Social Status, Non-Expected Utility, Asset Pricing, and Growth

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ABSTRACT

This paper investigates testable restrictions on the time-series behavior of consumption and asset returns implied by a representative agent model with the spirit of capitalism in which intertemporal preference is represented by a utility function that generalizes conventional, time-additive, expected utility. In the recursive structure of preference, we examine the implication for consumption, portfolio holdings, stock-market prices, and volatility of consumption and wealth when investors accumulate wealth not only for the sake of consumption but also for wealth-induced social status. When investors care about relative social status, the propensity to consume and risk-taking behavior will depend on social standards, and stock prices will be volatile. Hence, the spirit of capitalism seems to be a driving force behind stock-market volatility and economic growth. Because the elasticity of substitution and the coefficient of relative risk aversion are independent and the spirit of capitalism is introduced, the equity premium puzzle and the consumption smoothing puzzle can all be reassessed from the perspective of non-expected utility model with the social status.
1. Introduction

After Epstein and Zin (1989, 1991), Duffie and Epstein (1992), and Weil (1990), non-expected utility preferences often appear in asset pricing theory instead of conventional time-additive, expected utility. An attractive feature of this generalized specification is that intertemporal substitution and risk aversion can be partially disentangled, in contrast to the conventional case of an additive and homogeneous Von Neumann-Morgenstern intertemporal utility function, in which the elasticity of substitution and the coefficient of relative risk aversion are constrained to be reciprocals of one another. When applied to the consumption/portfolio choice problem of an infinitely lived, representative agent, this recursive structure of preference yields testable restrictions on observable variables, namely, real, per capita, consumption growth rates and real asset returns.

But for most asset pricing theories with non-expected preferences, wealth is no more valuable than its implied consumption rewards, see, for example, Epstein and Zin (1989, 1991), Svensson (1989), Weil (1990), Obstfeld (1994a, 1994b), in their models, wealth is clearly no more valuable than the maximum amount of consumption utility that it can bring. Because consumption rewards are the only thing that matter, everything has to be valued according to its relation with consumption. Thus, for instance, the equilibrium price of an asset is completely determined by its consumption beta. While the aforementioned motive is important—perhaps the most important—motive for wealth accumulation, it is, however, not the only important motive behind the sometimes relentless acquisition of wealth. In reality investors acquire wealth not just for its implied consumption, but for the resulting social status. Weber (1958) refers to this desire for wealth as the social status.

Building on work by Bakshi and Chen (1996) and Epstein and Zin (1989, 1991), in the recursive structure of preferences, we examine in this paper the implications for consumption, portfolio holdings and stock-market prices of the hypothesis that investors accumulate wealth not only for the sake of consumption but also for wealth-induced social status. Economies populated with status-conscious investor exhibit characteristics distinct from those with the standard agents. To mention a few examples, optimal consumption-portfolio plans will be functions of not only one’s own wealth and preference parameters but also social-wealth standard. Under the parameterized-preference model in this paper, the optimal propensity to consume is increasing in both one’s relative social standing and own wealth but decreasing in: (1) social-wealth standards, (2) the degree to which the investor cares about status, and (3) the investor’s aversion to poverty. In addition, the investor is more averse to wealth risk (1) the more he cares about status, (2) the higher the social-wealth standards, or (3) the lower the investor’s social standings. In such economies, even if the consumption process is smooth, stock prices can be quite volatile. The spirit of capitalism is a driving force behind stock-market volatility.

Because the elasticity of substitution and coefficient of relative risk aversion are independent and the social status is introduced, the model we described
in this paper can be used to address numerous outstanding empirical issues in macroeconomics and finance. For example, the equity premium puzzle of Mehra and Prescott (1985), the consumption smoothing puzzle of Hansen and Singleton (1983) can all be reassessed from the perspective of non-expected utility model with the social status.

We derive explicitly the optimal rules of consumption, asset holdings, wealth accumulation with both social status and non-expected utility in this paper. When we compare our model to the traditional one, we can see that our ratio of the volatility of consumption to the volatility of wealth is generally smaller than the base case. In addition, we derive two asset pricing formula and solve partially the risk premium puzzle.

This paper is organized as follows: In section 2, we introduce the preference structure and then define the investor’s consumption-portfolio problem. Section 3 studies closed-form solution to the consumption-portfolio problem, and then analyzes the effects of the parameters on the consumption and portfolio. Section 4 discusses the optimal wealth growth. In section 5, we studies the consumption volatility and wealth volatility. Section 6 analyzes the risk premium. Section 7 gives two asset pricing formula. Final section offers concluding remarks.

