A Solvency Based Multi-period Corporate Liquidity Crisis Prediction Model

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ABSTRACT

Many corporate failure prediction models have been proposed in the literature. They can be roughly grouped into two categories, “classical statistical models” and “stochastic intensity models”. The former focuses on searching for accounting-based measures as predicting variables to do firm failure prediction through statistical techniques basing upon the concept of hazard rate. The latter predict corporate failures by using exogenous information such as credit rating and recovery rate which are not directly related to asset values. Within the above two frameworks, few studies apply stochastic solvency ratio (similar to debt service coverage ratio) models to predict corporate liquidity crisis. Basing upon two significant characteristics of solvency ratio--“mean-reversion”, “non-negative value” and the concept of varying coefficient model, the study develops a “Time-dependent stochastic solvency ratio model”. To consider future industrial economic state changes’ impacts on a firm’s solvency ratio, we also construct a stochastic model of industrial economic state. The information forecasted from the state model is used as the base for adjusting the parameters of the time-dependent solvency ratio model. With the information of solvency ratio and the criteria of insolvency (when solvency ratio is less than one), a “Solvency Based Multi-period Liquidity Crisis Prediction Model” can be built. Above all, our solvency ratio model has the features: relating to firm-liquidity; following a stochastic process; being reasonably extended to multi-period with the stochastic model of industrial economic state; providing a simple and direct criterion for liquidity crisis. The above four merits are rarely simultaneously provided by other failure prediction models. Moreover, to perform a multi-period firm’s liquidity crisis prediction, this solvency ratio model needs only publicly available information of corporate finance and the industrial economic state (i.e. the industrial cyclicality information)

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I. Introduction

Many corporate failure prediction models have been proposed in the literature. According to modeling techniques, they can be roughly grouped into two categories, “classical statistical models”\(^1\) and “stochastic intensity models”\(^2\). The former focuses on searching for accounting-based measures as the predicting variables to forecast corporate failures through statistical techniques\(^3\). Among them, three major statistical techniques are worth noting, multivariate discriminant analysis (Altman’s Z-score, 1968), qualitative-response (Ohlson’s O-score, 1980), and duration analysis (Shumway, 2001). The latter on the other hand stress the “non-asset-value related information” such as credit rating (Litterman and Iben, 1991; Jarrow, Lando and Turnbull, 1997) or other default-related proxies. They estimate a firm’s bankruptcy probability through the constructing stochastic models of default-related proxies.

Although that “stochastic intensity models” better match market current reality, they rely on exogenous information such as credit rating rather than on a firm’s financial information such as liquidity related ratios. However, though the “classical statistical models” employ corporate historical financial data to do failure prediction, few of them propose stochastic models to estimate future liquidity measure\(^4\). Within

\(^{1}\) Classical statistical models include the first-generation, the second-generation and the latest-generation models. The first-generation classical statistical models based on multivariate discriminant analysis originated with Beaver(1966), Beaver(1968a,1968b), and Altman(1968). The second-generation classical statistical models are represented as Ohlson(1980) and based on qualitative-response models, such as logit and probit. Above these two models, Altman (1968) use the Z-Score and Ohlson(1980) employ the O-Score to be the composite measures of bankruptcy probability. And the failure predictions are both one-period. However, the latest-generation models extend to multi-period failure prediction in empirical works by using duration analysis. Duration analysis is to add “default time-related” variables (e.g. age) to be a time-dependent covariate in original one-period model (Lee and Urrutia, 1996; Shumway, 2001). The related duration models include Donald & Van de Ducht (1999), Kavvathas (2001), Chava and Jarrow(2002), and Hillegeist, Keating, Cram, and Lundstedt (2003).


\(^{3}\) Recently, many other new modeling techniques are mentioned. They include recursive partitioning analysis (or tree classification), neutral networks and general algorithms. These three classification methods are sometimes classed under the general denominator of “inductive learning”. It is therefore more difficult to validate this kind of models as an outsider so that this study will neglect these.

