Tax Deduction for Net Losses as Subsidizing Insurance Contract

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Abstract

This paper provides a rationale for a government to provide a tax deduction for net losses when the insurance market faces adverse selection problems and the direct compulsory insurance is unlikely. We adapt the setting of Rothschild and Stilts (1976) and show that when the equilibrium of the private insurance market is either a separating equilibrium or no equilibrium, the government may Pareto-improve the welfare of the insured by allowing a tax deduction for net losses. In such cases, a tax deduction for net losses in fact serves as a subsidizing contract of Miyazaki (1977) and Spence (1978). However, if the equilibrium of the insurance market is a pooling equilibrium, as in Wilson (1977), a tax deduction for net losses cannot Pareto-improve the welfare of the insured.

JEL classification: D82; G22; H24

Keywords: Tax deduction; Adverse selection; Social insurance
1 Introduction

It has long been prevalent in many countries that governments allow individuals a tax deduction for their net losses. Does such a policy make economic sense? Kaplow (1992) shows that the optimal tax deduction rate for net losses should be zero; thus, it seems to be a bad idea to allow individuals a tax deduction for net losses. However, this policy is continuously adopted notwithstanding Kaplow’s striking finding, in the robustness of which he shows that, from an efficiency perspective, governments should not allow such a tax deduction even after considering administration costs, moral hazard problems, and other imperfections in the private insurance market.

Kaplow (1992) does not fully discuss adverse selection problems in an insurance market. Instead, he argues that, in an insurance market with adverse selection, a government should provide insurance directly rather than allow a tax deduction for net losses. Without a doubt, to ease the negative externality caused by adverse selection, the direct provision of insurance would be a cure. However, it may not be an easy job for a government to provide insurance directly; moreover, it could also be very costly. Thus, if it is not efficient to provide insurance directly, a government may have reason to allow a tax deduction for net losses as an indirect provision for insurance. In that case, a tax deduction for net losses can be a Pareto-improving policy for a government to deal with adverse selection in insurance markets.

In this paper, we intend to show that, in an insurance market under adverse selection, it may be a Pareto improvement for a government to allow a tax deduction for net losses. Rothschild and Stiglitz (1976) show that, in an insurance market under adverse selection, no pooling equilibrium exists and a separating equilibrium
may exist. We will first demonstrate that, in the case where a separating equilibrium exists, a tax deduction for net losses might improve the welfare of both high- and low-risk individuals.

We find that a tax deduction for net losses, indeed, plays a role much like the subsidizing contract found by Miyazaki (1977) and Spence (1978). As Kaplow (1992) states, a tax deduction for net losses is like social insurance. If a government finances a tax deduction by means of a lump-sum tax, the lump-sum tax system actually creates a subsidy between individuals of high and low risks. A Pareto improvement may be possible if the low-risk individuals subsidize the high-risk ones and thereby mitigate the adverse selection problems the low-risk individuals face by relaxing the incentive compatibility conditions of the separating equilibrium. As our first result, we demonstrate that it is, indeed, the case and that a tax deduction for net losses may be a Pareto improvement when a separating equilibrium exists in the private insurance market under adverse selection.

We further show that a tax deduction for net losses can be a solution for coping with market failure caused by adverse selection. Rothschild and Stiglitz (1976) show that there may be no equilibrium, and the insurance market then fails under adverse selection problems. It is well-known that such a case gives a government a reason to provide compulsory insurance. We show that, in such cases, if direct compulsory insurance is infeasible, then a tax deduction for net losses can play the role of compulsory social insurance and might improve the lots of both the high- and low risk insured.

Finally, we look at whether it may be Pareto-improving for a government to provide a tax deduction for net losses if the equilibrium of private insurance is a pooling equilibrium, as in Wilson (1977), where it is adopted as an equilibrium
concept other than that of Rothschild and Stiglitz (1976). Recalling that a self-financing tax deduction for net losses is like a government-provided pooling contract, since a private insurance market can now provide similar pooling insurance contracts, there is no room for a tax deduction to improve the welfare of the insured.