2. Utility Function and Asset Return

2.1. Utility function

We consider an infinitely lived representative agent who receives utility from the consumption of a single good and wealth-induced social status. The economic decision interval has length $h$. At time $t$ the representative agent maximizes the intertemporal objective $U(t)$ defined by the recursion

$$
\begin{align*}
\frac{1}{1 - \delta} U(C(t), W(t), V(t)) &= \frac{1}{1 - \delta} C(t)^{1 - \delta} \left( \frac{W(t)}{V(t)} \right)^{-\lambda} h \\
&+ e^{-\delta h} f \left( [1 - R] E_t [U(C(t + h), W(t + h), V(t + h))] \right)
\end{align*}
$$

where the function $f(x)$ is given by

$$
f(x) = \frac{1 - R}{1 - \delta} x^{1 + \frac{1}{\lambda}}
$$

The recursive form in (1) is an analogy to the form in Obstfeld (1994b). $E_t$ is the mathematical expectation conditional on time $t$ information, $C(t)$ is time $t$ consumption, and $\delta (> 0)$ is the subjective rate of time preference. $V(t)$ is what determines "‘middle class’" within the investor’s reference group. We refer to $V(t)$ as the social-wealth index. It should be emphasized that for different consumers, their wealth references, $V(t)$, can be quite different, depending on the social or professional groups to which they compare themselves. The higher the incomes of the members in the reference group, the higher $V(t)$. The parameter $R(> 0)$ in (1) and (2) measures the agents relative risk aversion, and parameter $\varepsilon (> 0)$ is the intertemporal substitution elasticity. A higher $\varepsilon$
implies that, other things equal, the marginal utility of consumption is falling more slowly when consumption grows (see, Obstfeld (1994a)). When \( R = \frac{1}{\epsilon} \), so that \( f(x) = x \), this is the standard state-and time-separable expected-utility setup, which does not allow independent variation in risk aversion and consumption substitutability over time. The more general preference setup assumed in (1) is proposed by Epstein and Zin (1989, 1991), and by Weil (1990). Roughly speaking, intertemporal substitution is encoded in aggregator function, while the certainty equivalent function reflects the degree of risk aversion. There are two main reasons for considering such preference. First, dynamic comparisons of asset returns that confound risk aversion and intertemporal substitutability can be misleading. Second, one would like to answer the positive question of how preferences parameter influence growth.

In (1), \( W(t) \) is time-\( t \) wealth, the investor accumulates wealth not only for consumption but also for the sake of wealth-induced social status, the ratio of one’s own wealth to the social-wealth index, \( \frac{W(t)}{V(t)} \), determinies status, see Zou (1992, 1994), Bakshi and Chen (1996) for details. \(|\lambda|\) measures the extent to which the investor cares about status. We call it the social status, \( \lambda \geq 0 \) when \( \epsilon < 1 \) and \( \lambda < 0 \) otherwise. This specification is consistent with those in Kurz (1968), Chen (1990), Bakshi and Chen (1996), Carroll (2000), and Zou (1992, 1994) as well as with those in Weber (1958) and Keynes (1971).

### 2.2 The consumption-portfolio problem

To introduce the investor’s consumption portfolio problem, we assume that traded in this frictionless economy is one risk-free asset, with its constant rate of return given by \( r_0 \), and one risky asset with its price at time \( t \) denoted by \( P_t(t) \), and \( t \in [0, \infty) \). The asset price \( P_t \) follows a diffusion process:

\[
\frac{dP(t)}{P(t)} = \mu(t) dt + \sigma(t) dz(t)
\]

where \( \mu(t) \) and \( \sigma(t) \) are, respectively, the conditional expected value and standard deviation of the rate of return on risky asset per unit time, and \( z(t) \) is a standard Wiener process. The variables \( \mu(t) \) and \( \sigma(t) \), generally depend on the time \( t \) state of the economy. The social-wealth index follows a diffusion process:

\[
\frac{dV(t)}{V(t)} = \mu_V(t) dt + \sigma_V(t) dz_V(t)
\]

where \( \mu_V(t) \) and \( \sigma_V(t) \) generally depend on the time \( t \) state of the economy, and \( z_V(t) \) is a standard Wiener process. A justification for this assumption is that when asset prices and optimal consumption follow diffusion processes, the resulting social-index should be expected to follow a diffusion as well. Let \( \sigma_{1V} \) be the covariance between the return on the risky stock and the growth rate of \( V(t) \).

Consumption-portfolio rebalancing by the investor takes place at time \( t \). Let \( \alpha(t) \) be the fraction of time \( t \) savings invested in the risky asset, and \( 1 - \alpha(t) \)
the fraction of time $t$ savings invested in the riskless asset. The infinitely-lived capitalistic investor then chooses a plan, $(C(t), \alpha(t))$, so as to

$$
\max_{C(t), \alpha(t)} U(C(t), W(t), V(t))
$$

subject to the budget constraint

$$
dW(t) = \{W(t)[r_0 + \alpha(t)(\mu(t) - r_0)] - C(t)\} \, dt + \alpha(t)W(t)\sigma(t)dz(t)
$$

and (4).

3. Consumption, Savings, and Portfolio choice

This section uses a parameterized preference to study optimal consumption, savings, and portfolio rules in detail. To maintain simplicity, we assume that the price of the risky asset and the social-wealth index follows geometric Brownian motions, that is, the coefficients in (3) and (4) are constants: $\mu(t) = \mu$, $\sigma(t) = \sigma$, $\mu_V(t) = \mu_V$, $\sigma_V(t) = \sigma_V$.