\(^{4}\) Most of classical statistical models are dedicated to search for the probable solvency measures from financial information and then construct the failure prediction model with statistical techniques. Few
the frameworks of the above two failure prediction models, very few studies employ stochastic solvency ratio model to do corporate failure prediction, either.\(^5\) In addition, there was no literature that had ever developed an applicable model to describe the stochastic characteristics of solvency ability. In this study, we define a measure for corporate solvency ability (later denoted as solvency ratio) based on the concept of a firm’s solvency reality. The solvency ratio is defined as the ratio of a firm’s disposable cash to its net payment obligations in a same period\(^6\). When the ratio of a firm is less than one, it will fail to fulfill its payment obligation and enter into a situation of corporate failure. The solvency ratio is more meaningful than other liquidity measures in that it not only directly reflects a firm’s true periodic liquidity but also provides a straight indicator of a firm’s failure.\(^7\) Further through our observations of solvency ratio, we discover that the behavior of solvency ratio exhibits some stochastic characteristics, such as mean-reversion and non-negative values. In addition, these time-varying solvency ratio behaviors are influenced by changes of industrial economic states.

In order to obtain a firm’s liquidity distributions, this study starts in building a stochastic solvency ratio model that can appropriately describe aforementioned characteristics of the solvency ratio. With the aim of allowing the solvency ratio model to reflect the changes of industrial economic states, the solvency ratio model is designed to be time dependent. That is the parameters of the stochastic solvency ratio model are time varying and alter according to the changes of the states of industrial economy. Adopting the concept of varying coefficient model, we construct a stochastic model of industrial economic state\(^8\), using industrial cyclical

\(^5\) To our best knowledge, there is no publicly distributed solvency ratio based liquidity crisis prediction model.

\(^6\) The numerator of the solvency ratio contains three parts: operating cash inflow by moving average every four quarters, the beginning balance of cash and short-term investments. On the other side, the denominator of the solvency ratio includes four parts: operating cash outflow by moving average every four quarters, the debt amortization of principals, the interest payments and tax expenses. Please refer equation (1). The solvency ratio is equivalent to debt service coverage ratio (usually denoted as SR)

\(^7\) When solvency ratio is less than one, it stands for current holding cash balance is not enough to cover the net payment obligations.

\(^8\) The characteristics of economic state (business cyclical factor) can be referred in appendix I. We discover that its fluctuation obviously has the nature of mean-reversion.
factors as proxies for the industrial economic states. The information forecasted by the industrial economic state model is used as the base for adjusting the parameters of the time-dependent solvency ratio model, which we call a “Time-dependent stochastic solvency ratio model”. With the solvency ratio model, we can generate a firm’s liquidity distributions in future periods. Knowing a firm’s multi-period liquidity distributions and the criteria of insolvency (when solvency ratio is less than one), we are able to build a multi-period liquidity crisis prediction model, which we call a “Solvency Based Multi-period Liquidity Crisis Prediction Model”.

Compared with the classical statistical models, our model is different in three aspects. First, we define a new measure liquidity measure that can directly reflect a firm’s solvency. Second, our model can incorporate the state of industrial economy to reflect its impact on a firm’s solvency. Third, our solvency model is a mean-reverting and non-negative stochastic model that matches the common firm management principles of maintaining an optimal (or appropriate) firm liquidity, neither too high nor too low.

Comparing with stochastic intensity models, our model is different in two aspects. First, we use the firm-liquidity related information to be a stochastic variable instead of exogenous information such as credit rating. Second, comparing to those structural-form related corporate failure prediction models, our solvency ratio model can directly measure the probability of a firm’s insolvency rather than indirectly relies on the relationship between debt and asset value.

Duffie and Wang (2004) primarily employ the two stochastic covariates, distance-to-default and personal income, to explain the corporate failure probability. However, the variable, distance-to-default, is calculated by Merton model (1976) that belongs to structural-form credit risk models. In structural-form models, the decision criterion of default is based upon the relationship between assets value and liabilities value. However, corporate failures may happen due to liquidity crunch even when asset value is higher than liabilities value.