The rest of the paper is organized as follows: Section 2 describes the model. The main results are given in Section 3. Section 4 concludes and suggests some further extension.

2 Model

Let individuals face a binominal property risk with either a fixed loss or no loss. Assume that there exist two risk types—one, denoted \( H \), with high loss probability \( \pi_H \) and the other, \( L \), with low loss probability \( \pi_L \), with \( 1 > \pi_H > \pi_L > 0 \). For the sake of simplicity, let both types have the same initial wealth \( W \), loss amount \( S \), and a utility function \( U \) with \( U' > 0 \) and \( U'' \leq 0 \). The expected utility of the type \( i \) insured, \( i \in \{H, L\} \), is given by \( (1 - \pi_i)U(Z_N) + \pi_i U(Z_L) \), where \( Z_N \) and \( Z_L \) are, respectively, the final wealth of the type \( i \) insured in the no-loss and loss states. The types are the insured’s private information and unobservable to the insurer; hence, the insurer cannot directly set an insurance premium on the basis of risk types. An actuarially fair insurance contract is specified as \( (p\alpha, \alpha) \), where \( p\alpha \) is the insurance premium and \( \alpha \) is the indemnity.

Following Rothschild and Stiglitz (1976), the separating equilibrium can be defined as \( (p_i\alpha_i, \alpha_i), i \in \{H, L\} \), such that the following conditions are satisfied: (1)

Both types of insured are willing to purchase insurance (IR conditions):
\[
(1 - \pi_i)U(W - p_i\alpha_i) + \pi_i U(W - S + \alpha_i - p_i\alpha_i) \geq (1 - \pi_i)U(W) + \pi_i U(W - S), i \in \{H, L\},
\]
and (2) each type of insured will self-select the insurance policy designed for it (IC conditions). Specifically, for the type $H$ insured,

$$
(1 - \pi_H)U(W - p_H\alpha_H) + \pi_H U(W - S + \alpha_H - p_H\alpha_H) \\
\geq (1 - \pi_H)U(W - p_L\alpha_L) + \pi_H U(W - S + \alpha_L - p_L\alpha_L),
$$

and for the type $L$ insured,

$$
(1 - \pi_L)U(W - p_L\alpha_L) + \pi_L U(W - S + \alpha_L - p_L\alpha_L) \\
\geq (1 - \pi_L)U(W - p_H\alpha_H) + \pi_L U(W - S + \alpha_H - p_H\alpha_H).
$$

Now consider the situation where a government provides tax deduction $t$ for net losses, such that for net loss $l$ the tax deduction is $tl$. Let the separating insurance contract under a tax deduction, if it exists, be specified as $(p, \beta, \beta), i \in \{H, L\}$. We will consider a self-financing tax deduction system where a government’s budget for a tax deduction is collected by a lump-sum tax $\tau$ for each individual. Thus, the final wealth of the type $i$ insured in the no-loss and loss states are, respectively, $W - p_i\beta_i - \tau$ and $W - (S - \beta_i)(1 - t) - p_i\beta_i - \tau$. If the separating equilibrium still exists under the tax deduction system, then the new IR and IC conditions are now

$$(1 - \pi_i)U(W - p_i\beta_i - \tau) + \pi_i U(W - (S - \beta_i)(1 - t) - p_i\beta_i - \tau) \\
\geq (1 - \pi_i)U(W - \tau) + \pi_i U(W - S(1 - t) - \tau), i \in \{H, L\},$$

and

$$(1 - \pi_H)U(W - p_H\beta_H - \tau) + \pi_H U(W - (S - \beta_H)(1 - t) - p_H\beta_H - \tau) \\
\geq (1 - \pi_H)U(W - p_L\beta_L - \tau) + \pi_H U(W - (S - \beta_L)(1 - t) - p_L\beta_L - \tau),$$

and

$$(1 - \pi_L)U(W - p_L\beta_L - \tau) + \pi_L U(W - (S - \beta_L)(1 - t) - p_L\beta_L - \tau) \\
\geq (1 - \pi_L)U(W - p_H\beta_H - \tau) + \pi_L U(W - (S - \beta_H)(1 - t) - p_H\beta_H - \tau).$$

Let the number of high-risk and low-risk individuals be, respectively, $N_H$ and $N_L$. The budget constraint for the government is

$$[N_H\pi_H(S - \beta_H) + N_L\pi_L(S - \beta_L)]\tau = (N_H + N_L)\tau.$$
3 Propositions

First we show that a tax deduction for net losses may bring a Pareto improvement for an insurance market under adverse selection.