Epstein and Zin (1989, 1991), and Weil (1990) assume that time is discrete in their expositions of nonexpected utility preferences. But continuous-time extensions by Svensson (1989), and Duffie and Epstein (1992) provide formulations that are readily applied to the problem of maximizing the continuous-time limit of $U(C(t), W(t), V(t))$ in (1) subject to (4), (6) and an initial wealth endowment $W(t) \equiv W_t$ and the social-wealth index $V(t) \equiv V_t$. Let $J(W_t, V_t)$ denote the maximum feasible level of lifetime utility when wealth at time $t$ equals $W_t$ and the social-wealth index $V_t$.

As in Svensson (1989), the indirect utility function $J(W_t, V_t)$ satisfies the following recursive equation:

$$
f([1-R]J(W_t, V_t)) = \lim_{h \to 0^+} \max_{C(t), \alpha(t)} \frac{1 - R}{1 - \frac{\mu}{\sigma^2}} C(t)^{1 - \frac{\mu}{\sigma^2}} \left(\frac{W_t}{V_t}\right)^{-\lambda} \left(\frac{W_t}{V_t}\right)^{-\lambda} - e^{-\delta h} f \left(\left[1 - R\right]E_t \left[J(W_{t+h}, V_{t+h})\right]\right)
$$

It's lemma shows that in continuous time the stochastic Bellman equation resulting from maximizing $U(C(t), W(t), V(t))$ in (1) is given by the following equation:

$$
0 = \max_{C(t), \alpha(t)} \frac{1 - R}{1 - \frac{\mu}{\sigma^2}} C(t)^{1 - \frac{\mu}{\sigma^2}} \left(\frac{W_t}{V_t}\right)^{-\lambda} - \delta f \left(\left[1 - R\right]J(W_t, V_t)\right)
$$

$$
+(1 - R)f' \left(\left[1 - R\right]J(W_t, V_t)\right) \left[J_W W_t \{r_0 + \alpha(t)(\mu(t) - r_0)\} - C(t)\right] + J_V V_t \mu_V + \frac{1}{2} J_W W_t \sigma_V^2 + \frac{1}{2} J_V V_t \sigma_V^2
$$

$$
+ J_W V_t \sigma_W \alpha(t) W_t V_t
$$

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where $J_W$, $J_V$, $J_{WW}$, $J_{VV}$, $J_{WV}$ denote the first and second partial differential of $J$ with respect to $W$, $V$, respectively.

From (8), the first-order conditions with respect to $\alpha(t)$ and $C(t)$ follow:

$$C(t) \frac{1}{\lambda} \left( \frac{W_t}{V_t} \right)^{-\lambda} = f' (\cdot) J_W$$  \hspace{1cm} (9)

$$J_W (\mu - r_0) + J_{WW} \alpha (t) \sigma^2 W_t + J_{WV} \sigma_1 V_t = 0$$ \hspace{1cm} (10)

Equation (8)'s form suggests a guess that the indirect utility function is given by

$$J(W, V) = AW \frac{(1-\varepsilon)(\lambda \varepsilon + 1 - \varepsilon)}{\sigma^2 (R \lambda \varepsilon + R - \lambda \varepsilon - R \varepsilon)} V \frac{\lambda \varepsilon (R - 1)}{\lambda \varepsilon + 1 - \varepsilon}$$

Given this functional form for $J(W, V)$, (9) and (10) can be simplified. Equation (10) implies that demand for the risky asset is a constant fraction of wealth:

$$\alpha (t) = \frac{(\mu - r_0)(1-\varepsilon) + \lambda \varepsilon (R - 1) \sigma_1 V}{\sigma^2 (R \lambda \varepsilon + R - \lambda \varepsilon - R \varepsilon)}$$ \hspace{1cm} (11)

Equation (9) becomes:

$$C(t) = A \frac{1}{\lambda} \left( \frac{1}{1-\varepsilon} \right) \left( \frac{\lambda \varepsilon + 1 - \varepsilon}{1-\varepsilon} \right)^{-\varepsilon} W_t$$ \hspace{1cm} (12)

The substitution of (11) and (1) into (8) yields:

$$A \frac{1}{\lambda} \left( \frac{1}{1-\varepsilon} \right) \left( \frac{\lambda \varepsilon + 1 - \varepsilon}{1-\varepsilon} \right)^{-\varepsilon} = \frac{\varepsilon \delta + B}{(1-\varepsilon) \frac{1}{\lambda} \left( \frac{\lambda \varepsilon + 1 - \varepsilon}{1-\varepsilon} \right)^{-\varepsilon}}$$ \hspace{1cm} (13)

where

$$B = \frac{(\lambda \varepsilon + 1 - \varepsilon) r_0 - \lambda \varepsilon \mu V - \lambda \varepsilon (\lambda \varepsilon R - \lambda \varepsilon - 1 + \varepsilon) \sigma_V^2}{2(1-\varepsilon)}$$

