}\), which is the expected secondary market price of the firm when it is of type B. This completes the proof of part (ii).

The total revenue to the type G firm is \(E[R^{G}_{\alpha}(n)] = E[IR^{G}(n)] + (1-\alpha)[1 - (1-\theta)(1-p)^{n}]\). We just proved that \(E[IR^{G}(n)]\) is increasing in \(n\) and goes to \(\alpha\) as \(n \to \infty\). \((1-\alpha)[1 - (1-\theta)(1-p)^{n}]\) is also increasing in \(n\) and goes to 1 as \(n \to \infty\), so \(E[R^{G}_{\alpha}(n)]\) is increasing in \(n\) and goes to 1 as \(n \to \infty\). It follows directly that for any \(R < 1\), there exist \(C\) such that for \(C < \alpha\), \(n\) is large enough such that \(E[R^{G}_{\alpha}(n)] > R\).

Proof of Proposition 5: First note that the value of the fraction of the firm offering in the IPO goes to two sources: part of it goes to the issuing firm, which is the revenue to the issuing firm in the IPO; part of it goes to the bidders, i.e.,

\[
\theta \alpha + (1-\theta) * 0 = \theta E[IR^{G}(n)] + (1-\theta)E[IR^{B}(n)] + n_{a} \pi(n_{a}).
\]

(A.19)
Note that in equilibrium we have \( \pi(n_\alpha) = C \), so the average offering price is given by

\[
\theta E[IR^G(n)] + (1 - \theta)E[IR^B(n)] = \theta \alpha - n_\alpha C. \tag{A.20}
\]

In Proposition 1 we have proved that the average secondary market price (of the whole firm) is \( \theta \), so a fraction \( \alpha \) of the firm will have an average price of \( \theta \alpha \) in the secondary market. Hence the underpricing is given by

\[
\theta \alpha - (\theta E[IR^G(n)] + (1 - \theta)E[IR^B(n)]) = n_\alpha C. \tag{A.21}
\]

The percentage underpricing is thereby \( \frac{n_\alpha C}{\theta \alpha - n_\alpha C} \).

**Proof of Proposition 6:** Suppose bidders 2, 3, ..., \( n \) use the specified bidding strategy. When bidder 1 observes L, he knows that the firm is a type B firm, the expected payoff from bidding is \( (0 - F)\sum_{j=k-1}^{n-1} \frac{(n-1)}{j} (1 - p)^j \frac{k}{j+1} < 0 \), so he will not bid. When bidder 1 observes H, the payoff from bidding is \( \left( \frac{\alpha}{k} - F \right) \frac{k}{n} > 0 \), so he will bid. When bidder 1 observes M, the payoff from bidding is \( \theta \left( \frac{\alpha}{k} - F \right) \frac{k}{n} (1 - \theta) F \frac{k}{n(1-p)} \sum_{i=k}^{n} \frac{(n)}{i} (1 - p)^i \frac{n}{x} > \theta \left( \frac{\alpha}{k} - F \right) \frac{k}{n} (1 - \theta) F \frac{k}{n(1-p)} \frac{n}{x} \), which is nonnegative if \( \frac{\alpha(1-p)}{1-P_x} > \frac{\alpha}{k} \), so it is optimal for bidder 1 to bid if he observes M. This means that it is also optimal for bidder 1 to play the specified strategy.

In Proposition 8, we proved that \( \frac{\theta \alpha/k - F}{F} \) is decreasing in \( \alpha \), which is equivalent to \( F/\alpha \) is increasing in \( \alpha \). Since what we need is \( F \leq \frac{\theta(1-p)}{\alpha} \frac{\alpha}{k} \frac{1}{F} \), or equivalently, \( F/\alpha \leq \frac{\theta(1-p)}{\alpha} \frac{1}{k} \), which can be satisfied for low value of \( \alpha \).