Proposition 1

If a separating equilibrium exists, as defined by Rothschild and Stiglitz (1976), then there may exist a non-zero \( t \in (0,1) \) and a corresponding \( \tau \) such that the budget constraint is satisfied and the new separating equilibrium is a Pareto improvement for the original one where \( t = 0 \).

Proof

Since we intend to show only that providing a tax deduction could result in a Pareto improvement in the insurance market under adverse selection, one example is enough. If we let the utility function be \( U(z) = z - 0.01z^2 \) and choose \( W = 50 \), \( S = 20 \), \( \pi_H = 0.1 \), \( \pi_L = 0.001 \), \( N_H = 1 \), \( N_L = 10 \), then we find that a new Pareto-improving separating equilibrium exists under certain non-zero tax deduction rates. Table 1 provides some simulation results on basis of Equations (1)-(7).

[Insert Table 1 about here.]

In the first case, we assume that the price of private insurance is actuarially fair and a tax deduction system needs no expense. Under all the situations listed in Panel A, the expected utilities of both types of individuals at \( t > 0 \) are no less than those at \( t = 0 \). Notice that the expected utilities of both types of individuals are strictly improved at \( t = 0.7 \).

In the second case, we assume that both private insurance and tax deduction need expenses, and they have the same expense loading, 20%. Panel B shows that as long as \( t \geq 40\% \), both types of insured are strictly better off compared with their welfare
at $t = 0$.

In the third case, we assume that a tax deduction requires 20% expense loading, which is higher than private insurance loading, 10%. Panel C shows that a government could please both types of individuals by setting a high tax deduction rate, e.g., $50\% \leq t \leq 70\%$.

Q. E. D.

[Insert Figure 1 about here.]

A strict Pareto improvement occurs when both types of the insured are better off. The welfare of high-risk individuals could be improved when a self-financing tax deduction system creates a material subsidy from low-risk ones. On the other hand, the low-risk individuals, while subsidizing high-risk ones, could be better off if the adverse selection problem they faced is mitigated enough whereby.

Figure 1 illustrates that the existence of a non-zero tax deduction rate could improve the lots of all individuals. Point $A$ with the coordinate $(W, W - S)$ is the wealth of individuals without insurance in the no-loss and loss states, respectively.

$e_H$ and $e_L$ are a separating equilibrium defined by Rothschild and Stiglitz (1976) at $t = 0$ in the case of no-expense loading. The high-risk insured is indifferent about choosing contract $e_H$ or $e_L$, full coverage or partial coverage, respectively, while the low-risk insured will stick with $e_L$ in the equilibrium. Now consider a tax deduction system. The wealth of an agent without insurance becomes $W - \tau$ and $W - S(1 - t) - \tau$ in the no-loss and loss states, respectively, and is located at point $B$. If a government could provide a proper tax deduction such that line $AB$ is closer to the low-risk pricing line $P_L$, as shown in Figure 1, then we may find a new separating equilibrium $e'_H$ and $e'_L$ that improves the lots of both types of insured.

A fortiori, Proposition 1 can be extended into the case where the transaction
costs for private insurance are more costly than those for a tax deduction. Indeed, Kaplow (1992) has shown that, if the transaction cost for a private market is higher than that for a tax deduction system, then an individual may be better off when a government allows a tax deduction for individual losses.

Proposition 1 provides a rationale for a government to provide a tax deduction to supplement the function of the insurance market under adverse selection. A tax deduction system can be regarded as a subsidizing contract, as in Miyazaki (1977) and Spence (1978), which improves the welfare of all the insured. If a negative lump-sum tax is allowed for high-risk individuals, then a tax deduction system is even more like a subsidizing contract, as suggested by Miyazaki (1977) and Spence (1978).