Proposition: Let the utility be as given in (1). Then, the optimal solution to the consumption-portfolio problem is:

$$\alpha^* (t) = \frac{(\mu - r_0)(1-\varepsilon)}{\sigma^2 (R \lambda \varepsilon + R - \lambda \varepsilon - R \varepsilon)} + \frac{\lambda \varepsilon (R - 1) \sigma_1 V}{\sigma^2 (R \lambda \varepsilon + R - \lambda \varepsilon - R \varepsilon)}$$ \hspace{1cm} (11')
$C^* (t) = A^{\frac{1-\varepsilon}{\lambda \varepsilon + (1-\varepsilon)}} \left( \frac{\lambda \varepsilon + 1 - \varepsilon}{1 - \varepsilon} \right)^{-\varepsilon} W_t \Delta \eta W_t$ \hspace{1cm} (12')

$J (W, V) = AW^{\left(1-\frac{R(1+\lambda \varepsilon - \varepsilon)}{\lambda \varepsilon + (1-\varepsilon)}-\frac{R}{\lambda \varepsilon + (1-\varepsilon)}\right)} \left(\lambda \varepsilon + (1-\varepsilon)\right)$ \hspace{1cm} (14)

where $A$ is given in (13).

Given the utility of wealth in (14), the relative risk aversion in wealth is simply \( RRA = \frac{\mu - r_0}{\sigma^2} \). When $\lambda = 0$, RRA reduces to R, as obtained in Obstfeld (1994b), and $\alpha^* (t)$ reduces to $\mu - r_0 R \sigma^2$. Furthermore, when $R = \frac{1}{\varepsilon}$ and $\lambda = 0$, (12)' reduces to the formula derived by Merton (1971) in the expected-utility base case, which serves as a convenient benchmark:

$C (t) = \frac{1}{R} \left\{ \delta + (R - 1) r_0 + \frac{(\mu - r_0)^2 (R - 1)}{2R \sigma^2} \right\} \left(\lambda \varepsilon + (1-\varepsilon)\right)$ \hspace{1cm} (15)

We can tell more from (9)-(14) than from standard utility function.

When making consumption-portfolio decisions, investors will have to take into account what happens to the social-wealth index so that their relative status will not suddenly sink below a certain level. The first term in (11'),

$$\frac{(\mu - r_0)(1-\varepsilon)}{(\lambda \varepsilon + (1-\varepsilon))} \frac{1}{\sigma^2} \text{, is dictated by the investor's aversion to wealth risk and the independence between risk aversion and consumption substitutability. In particular, since caring about status makes the investor more risk averse, he will hold less of the risky asset than someone who does not care about status(\(\lambda = 0\))(compare to eq.(11) in Obstfeld(1994b)), furthermore, if $\varepsilon R > 1$, the effect when risk aversion and consumption substitutability change independently is much more than when risk aversion and consumption substitutability have reciprocal relationship, and if $\varepsilon R < 1$, the case is exactly converse(compare to eq.(23) in Baski and Chen(1996)).

The second term in (11'),

$$\frac{\lambda \varepsilon (R - 1) \sigma_{1V}}{\sigma^2 (\lambda \varepsilon + (1-\varepsilon))} = \frac{\lambda \varepsilon (R - 1)}{\sigma^2 (\lambda \varepsilon + (1-\varepsilon))} \frac{1}{\lambda \varepsilon + (1-\varepsilon)}$$

deserves more comments. This part of the optimal holding depends critically on how the the risky asset is correlated with the social-wealth index $V_t$. (i) Suppose $\sigma_{1V} > 0$, that is, the risky asset is positively correlated with the index $V_t$. Then, we can derive the analogous result with that in Baski and Chen(1996), that is, adding this risky asset to the portfolio will increase the correlation between $W_t$ and $V_t$, which serves to insure against future uncertain declines in status that can result from rises in social-wealth standards. Consequently, the second term in (11') is positive and increasing in $\sigma_{1V}$, and the investor puts a higher proportion into the risky asset than dictated by risk aversion alone. (ii) Suppose $\sigma_{1V} = 0$, that is, the risky asset is uncorrelated with $V_t$. Then, the risky asset is of no status-insurance value. As a result, the second term is zero and the investor’s holding is complete dictated by the investor’s aversion to wealth risk.

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and the independence of risk aversion and consumption substitutability. (iii) Finally, suppose $\sigma_{1V} < 0$. In this case, holding too much of the risky asset will only work toward reducing the investor’s status some further when $V_t$ rises. To avoid such a double penalty, the investor will hold less of the risky asset than determined by risk aversion. From the above analysis, we find that the spirit of capitalism seems to be a driving force behind stock-market volatility.

From (9)-(14), we obtain the following propositions:

(Insert figure 1a and figure 1b about here)

Fig.1a plots $\alpha^*_t$ as a function of $R$ for a specific set of parameter values. The demand for the risky asset is an increasing function of the relative risk aversion. The correlation between the risky asset and the index is large, then, adding this risky asset to the portfolio will increase the correlation between $W_t$ and $V_t$, furthermore, the increased correlation is so large that the effect dominates the effect of the investor’s risk aversion, so the more the risk aversion is, the more the demand for the risky asset.

Fig.1b plots $\alpha^*_t$ as a function of $R$ for a different covariance $\sigma_{1V}$ between the return on the risky asset and the growth rate of $V(t)$. The demand for the risky asset is a decreasing function of the relative risk aversion. The correlation between the risky asset and the index is small, then, as long as adding this risky asset to the portfolio will increase the correlation between $W_t$ and $V_t$, but the increased correlation is so small that the effect is dominated by the effect of the investor’s risk aversion, so the more the risk aversion is, the less the demand for the risky asset.