In sum, solvency ratio has the following features: First, it is firm-liquidity related; second, it owns stochastic characteristics of mean-reversion and non-negative value; third, it can be reasonably extended to multi-period with the stochastic model of
industrial economic state; fourth, it directly provides a criterion for liquidity crisis (when solvency ratio is less than one). The above four merits are rarely simultaneously provided by other failure prediction models. Moreover, to perform our model needs only publicly available information (e.g., corporate financial data and industrial economic data). Our empirical analysis shows that the stochastic solvency ratio model is preliminarily supported.

The rest of the paper is divided into four sections: First, we construct a time-dependent stochastic solvency ratio model, including a discussion on the stochastic characteristics of solvency ratio, the time-dependent stochastic solvency ratio model, and a stochastic industrial economic state model; Second, we present a “Solvency Based Multi-period Liquidity Crisis Prediction Model”; Third, we empirically examine effectiveness of our model. In the last section we conclude this study.

II. The Time-Dependent Solvency Ratio Model

In this section, firstly, define solvency and solvency ratio. Second, we explore the characteristics of firm’s solvency ratio. Third, we discuss stochastic models that can appropriately describe solvency ratio’s characteristics. Fourth, we construct our solvency ratio models based upon previous discussion. Finally, to consider future industrial economic state changes’ impacts on a firm’s solvency ratio, we introduce a stochastic model of industrial economic state. The information forecasted from the state model is used as the base for adjusting the parameters of the solvency ratio model.

1. Solvency and solvency Ratio

Solvency ratio is calculated from the current holding cash balance (including the periodic cash inflow and the beginning balance of cash and short-term investment) relative to current payment obligations so that it can truly reflect a firm’s liquidity in a short run. And liquidity has been empirically confirmed to be a sign of a firm’s solvency. Therefore solvency ratio can be viewed as a better measure to predict corporate failures. The definition of solvency ratio used here as follows:

\[
SR_i = \frac{OCIF_i^{MD} + C_{t-1} + SI_{t-1}}{OCOF_i^{MD} + DA_i + Int_i + Tax_i}
\]  (1)
In equation (1),

- $OCIF_t^{MA}$: stands for operating cash inflow by moving average every four quarters. It contains EBIT (earnings before interest and tax expenses), non-operating-related adjustment items (latter denoted as $NOR\ Adj.items$) and operating-related adjustment items (latter denoted as $OR\ Adj.items$). For “$NOR\ Adj.items$”, it covers depreciation & amortization expenses, net losses from investment or assets disposal and so on; for “$OR\ Adj.items$”, it primarily includes the net decreases of accounting receivables, inventories and other operating-related items\(^9\) besides accounting payables. In this study, we divide accounting payables (latter denoted as $AP$) into two parts: its decreases is proxy for real cash outflow so that it should be classified into $OCOF_t^{MA}$; its increases stands for cash inflow so that we take it as one plus item for $OCIF_t^{MA}$. In addition, we also take the moving-average method to eliminate the influencing effects of credit policies and seasonality. Based on the above, $OCIF_t^{MA}$ can be showed as equation (2):

$$OCIF_t^{MA} = MA(EBIT_t + NOR\ Adj.items_t + OR\ Adj.items_t + Increase\ on\ AP_t)$$  \hspace{1cm} (2)$$

- $OCOF_t^{MA}$: stands for operating cash outflow by moving average every four quarters. It primarily includes account payables. Based on the same reasons for $OCIF_t^{MA}$, moving-average method is indispensable. As for other accrual expenses, they have to be viewed as the adjustment for EBIT firstly because the characteristics of internal expenses.

- $C_{t-1}, SI_{t-1}$: stands for the beginning balance of cash and short-term investments.

- $DA_t$ : stands for the amortization of debt principals\(^10\). It is calculated from the net decreases both of total short-term debts and long-term debts in the period $t$.

- $Int_t, Tax_t$ : stands for the interest payments and tax expenses in the period $t$.