Obviously, providing a tax deduction does not always result in a Pareto improvement. An individual, indeed, receives extra coverage provided by a government; but, at the same time, a tax deduction system crowds out private insurance, as Kaplow (1992) notes. An individual may be better or worse off, depending on the net effect of the private insurance contract and the social subsidizing contract on the insured’s welfare.

From this simulation, we find that the degree of adverse selection is an important factor for determining the optimal tax deduction rate. The Appendix gives a glimpse of the result, where the number of high- and low-risk insured (expressed as percentages) can be taken as a proxy of the degree of adverse selection. When the adverse selection problem is not severe, e.g., the percentage of high-risk insured in the market is below 10%, the expected utility of the high-risk insured increases as $t$ increases. The low-risk insured under a tax deduction system is at least as happy as he or she is under $t = 0$. Thus, a non-zero tax deduction rate provides a Pareto improvement in the equilibrium. On the other hand, when the percentage of
high-risk insured to the population is high, or the adverse selection problem in an
insurance market is serious, the pattern of expected utility movement reverses. An
individual has the highest expected utility when $t = 0$. Thus, a government should
not allow a tax deduction to the public.

A famous result in Rothschild and Stiglitz (1976) is that, due to adverse selection,
the insurance market may even fail—that is, no equilibrium exists. Let us further
analyze whether a tax deduction for personal losses could be a Pareto improvement
for the insured when the private insurance market fails. For the sake of simplicity,
we assume that a tax deduction system does not heal an insurance market failure
problem—that is, a private insurance market still fails under a tax deduction system.

**Proposition 2**

*Given that there are no transaction costs in either a private insurance market or
a tax deduction system, a tax deduction system can be a Pareto improvement when a
private insurance market fails because of adverse selection.*

**Proof**

Consider a social planner who chooses an optimal tax deduction rate to
maximize a weighted expected utility of both high and low risks,

$$wEU_H + (1 - w)EU_L,$$

where the weights have incorporated the number of individuals
of each type. Formally, the social planner solves

$$\max_w wEU_H + (1 - w)EU_L,$$

where

$$EU_i = \pi_i U(W - S(1 - t) - \tau) + (1 - \pi_i)U(W - \tau), i \in \{H, L\},$$

and

$$\tau = t\left(\frac{N_H \pi_H + N_L \pi_L}{N_H + N_L}\right)S.$$
Denote $\theta = \frac{N_H}{N_H + N_L}$. The first-order condition of the above problem is

$$
\left(\frac{w\pi_H + (1 - w)\pi_L}{1 - [w\pi_H + (1 - w)\pi_L]}\right)\left(1 - \frac{[\theta\pi_H + (1 - \theta)\pi_L]}{\theta\pi_H + (1 - \theta)\pi_L}\right) = \frac{U'(W - \tau)}{U'(W - S(1 - t) - \tau)}. 
$$

(9)

This model is like that of the demand for insurance first proposed by Mossin (1968). It is interesting to note that if $w = \theta$, then the optimal tax deduction rate is one; and if $w < \theta$, then the optimal tax deduction rate is less than one. There exists an optimal tax deduction rate between zero and one if the first-order condition holds and $w < \theta$.

Q. E. D.

A tax deduction for losses can be seen as an insurance contract provided by a government. Since a private insurance market cannot provide coverage due to adverse selection (under the situations now in question), it could be Pareto welfare-enhancing for a government to provide social insurance through a tax deduction system. In fact, a tax deduction system provides an average-pricing and compulsory insurance. It is well-known that there is no pooling equilibrium in the model of Rothschild and Stiglitz (1976), as insurance companies can always offer a profitable contract to attract low-risk insured to deviate from the average pricing contract. However, in the case in question, a tax deduction system is a compulsory social insurance; thus, insurance companies cannot offer profitable contracts to make low-risk individuals deviate.