**Proof of Proposition 7:** When the offering price is set at \( F \), the expected payoff to each participant as a function of total number of participants, \( n \), is

\[
\pi(n, F) = \theta \left( \frac{\alpha}{k} - F \right) \frac{k}{n} (1 - \theta) F \sum_{j=k-1}^{n-1} \frac{(n-1)}{j} (1 - p)^j \frac{k}{j+1}. \tag{A.22}
\]

Since \( \sum_{j=k-1}^{n-1} \frac{(n-1)}{j} (1 - p)^j \frac{k}{j+1} = \frac{k}{n(1-p)} \sum_{i=k}^{n} (1 - p)^i \frac{n}{x} \), we have \( \pi(n, F) = (\frac{\alpha}{k} - F) \frac{k}{n} \).\(^{39}\)

We can see that the expected payoff to each participant is a decreasing function of total number of participants, and the equilibrium number of participants is determined by the following equation \( (\frac{\alpha}{k} - F) \frac{k}{n} = C \). By implicit function theorem we have

\[
\frac{\partial \pi}{\partial F} = - \frac{-k/n}{-\frac{\alpha}{k} - F} \frac{k}{n^2} = \frac{-n}{-\frac{\alpha}{k} - F} < 0, \tag{A.23}
\]

which completes the proof.

**Proof of Proposition 8:** The average secondary market price of the whole firm is \( \theta \), as we proved in Proposition 1. Since a fraction \( \alpha \) of the firm is sold in the IPO, and that fraction is divided into \( k \) shares, the average secondary market price of each share is \( \alpha \theta/k \). If the offering price of each share, \( F < \alpha \theta/k \), it is an underpricing equilibrium.

We proved in Proposition 6 that for \( \alpha \leq \pi \), it is optimal for the type G firm to set \( F \leq \frac{\theta(1-p)}{\alpha} \frac{\alpha}{k} \), which is an underpricing equilibrium.

In order to prove \( \frac{\alpha/d}{F} \) is decreasing in \( \alpha \), it is sufficient to prove \( \frac{F}{\alpha} \) is increasing in \( \alpha \). In the Lemma 7 we proved that the number of participants in the fixed-price offering is \( (\frac{\alpha}{k} - F) \frac{k}{n} = C \), or \( \theta \alpha = kF + nC \). The objective of the type G firm is to

\[
\max_F \quad kF + (1 - \alpha)[1 - (1 - \theta)(1 - p)^n] \tag{A.24}
\]

\[
s.t. \quad \theta \alpha = kF + nC
\]

\(^{39}\) We make the approximation since \( \sum_{i=k}^{n} C^i_n (1 - p)^i \) is very close to 1. For example, when \( k = 13 \), and \( n = 14 \) (we assume \( n \geq k + 1 \) in either auction or fixed-price offering), we found that \( \sum_{i=k}^{n} C^i_n (1 - p)^i \approx 0.9916 \). For \( n > 14 \), it is even closer to 1. When \( n = 15 \), \( \sum_{i=k}^{n} C^i_n (1 - p)^i \approx 0.9999842 \). For \( n = 16 \), \( \sum_{i=k}^{n} C^i_n (1 - p)^i \approx 0.9999835 \). We find error of similar magnitude for other values of \( k \).
Solve for the optimization problem, we find that

\[ F = \frac{\theta \alpha}{k} - C \ln\left\{ (1 - \theta)(1 - \alpha) \ln[1/(1 - p)] \right\} / \ln[1/(1 - p)] . \]  \hfill (A.25)

This means $F = \frac{1}{k} \ln\left\{ (1 - \theta)(1 - \alpha) \ln[1/(1 - p)] \right\} / \ln[1/(1 - p)]$, which is indeed increasing in $\alpha$.

**Proof of Proposition 9:** In section 3.2.1 and 3.2.2 we assume that the type G firm choose the IPO mechanism to maximize its total revenue, so part (i) is automatically satisfied. We have to show that it is indeed optimal for the type B firm to mimic the type G firm. From equation 6 we know that the revenue to the type B firm from IPO auction is positive. In a fixed-price offering, the revenue to the type B firm is $F$ when the IPO succeeds, and 0 when the IPO fails, so the expected revenue is also positive. In contrast, if the type B firm chooses not to mimic the type G firm, the market believes that it is a type B firm and the revenue would be 0. So it is optimal for the type B firm to mimic the type G firm.

Since in equilibrium, both types of the issuing firm pool together, the specified equilibrium beliefs are consistent with equilibrium strategies of all parties. Finally, the secondary market price is also consistent with equilibrium strategies and beliefs of all parties from the proof of Proposition 1.