\(^9\) Other operating-related items include other accrual expenses such as wage payables, tax payables and so on. Because these items belong to internal expense necessary for firm’s continuous operation, it is required to take it as adjustment to EBIT firstly.

\(^10\) Because the debt amortization is showed as the net decreases of total debts in the period $t$, the amount already reflects a firm’s short-term financing capacity.
From equation (1), we know that solvency ratio is mainly influenced by operating cash inflow and outflow, holding cash balance, and debt payment obligations. Because a firm’s operating performance and solvency ability are both primarily influenced by industrial economic state (namely business cyclical factor), there must exist a close relationship between solvency ratio and economic state. Moreover, as for the investing net cash flow, we can reasonably neglect it in this study. The main reasons will state as follows. First, the essence of investing net cash flow belongs to the results of strategic plans and it will experience quite a long-term time period. Therefore it is not disposable in the short-term period (e.g. a quarter)$^{11}$. Second, firm real value is primarily decided by operation-related cash flow so that it is not necessary to consider disposals of assets and long-term investments for normal operating firm$^{12}$. As a result, it will be little influencing on the cash inflow items when proceeding firm’s liquidity evaluation.

Next, we will introduce the stochastic economic state model and its relationship with parameters’ adjustments of the stochastic solvency ratio model.

2. The characteristics of firm’s solvency ratio

Through our observations of solvency ratio, we discover that the behaviors of solvency ratio exhibit some stochastic characteristics, including mean-reversion and non-negative values. Figure 1-4 display these characteristics. It is understandable that a normally managed firm tends to maintain its solvency ratio stable or stable with an upward trend$^{13}$. Since solvency ratio’s calculation is related to operating cash inflow and outflow and we can only acquire the quarterly data of a firm’s cash flow, we employ the moving-average cash inflow and outflow per operating cycle (usually one year) in order to eliminate the influence management manipulation in credit policy$^{14}$. In sum, based on the solvency ratio natures we found above, a

$^{11}$ On the part of investing cash inflow, it primarily contains disposals of long-term investments and assets; however, it is not only non-operating related but also lower liquid. On the other part of investing cash outflow, its mainly component is capital expenditures. Therefore, investing net cash flow will be mostly negative in general cases because capital expenditures are necessary for maintaining stable growth trend and the disposals of assets are not usual activities. As a result, it has little influence on the cash inflow items.

$^{12}$ It is usual for hazard firms to dispose fixed assets and long-term investments. Therefore, in this study, we can neglect investing cash inflow because we focus on normal firms.

$^{13}$ Solvency ratio stands for the value of current holding cash balance relative to current payment obligations. For corporate managements, they will tends to maintain it in stable level in order to utilize funds efficiently, time-optimally or avoid agency problems. For example, financial slack (time-optimally) and cash flow hypothesis (agency problems) are the reasons.


6
“mean-reversion stochastic process” seems appropriate to depict solvency ratio’s characteristics\textsuperscript{15}. In addition, by examining historical solvency ratios of sampled companies, we find that they comply with lognormal distributions\textsuperscript{16}.

\[dX = a(b - X) \cdot dt + \sigma \cdot X^\beta \cdot dz, \quad dz = e^{\sqrt{dt}} \cdot N(0,1)\]  \hspace{1cm} (3)

where,

\textsuperscript{15} In financial literatures, mean-reversion stochastic models are often applied to model interest rate. We therefore observe interest rate illustrated as figure A2-1 and A2-2 (refers appendix II). Comparing figures 1~4 with figures A2-1,A2-2, we discover that the mean-reversion of solvency ratio appear more obviously than interest rate. This is mainly because that interest rate may be disturbed by many macroeconomic noises, such as the liquidity trap.

\textsuperscript{16} The examinations for solvency ratio’s historical distributions please refer Appendix III.
$dX$: stochastic variable $X$'s changes in a short time  

$a$: stochastic variable $X$'s mean-reversion speed  

$b$: the long-term average of stochastic variable $X$  

$\sigma \cdot X^\beta$: the standard deviation of stochastic variable $X$'s changes in a short time (dt).  