[Insert Figure 2 about here.]

Figure 2 shows the case where both types of insured are better off in a tax deduction system under market failure. Point $A$ with $(W, W - S)$ denotes the insured’s wealth without insurance. The expected utility of the high-risk (low-risk)
insured is \( U_H \) (\( U_L \)). Point \( B \) with \((W - \tau, W - S(1 - t) - \tau)\) denotes the wealth of an individual still without insurance while under a tax deduction system. When transaction costs are absent, \( B \) would lie upon the average pricing line \( \theta P_H + (1 - \theta)P_L \). As long as \( \theta P_H + (1 - \theta)P_L \) is close enough to line \( P_L \), we would find that both types of insured are better off at \( B \) than at \( A \).

In conclusion, when the private insurance market has, respectively, a separating equilibrium and no equilibrium, Propositions 1 and 2 show that a government-provided pooling contract (tax deduction) can improve the welfare of all individuals. There is space for Pareto improvement since a private insurance market does not function well due to adverse selection.

Wilson (1977) defines the anticipatory equilibrium as that wherein insurance companies offer only policies that, after other companies anticipate reactions, at least break even. That is, a Wilson equilibrium exists if no insurance companies can offer a new policy, such that this policy earns a profit and remains profitable even after other companies have dropped their now-unprofitable policies. As shown by Wilson (1977), a pooling equilibrium exists in the market when the Rothschild-Stiglitz separating equilibrium does not exist. We adopt below Wilson’s “anticipatory” equilibrium to examine whether a government should provide a tax deduction for net losses.

**Proposition 3**

*Given that there are no transaction costs in either a private insurance market or a tax deduction system, a tax deduction system cannot be a Pareto improvement when a private insurance market has a pooling equilibrium, as in Wilson (1977).*

**Proof**
The pooling equilibrium $e$ is located at the point where the low-risk insured’s indifference curve $U_L$ is tangent to the average pricing line $\theta P_H + (1-\theta)P_L$, as shown in Figure 3. After a government provides a tax deduction for net losses, the new initial point of the insured will shift from $B$ along the average pricing line $\theta P_H + (1-\theta)P_L$ (assume for simplicity that the government is as efficient as the insurer in handling “insurance” affairs). However, the market equilibrium is located at exactly the same point as that without a tax deduction if a private insurance market has a pooling equilibrium where the pricing line of the insurance remains the same. Thus, a tax deduction for net losses cannot improve the welfare of the insured. Q.E.D.

Kaplow (1992) shows that a tax deduction for net losses cannot Pareto-improve the individuals’ welfare if a private insurance market functions well. Proposition 3 is consistent with Kaplow’s finding (1992). Since a private insurance market can provide similar pooling contracts as those generated by a tax deduction, this leaves no room for a government to improve the welfare of the insured, unless the government is more cost-efficient than the insurer.

4 Conclusion

This paper provides a rationale for a government to provide a tax deduction for net losses when the insurance market faces adverse selection problems. Kaplow (1992) shows that the optimal tax deduction rate for net losses should be zero; but his discussion does not address an adverse-selection situation. In this paper, we show that the optimal tax deduction rate for net losses may not be zero in the insurance market under adverse selection. When the equilibrium of the insurance market is
either a separating equilibrium or no equilibrium, as in the setting of Rothschild and Stiglitz (1976), the government may Pareto-improve the welfare of the insured by allowing a tax deduction for net losses. On the other hand, if the equilibrium of the insurance market is a pooling equilibrium, as in Wilson (1977), a tax deduction for net losses cannot Pareto-improve the welfare of the insured. A natural extension of this paper is to discuss whether the government should provide a tax deduction for individuals’ net losses on the basis of other definitions of market equilibrium.
References


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Wealth in the loss state

Wealth in the no-loss state

Figure 1 Separating equilibrium
Figure 2 No equilibrium in the insurance market
Wealth in the loss state

\[ \theta P_H + (1 - \theta) P_L \]

Wealth in the no-loss state

Figure 3 Pooling equilibrium in the insurance market
Appendix

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