(Insert figure 2a and figure 2b about here)

Fig. 2a plots $\alpha^*_t$ as a function of $\varepsilon$ for a specific set of parameter values. The demand for the risky asset is the increasing function of the intertemporal substitution elasticity. The risky asset is positively correlated with the index $V_t$. The more $\varepsilon$ is, the more slowly the marginal utility is declining over time when consumption grows, the more the benefit from the future consumption is, the less the investor consumes, and the more the investor invests now. Because the risky asset is positively correlated with the index $V_t$, adding this risky asset to the portfolio will increase the correlation between $W_t$ and $V_t$, which serves to insure against future uncertain declines in status that can result from rises in social-wealth standards, so the larger the substitution elasticity, the more the investor invests in the risky asset.

Fig. 2b plots $\alpha^*_t$ as a function of $\varepsilon$ for a negative covariance $\sigma_{1V}$ between the return on the risky asset and the growth rate of $V(t)$. The demand for risky asset is the decreasing function of the intertemporal substitution elasticity. The risky asset is negatively correlated with the index $V_t$. The more $\varepsilon$ is, the more slowly the marginal utility is declining over time when consumption grows, the more the benefit from the future consumption is, the less the investor consumes,
and the more the investor invests now. Because the risky asset is negatively correlated with the index $V_t$, holding too much of the risky asset will only work toward reducing the investor's status some further when $V_t$ rises. To avoid such a "double penalty", the more $\varepsilon$ is, the investor will hold less of the risky asset and hold more the riskless security.

(Insert figure 3a and figure 3b about here)

Fig. 3a plots $\alpha^*_t$ as a function of $\lambda$ for a specific set of parameter values. The demand for the risky asset is the increasing function of the social status. Because the risky asset is positively correlated with the index $V_t$, adding this risky asset to the portfolio will increase the correlation between $W_t$ and $V_t$, which serves to insure against future uncertain declines in status that can result from rises in social-wealth standards, so the more the investor cares about the social status, the more the investor invests in the risky stock.

Fig. 3b plots $\alpha^*_t$ as a function of $\lambda$ for a negative covariance $\sigma_{1V}$ between the return on the risky asset and the growth rate of $V(t)$. The demand for the risky asset is the decreasing function of the social status. Because the risky asset is negatively correlated with the index $V_t$, holding too much of the risky asset will only work toward reducing the investor's status some further when $V_t$ rises. To avoid such a "double penalty", the more the investor cares about the social status, the investor will hold less of the risky asset and hold more the riskless security.

(Insert figure 4a and figure 4b about here)

Fig. 4a plots $C^*_t$ as a function of $R$ for a specific set of parameter values. The consumption is the increasing function of the relative risk aversion. Because the risky asset is positively correlated with the index $V_t$, adding this risky asset to the portfolio will increase the correlation between $W_t$ and $V_t$, which serves to insure against future uncertain declines in status that can result from rises in social-wealth standards, furthermore, the correlation between the risky asset and the index is so large that the investor can make sure that his relative status will not suddenly sink below a certain level, if only he invests certain amount wealth in the risky asset, and then he can consume more.

Fig. 4b plots $C^*_t$ as a function of $R$ for a different set of parameter values, in which the risk of the risky asset and the social status is smaller than that in Fig. 4a. The consumption is the decreasing function of the relative risk aversion. Because the risky asset is positively correlated with the index $V_t$, adding this stock to the portfolio will increase the correlation between $W_t$ and $V_t$, which serves to insure against future uncertain declines in status that can result from
rises in social-wealth standards, so the larger the investor is risk aversion, the less the investor consumes, and the more he invests.

(Insert figure 5a and figure 5b about here)

Fig. 5a plots $C^*_t$ as a function of $\varepsilon$ for a specific set of parameter values. The consumption is the decreasing function of the intertemporal substitution elasticity. The risky asset is positively correlated with the index $V_t$. The more $\varepsilon$ is, the more slowly the marginal utility is declining over time when consumption grows, the more the benefit from the future consumption is, the less the investor consumes, and the more the investor invests now.

Fig. 5b plots $C^*_t$ as a function of $\varepsilon$ for a different set of parameter values. When the intertemporal substitution elasticity is very small, the consumption is the decreasing function of it. But when the intertemporal substitution elasticity is larger and larger, consumption is the increasing function of it. When $\varepsilon$ is very small, the more $\varepsilon$ is, the more the benefit from the future consumption is, the less the investor consumes, and the more the investor invests now. When $\varepsilon$ excesses some point, because the risky stock is positively correlated with the index $V_t$, adding this risky asset to the portfolio will increase the correlation between $W_t$ and $V_t$, which serves to insure against future uncertain declines in status that can result from rises in social-wealth standards, furthermore, because $\varepsilon$ is so large that the investor can feel satisfied from the future if only he invests certain amount wealth in the risky asset, and then he can consume more.