$\beta$: positive constant. So, $\text{Var}(dX) = \sigma^2 \cdot X^{2\beta} \cdot dt$

While a generalized time-dependent mean-reversion stochastic model can be displayed as follows:

$$dX = [\theta(t) + a(t) \cdot (b - X)] \cdot dt + \sigma(t) \cdot X^\beta \cdot dz \quad (4)$$

In the equation (4), $\theta(t)$ is extra-added in drift term and it is a function of time.  

In fact, equation (4) is also a kind of mean-reversion stochastic models. It can be restated as follows:

$$dX = a(t) \cdot [b'(t) - X] \cdot dt + \sigma(t) \cdot X^\beta \cdot dz \quad (5)$$

In equation (5), $b'(t) = \theta(t)/a(t) + b$

$b'(t)$ stands for the long-term average of stochastic variable $X$ varying with time;  

$a(t)$ represents for stochastic variable $X$'s mean-reversion speed varying with time.

Equation (5) can better describe the stochastic variable $X$’s fluctuating behavior because of the increase on parameters’ degrees of freedom.

We adopt the time-dependent version of mean-reverting stochastic model since a firm’s solvency ratio severely influenced by changes of industrial economic states.

Applying the concept of varying coefficient model\textsuperscript{17}, the parameters in the solvency ratio model are time-varying to reflect the changes of future economic states. While the expected future economic state changes are obtained from a stochastic industrial economic state model. According to our analysis, it is appropriate to apply a time-independent mean-reverting stochastic model\textsuperscript{18} for modeling industrial state of economy.

4. Stochastic solvency ratio model

From above discussion, we set our solvency ratio model as a “Time-dependent

\textsuperscript{17} It is usually applied in time-series sample data. Its characteristic is that it takes the changes of the model’s coefficients as one or one more explainable variables in another regression model. And it makes the expected value of the coefficient be decided by a series of explaining variables.

\textsuperscript{18} That is the constant parameter stochastic model shown in equation (1).
stochastic model”. Since solvency ratio is lognormal-distributed, to simplify model setting, we take natural log on solvency ratio\(^{19}\) and then the natural log value of solvency ratio (later denoted as lnSR) becomes a normal distribution. Therefore, we can utilize Gaussian process to describe the future stochastic fluctuation of lnSR. In addition, since, in our model setting, lnSR’s stochastic fluctuation nature already varies with time (namely varies with economic state), the influence of its size on lnSR’s fluctuation should have been reflected in the changes of economic state. We can therefore assume that \(\bar{\beta}\) is equal to 0 in equation (5) and establish the “Time-dependent stochastic solvency ratio model” as equation (6):

\[
d(lnSR_t) = a(t) \cdot [b(t) - lnSR_{t-1}] \cdot dt + \sigma(t) \cdot dz, \quad dz = \sqrt{\sigma(t)} \cdot dt, \quad \varepsilon \sim N(0,1)
\] (6)

where,

\(d(lnSR_t)\): lnSR’s term variation (or instantaneous changes in continuous time)

\(a(t)\): lnSR’s mean-reversion speed.

\(b(t)\): lnSR’s long-term average level

\(\sigma(t)\): standard deviation of lnSR’s term variation, namely \(\sqrt{\text{Var}(d(lnSR_t))}\).

5. Stochastic economic state model and parameters’ adjustments of the stochastic solvency ratio model

In order to simplify our model and without loss of generalization, we assume that \(a(t)\) in equation (6) is equal to a constant\(^{20}\). The \(a(t)\) stand for long-run mean-reversion speed of a firm’s lnSR. While, \(b(t)\) and \(\sigma(t)\) represent for long-term average lnSR and standard deviation of lnSR’s term changes\(^{21}\) respectively. These three parameters can be estimated by the AR(1) method mentioned by Chen(1996).