**Proof of Proposition 10:** The total revenue to type G firm when fixed price offering is used is

\[ E[R^G_f(\alpha)] = kF + (1 - \alpha)[1 - (1 - \theta)(1 - p)^n] . \]  \hfill (A.26)

Since $F \leq \frac{\theta(1 - p)\alpha}{\theta(1 - p) + \alpha} < \theta\alpha/k$, and $1 - (1 - \theta)(1 - p)^n < 1$, we have $E[R^G_f(\alpha)] < 1 - (1 - \theta)\alpha$. From Proposition 4 we know that there exists a critical value $C'$ such that for all $C \leq C'$, $E[R^G_f(\alpha)] \geq 1 - (1 - \theta)\alpha$, hence $E[R^G_f(\alpha)] > E[R^G_f(\alpha)]$.

Define $C''$ as the maximum C such that there are at least $k+1$ information producers in the IPO auction for given $\alpha$: $C'' = p(1 - \theta)(1 - \theta)\frac{\pi}{k}I(n_{\min})$, then for $C > C''$, the issuing firm will choose fixed-price offering, since there will not be enough information producers if auction is used.

**Proof of Proposition 11:** We proved in Proposition 10 that for any $\alpha \leq \bar{\alpha}$, there exist $C'(\alpha)$ such that for all $C \leq C'(\alpha)$, auction dominates. For any $\alpha' < \bar{\alpha}$, we define $\bar{C} = \min\{C'(\alpha) : \alpha \in [\alpha', \bar{\alpha}]\}$, then as long as $C \leq \bar{C}$, auction dominates for $\alpha \in [\alpha', \bar{\alpha}]$. For $\alpha < \alpha^*\alpha$, there will be less than $k+1$ participants if auction is used, therefore the fixed-price offering is the equilibrium choice.

**Proof of Proposition 12:** The total revenue to type G firm when fixed price offering is used is

\[ E[R^G_f(\alpha)] = kF + (1 - \alpha)[1 - (1 - \theta)(1 - p)^n] . \]  \hfill (A.27)

Since $F \leq \frac{\theta(1 - p)\alpha}{\theta(1 - p) + \alpha} < \theta\alpha/k$, and $1 - (1 - \theta)(1 - p)^n < 1$, we have $E[R^G_f(\alpha)] < 1 - (1 - \theta)\alpha$. Since the issuing firm must sell at least $\alpha_{\min}$, i.e., $\alpha \geq \alpha_{\min}$, we have $E[R^G_f(\alpha \in [\alpha_{\min}, 1])] < 1 - (1 - \theta)\alpha_{\min}$. By Proposition 4 we know that there exists a critical value $\bar{C}(\alpha_{\min})$ such that for all $C \leq \bar{C}$, $E[R^G_f(\alpha_{\min})] \geq 1 - (1 - \theta)\alpha_{\min}$, hence $E[R^G_f(\alpha \in [\alpha_{\min}, 1])] \geq 1 - (1 - \theta)\alpha_{\min} > E[R^G_f(\alpha \in [\alpha_{\min}, 1])]$, i.e., the issuing firm will choose auction when $C$ is small, even when $\alpha$ is endogenous.

By definition of $C_{\max}$, when $C > C_{\max}$, auction is not feasible for any $\alpha$, so the issuing firm will choose fixed-price offering.

**Proof of Proposition 13:** The proof for parts (a) and (b) is similar to the proof in Lemma 1. So we only need to prove part (c). Suppose investors 2, 3, ..., $n$ will use the equilibrium bidding strategy described in the lemma. If investor 1 observes $M$, and if he submits a bid $b \in [r, 1]$, the expected payoff to him is

\[
\pi(b) = \theta \int_r^b \left( \frac{\alpha}{r} - x \right) \left( \frac{b - x}{r} \right)^{n-2} [p + (1 - p)(1 - M(x))]^{n-k-1} \left[ (1 - p)M(x) \right]^{n-k-1} dx
\]

\[
- (1 - \theta) \int_r^b x \left( \frac{b - x}{r} \right)^{n-2} \left[ (1 - p)(1 - M(x)) \right]^{k-1} [p + (1 - p)(1 - M(x))]^{n-k-1} dx.
\]

We know that if a player is willing to play mixed strategy, the payoff to him must be the same for any pure strategy with positive probability, which means $\pi'(b) = 0$ and $\pi(b) = 0$ for $b \in [r, \frac{r}{k}]$, these two conditions lead to the results we want directly.
Proof of Lemma 1: Part (i) of this lemma is similar to the proof of Proposition 3. Part (ii) follows directly from implicit function theorem. Note that the derivative of the left side of Equation 12 with respect to $r$ is positive.