(Insert figure 6 about here)

4. The optimal wealth growth

Combining (12') and (6), the optimal-wealth and consumption-growth dynamics are given below:

\[
\begin{align*}
\frac{dW(t)}{W(t)} &= \frac{dC^*_t(t)}{C^*_t(t)} = \mu_W \, dt + \frac{(\mu - r_0)(1 - \varepsilon) + \lambda \varepsilon (R - 1) \sigma_1 V}{\sigma (R \lambda \varepsilon + R - \lambda \varepsilon - \lambda \varepsilon)} \, dz(t) \quad (16) \\
&= \mu_W \, dt + \sigma_W \, dz(t)
\end{align*}
\]

where

\[
\mu_W(t) = r_0 + \frac{(\mu - r_0)^2(1 - \varepsilon) + \lambda \varepsilon (R - 1) \sigma_1 V (\mu - r_0)}{\sigma^2 (R \lambda \varepsilon + R - \lambda \varepsilon - \lambda \varepsilon)} - A \frac{1 - \varepsilon}{1 - \varepsilon} \left( \frac{\lambda \varepsilon + 1 - \varepsilon}{1 - \varepsilon} \right)^{-\varepsilon}
\]

The impact of an increasing in $\lambda$ and $R$ on $\mu_W$ is clouded by two opposite effects: the portfolio effect and the saving effect. On the one hand, when the investor cares more about status, his risk aversion in wealth RRA will increase,
which means holding less of the risky asset and a lower $\alpha_t^*$. Consequently, the increased risk aversion asserts a negative effect on wealth growth. On the other hand, an increasing in $\lambda$ and $R$ induces the investor to consume less and raise the savings rate. Consequently, the increased risk aversion asserts a positive effect on wealth growth. So, the net effect of an increasing in $\lambda$ and $R$ on $\mu_W$ will depend on the stronger one between the portfolio effect and the saving effect.

(Insert figure 7 about here)

Fig. 7 plots $\mu_W$ as a function of $R$ for a specific set of parameter values. The growth of wealth is the increasing function of the relative risk aversion. The risky asset is positively correlated with the index $V_t$. Because the risky asset is positively correlated with the index $V_t$, adding this risky asset to the portfolio will increase the correlation between $W_t$ and $V_t$, which serves to insure against future uncertain declines in status that can result from rises in social-wealth standards, furthermore, the increased correlation is so large that the effect dominates the effect of the investor’s risk aversion, and the saving effect dominates the portfolio effect, so the more the risk aversion is, the more the demand for the risky asset, the more growth the economy has.

(Insert figure 8a and figure 8b about here)

(Insert figure 9a, figure 9b, figure 9c and figure 9d about here)

Fig. 9a plots $\mu_W$ as a function of $\lambda$ for a specific set of parameter values. The growth of wealth is the increasing function of the social status. The risky asset is positively correlated with the index $V_t$. Because the risky stock is positively correlated with the index $V_t$, adding this stock to the portfolio will increase the correlation between $W_t$ and $V_t$, which serves to insure against future uncertain declines in status that can result from rises in social-wealth standards, and the saving effect dominates the portfolio effect, so the more the investor cares about the social status, the more the investor invests on the risky asset, and then the more growth the economy has.

Fig. 9b plots $\mu_W$ as a function of $\lambda$ for a negative covariance $\sigma_{1V}$ between the return on the risky asset and the growth rate of $V(t)$. The growth of wealth is the convex function of the social status. The risky asset is negatively correlated with the index $V_t$. Because the risky asset is negatively correlated with the index $V_t$, holding too much of the risky asset will only work toward reducing the investor’s status some further when $V_t$ rises. To avoid such a “double penalty”, the more the investor cares about the social status, the less the investor invests on the risky asset, and then the less growth the economy has. In this case, the portfolio effect dominates the saving effect.
From the above analysis, we find that the spirit of capitalism seems to be a driving force behind economic growth.

5. Consumption volatility and wealth volatility

From (12'), the ratio of the variability of consumption to the variability of wealth is

\[
\frac{\sigma^2(dC^*)}{\sigma^2(dW)} = \eta^2
\]

To sharply contrast these results with the base case, it is useful to restate the corresponding results for the base case. Note from equation (15) that the base model implies the following relationship between the volatility of changes in consumption and the volatility of changes in wealth:

\[
\frac{\sigma^2(dC)}{\sigma^2(dW)} = \theta^2
\]

Since Hall (1978,1988), Hansen and Singleton (1983), Deaton (1987) find consumption surprisingly smooth relative to income and prices, this outstanding ""consumption smoothing puzzle"" have attracted more and more theoretical and empirical study, such as liquidity constraints [Zeldes (1989)], habit formation [Constantinides (1988) Sundaresan (1989)], non-expected utility [Epstein and Zin (1989, 1991)], and mean reversion [Black (1990)], etc.. In our model, we will discuss the combined effect of the non-expected utility and the social status. It will be shown that our model has many important implications for consumption, savings, and portfolio choice behavior. In such economies, even if the consumption process is smooth, stock prices can be quite volatile. The spirit of capitalism is a driving force behind stock-market volatility.