\(^{19}\) This transformation belongs to a monotonic transformation. A monotonic transformation still maintains the original solvency ratio such as the characteristic of mean-reversion. As a result, lnSR is a representative indicator for solvency ratio. The details please refer to Appendix IV.

\(^{20}\) Actually \(a(t)\) will be influenced by the growth trend of individual enterprise. In this study, we assume that \(a(t)\) is a fixed constant in order to simplify model. We therefore only consider the business life cycle of individual firm when applying so that the general form of our model will be maintained.

\(^{21}\) In this study, we will consider macroeconomic cycle and industrial maturity and regard them as the adjustment basis of parameters’ term-changes in stochastic solvency ratio model. The basic concept of this idea is that industrial states will influence a firm’s operating performance and its periodic liquidity. However, these two considerable factors will reflect on the industrial “the growth rate of coincident indictors” or “the growth rate of leading indictors”. We therefore lead the estimates of the future coincident or leading indictors’ growth rate into stochastic solvency ratio model and then it can be reflected on the changes of economic state (time).
In this study, we use a firm’s industry coincident indicators\textsuperscript{22} to proxy economic state and build a stochastic industrial economic state model as equation (7) below\textsuperscript{23}. With this state model, the economic states in the future periods can be estimated.

\begin{equation}
    d(\eta_t) = a_\eta \cdot [b_\eta - \eta_{t-1}] \cdot dt + \sigma_\eta \cdot dz
\end{equation}

where,
- $\eta_t$: the growth rate of industrial coincident indicator in time $t$.
- $a_\eta$: the mean-reverting speed of industrial coincident indicator’s growth rate
- $b_\eta$: the long-term average of industrial coincident indicator’s growth rate
- $\sigma_\eta$: the standard deviation of changes of industrial coincident indicator’s growth rate

Consequently we can fine-tune the parameters of the stochastic solvency ratio model according to the estimates of future industrial economic states that are derived from the stochastic economic state model. The parameters $b(t)$ and $\sigma(t)$ in equation (6) are shown as bellow\textsuperscript{24}:

\begin{align*}
    b(t) &= b \cdot (1 + \psi_t^b) \quad \text{(8)} \\
    \sigma(t) &= \sigma \cdot (1 + \psi_t^\sigma) \quad \text{(9)}
\end{align*}

In equations (8) and (9),
- $b$: the long-term average of lnSR estimated by AR(1) method (Chen, 1996).
- $\sigma$: the fluctuating parameter of lnSR estimated by AR(1) method (Chen, 1996).

When industrial economic state’s proxy is a coincident indicator\textsuperscript{25}:

\begin{align*}
    \psi_t^b &= \frac{\dot{\omega}_{t-1} - \overline{\omega}}{\overline{\omega}} \cdot \alpha_1 \\
    \psi_t^\sigma &= \frac{\dot{\omega}_{t-2} - \overline{\omega}}{\overline{\omega}} \cdot |\epsilon_t| \quad \text{\textsuperscript{26}}
\end{align*}

\textsuperscript{22} It can be also applied in leading indicators.
\textsuperscript{23} The characteristics of economic state (business cyclical factor) can be referred in appendix I. We discover that its fluctuation obviously has the nature of mean-reversion.
\textsuperscript{24} In our models, we assume that a firm’s lnSR reflect the state of industrial economy and then we let lnSR’s mean-reversion speed be equal to a constant because the solvency ratio model’s parameters (b, $\sigma$) have been adjusted by the future industrial economic states. The details of parameters’ adjustments in stochastic solvency ratio model are thoroughly discussed in appendix V (It discusses the relationship between stochastic solvency ratio model’s estimation of parameters and economic states and introduces the adjustment method.).
\textsuperscript{25} When economic state’s proxy is a leading indicator:

\textsuperscript{26} The nature of mean-reversion is a leading indicator.
\[ \psi_i^\alpha = \frac{\hat{\omega}_i - \bar{\omega}}{\bar{\omega}} \cdot \alpha_i \]  

\[ \psi_i^\sigma = \frac{\hat{\omega}_i - \hat{\omega}_{i-1}}{\bar{\omega}} \cdot |\alpha_i| \]

In equation (10) and (11),
\( \hat{\omega}_i \) : the estimates of industrial economic state in future periods from stochastic industrial economic state model.
\( \bar{\omega} \) : the long-term average of industrial economic state calculated from historical data
\( \alpha_i \) : the sensitivity of lnSR relative to industrial economic state (namely the regressive coefficient of \( \ln SR = \alpha_0 + \alpha_1 \cdot \omega_i + \varepsilon \), and \( \ln SR_t \) stands for the natural log value of solvency ratio in time \( t \)).