Proof of Proposition 14: In the basic model, we proved that when the firm sells a small fraction of its equity in IPO, the firm will choose fixed-price offering because there will less than $k+1$ participants in auction. If the issuing firm has the choice to set a reservation price, this can decrease the number of information producers, but cannot increase the number of information producers since we proved that the revenue of each participant is decreasing in $r$. So the firm will still choose fixed-price offering even when the firm has the choice of setting a reservation price.

We also proved that for large values of $\alpha$, the firm chooses an IPO auction. When the firm has the choice to set a reservation price, the total revenue is no less than the case without reservation price, so the firm will still choose an IPO auction.

Proof of Proposition 15: We proved in the basic model that when the $C$ is large, the firm will choose fixed-price offering because there will less than $k+1$ information producers in auction. If the issuing firm has the choice to set a reservation price, this can decrease the number of information producers, as we just proved in the last proposition. So the firm will still choose fixed-price offering even when the firm has the choice of setting a reservation price for large values of $C$. We also proved that for small value of $C$, the firm chooses an IPO auction without reservation price. When the firm has the choice to set a reservation price, the total revenue is no less than the case without reservation price, so the firm will still choose an IPO auction.
## B Summary of Notation

<table>
<thead>
<tr>
<th>Category</th>
<th>Symbol</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Issing Firm</td>
<td>(j = G, B)</td>
<td>(G): Good firm, (B): Bad firm</td>
</tr>
<tr>
<td>IPO Method</td>
<td>(q = a, f)</td>
<td>(a): auction, (f): fixed-price offering</td>
</tr>
<tr>
<td>Signal</td>
<td>(S = H, M, L)</td>
<td>(H): High, (M): Middle, (L): Low</td>
</tr>
<tr>
<td>Price</td>
<td>(F)</td>
<td>Offering price for <em>each share</em> in fixed-price offering</td>
</tr>
<tr>
<td></td>
<td>(E[SP^j])</td>
<td>Expected secondary market price of the whole firm when the issuing firm is of type (j \in {G, B})</td>
</tr>
<tr>
<td></td>
<td>(E[SP])</td>
<td>Expected secondary market price across types</td>
</tr>
<tr>
<td>Revenue</td>
<td>(E[IR^j])</td>
<td>Expected revenue to type (j) firm from IPO auction</td>
</tr>
<tr>
<td></td>
<td>(E[R_q^j])</td>
<td>Expected total revenue (IPO+seasoned equity offering) to type (j) firm under mechanism (q \in {a, f})</td>
</tr>
<tr>
<td>Probability</td>
<td>(\theta)</td>
<td>Outsider’s prior probability that an issuing firm is of type (G)</td>
</tr>
<tr>
<td></td>
<td>(p)</td>
<td>Probability that a signal reveals true firm type</td>
</tr>
<tr>
<td></td>
<td>(M(b))</td>
<td>cdf from which investors who observe (M) draw a bid</td>
</tr>
<tr>
<td></td>
<td>(m(b))</td>
<td>pdf associated with (M(b))</td>
</tr>
<tr>
<td>Others</td>
<td>(C)</td>
<td>Cost for information producers to obtain a signal</td>
</tr>
<tr>
<td></td>
<td>(\alpha)</td>
<td>Fraction of equity sold in the IPO</td>
</tr>
<tr>
<td></td>
<td>(n)</td>
<td>Number of information producers in the IPO</td>
</tr>
<tr>
<td></td>
<td>(k)</td>
<td>Number of shares sold in the IPO</td>
</tr>
<tr>
<td></td>
<td>(r)</td>
<td>Reservation price in the IPO auction</td>
</tr>
<tr>
<td></td>
<td>(R_{fail}^f)</td>
<td>Revenue to the issuer from alternative sources if IPO fails</td>
</tr>
<tr>
<td></td>
<td>(\pi)</td>
<td>payoff to each investor if he produces information</td>
</tr>
</tbody>
</table>