In any period t for the non-expected preference, current consumption \( C_t \) is deterministic but future consumption is uncertain. There are two key features underlying the specification of intertemporal utility. First, the agent forms a certainty equivalent of random future utility using his risk preferences. Second, to obtain current-period lifetime utility, this certainty equivalent is combined with deterministic current consumption via an aggregator function. It is shown in Epstein and Zin (1989) that this class of preference allows a separation of risk aversion from substitution that is not possible in the expected utility framework. This separation leads the richness of explanation about the behavior of asset price and the volatility of consumption.
Furthermore, in our model, the optimal propensity to consume is increasing in both one’s relative social standing and own wealth but decreasing in: (1) social-wealth standards, (2) the degree to which the investor cares about status, and (3) the investor’s aversion to poverty. In addition, the investor is more averse to wealth risk (1) the more he cares about status, (2) the higher the social-wealth standards, or (3) the lower the investor’s social standings. So an increasing in consumption in response to an increase in wealth has two effects: First, it increases utility. Second, it decreases the wealth and then decreases the relative social standing, this also decreases the utility. Thus the response to an increase in wealth is more moderate increase in consumption. By the same token, the response to a decrease in wealth will also be shown to be more moderate in our framework. Thus, the marginal propensity to consume is generally lower in the class of models in which the investors care about the social status. This implies that any “shock” in the system must have a relatively greater impact on the dynamics of wealth than it would have in a model without concerning the social status. The ratio of the variability of changes in consumption to the variability of changes in wealth is shown to be strictly smaller in our model than in the time-separable utility case. The simulations presented confirm this result.

In order to analyze the effect of social status on the volatility of consumption and the volatility of the wealth, we select $\pi^2$ as the base case. Figure 13a is the relationship between $\pi^2$ and the relative risk aversion coefficient $R$.

(In Insert figure 13a to figure 13k about here)

In Figures 13a-13k, except Figures 13d, 13f, 13g, and 13j, our ratio of volatility is generally smaller than the base case. Note in the Figures 13d, 13f, 13g, and 13j, the correlation between the risky asset and the index is very large, or the intertemporal substitution elasticity is large. Because the correlation between the risky stock and the index is so large that the investor only need add very little the risky asset to the portfolio in order to insure against future uncertain declines in status that can result from rises in social-wealth standards, so the investor can consume more than the $\sigma_{1V}$ is little when the wealth increase.

6. Risk premium
In order to research the risk premium, we consider the partial equilibrium, in which the risk asset demand of representative agent is 1, $\alpha_t^* = 1$. From (11'), we have

$$1 = \alpha^* (t) = \frac{(\mu - r_0)(1 - \varepsilon)}{\sigma^2 (R\lambda \varepsilon + R - \lambda \varepsilon - R\varepsilon)} + \frac{\lambda \varepsilon (R - 1) \sigma_{1V}}{\sigma^2 (R\lambda \varepsilon + R - \lambda \varepsilon - R\varepsilon)}$$

that is,

$$\mu - r_0 = \frac{\sigma^2 (R\lambda \varepsilon + R - \lambda \varepsilon - R\varepsilon) - \lambda \varepsilon (R - 1) \sigma_{1V}}{1 - \varepsilon}$$

(17)
Now, we discuss the effects of the parameters on the risk premium, $\mu - r_0$.

For the United States during the period 1890-1979, the difference between the average return on the stock market and the return on short-term government debt—the equity premium—is about six percentage points. Thus if we take the average return on short-term government debt as an approximation to the average risk-free rate, the quantity $\mu - r_0$ is about 0.06. In a famous paper, Mehra and Prescott (1985) show that it is difficult to reconcile this observed equity premium $\mu - r_0$ with the theoretical value in the conventional models. They show that the coefficient of relative risk aversion needed to account for the equity premium is 25. This is an extraordinary level of risk aversion. Bakshi and Chen (1996) suggest that the relative risk aversion is in the range 3.04-4.24. We will research the range the relative risk aversion will belong to in order to explain the risk premium in our model.

First, we will select the ranges for all of the parameters. Over the period 1890-1979, the standard deviation of the return on the market is 16.7 percentage points, so we select that $\sigma = 0.17$. Hansen and Singleton (1982,1983), Epstein and Zin (1991) analyzed a number of monthly data sets (for the United States) differing in the measurement of consumption, asset returns, and the time period, their empirical results imply that the elasticity of substitution is typically small (i.e. always less than one), this is consistent with our spirit-of-capitalism hypothesis too, so we select that $\varepsilon < 1$. Table1-3 are the results we derived when $\varepsilon = 0.05$, and Table 4-6 are the result when $\varepsilon = 0.5$.

Table 1: The risk premium $-\sigma_{1V} = 0.01$, $\varepsilon = 0.5$, $\sigma = 0.17$

Table 2: The risk premium $-\sigma_{1V} = 0.04$, $\varepsilon = 0.5$, $\sigma = 0.17$

Table 3: The risk premium $-\sigma_{1V} = 0.1$, $\varepsilon = 0.5$, $\sigma = 0.17$

Table 4: The risk premium $-\sigma_{1V} = 0.01$, $\varepsilon = 0.05$, $\sigma = 0.17$

Table 5: The risk premium $-\sigma_{1V} = 0.04$, $\varepsilon = 0.05$, $\sigma = 0.17$

Table 6: The risk premium $-\sigma_{1V} = 0.1$, $\varepsilon = 0.05$, $\sigma = 0.17$
From Table 1-6, we derive that it is needed to be less than 5 for the relative risk aversion coefficient in order to explain the risk premium.