In the above adjustment methods, \( \alpha_i \) reflects the sensitivity of solvency ratio to the fluctuation of industrial economic state. When the industrial economic state changes on unit, solvency ratio will change \( \alpha_i \) unit accordingly.

On the other side, the mean-reverting speed of industrial economic state’s growth rate \( (a_\eta) \), the long-term average growth rate of industrial economic state’s \( (b_\eta) \) and the standard deviation of changes of industrial economic state’s growth rate \( (\sigma_\eta) \) are both constants and are estimated by AR(1) method (Chen, 1996).

### 6. Parameters estimation of stochastic economic state model

In equation (7), both the parameters, \( a_\eta \), \( b_\eta \) and \( \sigma_\eta \), are estimated by using the AR(1) method (Chen, 1996). \( a_\eta \), \( b_\eta \) and \( \sigma_\eta \) individually stands for the mean-reverting speed, long-term average, and the changes’ standard deviation of industrial economic state’s change rate.

Chen’s estimate method is under the O-U process, and the conditional density of any future industrial economic state is a normal distribution with the mean and variance as follows:

\[ E_s[\eta(s)] = \eta(t) \cdot e^{-a(s-t)} + b_\eta \cdot (1 - e^{-a(s-t)}) \]  

\[ Var_s[\eta(s)] = \frac{\sigma_\eta^2 [1 - e^{-2a(s-t)}]}{2a} \]

In equation (12) and (13), \( s \) stands for the observed time point in the future.
With this result, we can write the equation as a discrete autoregressive process for order 1, i.e., AR(1) process:

\[
\eta(s) = \eta(t) \cdot e^{-a(s-t)} + b_\eta \cdot (1 - e^{-a(s-t)}) + \xi(s) \tag{14}
\]

\[
\eta_{t+\Delta t} = \eta_t \cdot e^{-a\Delta t} + b_\eta \cdot (1 - e^{-a\Delta t}) + \xi_{t+\Delta t} \tag{15}
\]

Where the error term \(\varepsilon\) is normal distributed with mean 0 and variance as described in equation (14). And \(\Delta t\) is a length of time interval. The AR(1) process allows \(\eta_t\) to satisfy all three properties of the OU process, i.e., mean, variance, and white noise with normal density. Obtaining this exact form from discretization is essential for simplifying the estimation process of the parameters. Equation (15) can be written as the following regression model:

\[
\eta_t = \alpha + \beta \cdot \eta_{t-\Delta t} + e_t \tag{16}
\]

where \(\alpha = b_\eta (1 - \beta), \beta = e^{-a\Delta t}\), so all the three parameters can be solved from equation (16).

\[
a = -\frac{\text{Ln}\beta}{\Delta t} \quad b_\eta = \frac{\alpha}{1 - \beta} \quad \sigma^2 = \frac{2a \cdot \text{MSE}}{1 - e^{-2a\Delta t}} \tag{17}
\]

According to equation (17), we therefore estimate the three parameters; that are \(a_\eta\), \(b_\eta\) and \(\sigma_\eta\), of the stochastic industrial economic state model.

\[
\text{7. Parameters estimation of stochastic solvency ratio model}
\]

Stochastic solvency ratio model\(^{26}\) is a kind of Gaussian process so that AR(1) method (Chen, 1996) can be also appropriately used to estimate its parameters. Under the assumption that the parameters of industrial state model are fixed, equation (6) can be rewritten as equation (18):