7. Estimation and empirical tests
In order to analyze the risk premium, we discuss the special case: the absolute wealth is the social status, that is, \( V(t) \equiv 1 \). In this special case, equation (10), (9), and (8) change to be

\[
J' (W_t) (\mu - r_0) + J'' (W_t) \alpha(t) \sigma^2 W_t = 0 \tag{18}
\]

\[
W_t^{-\lambda} C(t)^{-\frac{1}{2}} = f' ([1 - R] J (W_t)) J' (W_t) \tag{19}
\]

\[
0 = \max_{C(t), \alpha(t)} \frac{1 - R}{1 - \frac{1}{\varepsilon}} C(t)^{1 - \frac{1}{2}} W_t^{-\lambda} - \delta f ([1 - R] J (W_t)) + (1 - R) f' ([1 - R] J (W_t)) \{ J'(W_t [\alpha(t) (\mu(t) - r_0)] - C(t)) + \frac{1}{2} J'' (t)^2 \sigma^2 W_t^2 \} \tag{20}
\]

From (18) and (19), we have

\[
\frac{J''(W_t)}{J'(W_t)} = - \frac{\mu - r_0}{\alpha(t) \sigma^2 W_t} \tag{21}
\]

\[
((1 - R) J)^{\frac{\alpha(t)}{1 - \varepsilon}} J' (W_t) = W_t^{-\lambda} C(t)^{-\frac{1}{2}} \tag{22}
\]

Substituting (21) and (22) into (20), we have

\[
0 = \frac{C(t)}{\varepsilon - 1} - \frac{(1 - R) \delta}{1 - \frac{1}{2}} W_t r_0 + \frac{1}{2} W_t \alpha(t) (\mu - r_0) \tag{23}
\]

From (14), we have

\[
\frac{J}{J'} = \frac{1 - \varepsilon}{(1 - R)(\lambda \varepsilon + 1 - \varepsilon)} W_t \tag{24}
\]

Combining (23) and (24), we have:
Proposition: For any asset in equilibrium, the rate of return satisfies the following equation:

\[ \mu - r_0 = \frac{2C(t)}{(1-\varepsilon)W_t\alpha(t)} - \frac{2\delta\varepsilon}{(\lambda\varepsilon + 1 - \varepsilon)\alpha(t)} - \frac{2r_0}{\alpha(t)} \]  
(25)

Differentiating both sides of (19) about \( W_t \) gives:

\[ \frac{J''}{J'} + \frac{R - 1}{1 - R} \frac{J'}{J} = -\left( \frac{\lambda}{W_t} + \frac{C'(t)}{\varepsilon C(t)} \right) \]  
(26)

Combining (21), (24), and (26), we have:

Proposition: For any asset in equilibrium, the rate of return satisfies the following equation:

\[ \mu - r_0 = \alpha(t)\lambda\sigma^2 + \alpha(t)W_t\sigma^2 \frac{C'(t)}{\varepsilon C(t)} + \frac{(R\varepsilon - 1)\sigma^2(\lambda\varepsilon + 1 - \varepsilon)\alpha(t)}{\varepsilon(1 - \varepsilon)} \]  
(27)

Equation (27) appears to resemble the pricing equation of Epstein and Zin (1991 eq. 23), Duffie and Epstein (1992 eq. 21), and Bakshi and Chen (1996 eq. 12). It is useful for gaining insight into the static versus consumption CAPM debate. Heuristically, the static (CAPM) (see Jensen (1972)) measures the riskiness of an asset by means of the covariance of its return with the return on the market portfolio. In the intertemporal CAPM (CCAPM) (See Merton(1971)), the riskiness of an asset is measured by the covariance of its return with the marginal rate of substitution of consumption over time (most commonly as a function of the growth rate of consumption). In Bakshi and Chen (1996 eq.20) it is only relative to CCAPM, and Epstein and Zin (1991 eq. 23) does not consider the spirit of capitalism. In (27), the \( \alpha_t \) is market portfolio in equilibrium and \( \frac{C'(t)}{\alpha(t)} \) is the growth rate. Thus the riskiness of any asset is related to both the market portfolio (CAPM) and the growth rate (CCAPM). Because the elasticity of substitution and the coefficient of relative risk aversion are independent and because the spirit of capitalism is introduced, the equity premium puzzle in Mehra and Prescott (1985) can be partially explained in our model. Intuitively, for the given reasonable coefficient of relative risk aversion \( R \), we can choose the proper elasticity of substitution so that the equity premium, \( \mu - r_0 \), is not too low, furthermore, the more the spirit of capitalism, \( \lambda \), the more the equity of premium, \textit{ceterius paribus}. The empirical tests of equation (25) and (27) are our future works.

8. Conclusion

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