\(^{26}\) In this study, stochastic solvency ratio model is based on the natural log value of solvency ratio (lnSR). And lnSR complies with normal distribution so that the designation of Gaussian process is appropriate.
\[ d(\ln SR_t) = a \cdot [b \cdot (1 + \psi_t^b) - \ln SR_{t-1}] \cdot dt + \sigma \cdot (1 + \psi_t^\sigma) \cdot dz \] (18)

III. A Solvency Based Multi-Period Liquidity Crisis Prediction Model

A multi-period liquidity crisis prediction model focuses a firm’s multi-period solvency ratio. We therefore need to know the firm’s solvency ratio distributions in the future periods. With our “time-dependent stochastic solvency ratio model”, we first simulate appropriate number of lnSR paths. And then we switch these paths back to “solvency ratio paths” by taking exponential transformation to obtain the firm’s solvency ratio distributions in the future periods. As a result, the firm’s multi-period liquidity crisis predictions can be done.

In this study, we can get one future solvency ratio path of a firm by simulating once according to equation (18). Repeating above process for N times, we can have a firm’s N solvency ratio paths. Through a cross-sectional analysis in each period, we can obtain the firm’s multi-period solvency ratio distributions. A firm is deemed as having liquidity crisis when solvency ratio is less than one. It can be illustrated as figure 5.

Figure 5. Liquidity Crisis determination method
In figure 5, the solvency ratio distribution complies with lognormal distribution based upon the empirical results of goodness-of-fit tests on our sample firms\textsuperscript{27}. We therefore preliminary think it reasonable that solvency ratio is designed to be lognormal-distributed\textsuperscript{28}.

In addition, liquidity crisis will occur when the solvency ratio is less than one and the probability of liquidity crisis can be showed as below:

\[
Probability\ of\ liquidity\ crisis_t = \int_0^1 f(SR_t) \cdot d(SR_t) \tag{19}
\]

Based upon the equation (19), we can do the liquidity crisis predictions for the future periods.

In summary, the process of our “Solvency based multi-period liquidity crisis predictions model can be illustrated as figure 6.

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\textsuperscript{27} Lognormal distributions also satisfy the characteristics of non-negative value for solvency ratio.

\textsuperscript{28} The details please refer appendix III.
IV. Empirical Analysis

In this section, we use our “Solvency Based Multi-period Liquidity Crisis Prediction Model” to empirically assess seven companies from Taiwan’s stock market to preliminarily examine the validity of the model. In the following, we introduce our data, the results of parameter estimation of industrial economic state model and stochastic solvency ratio model, and last present the liquidity crisis prediction results.

1. Data

The sample companies are from Taiwan’s stock market. The industry distribution of the sample companies is illustrated in table 1. All company related financial information is from TEJ database\textsuperscript{29} and their credit rating data are from Taiwan Ratings Corporation (later denoted as TRC) and S&P website. The sources and information of industrial state (industrial cyclical factors), including coincident and leading indicators, exhibit in table 3\textsuperscript{30}. To sum up, all data sources are completely illustrated in table 2.

Table 1. The industrial categories’ distribution of empirical sample

<table>
<thead>
<tr>
<th>Industry</th>
<th>Steels</th>
<th>Cars</th>
<th>Plastics</th>
<th>Computers</th>
<th>Semiconductors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2. All related empirical data sources

<table>
<thead>
<tr>
<th>Items</th>
<th>Corporate financial data and ratings</th>
<th>Industrial economic state (business cyclical factors)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sources</td>
<td>TEJ, TRC and S&amp;P website</td>
<td>TEJ database</td>
</tr>
</tbody>
</table>

\textsuperscript{29} The estimation period for parameters in our solvency ratio model is from 1995 to 2004 Q2, except for FCFC (from 1998 to 2004 Q2).

\textsuperscript{30} The applicable proxies for industrial economic states in table 2 selected are according to the criteria of NBER and Council for Economic Planning and Development (in Taiwan).
Table 3: The applicable proxies for industrial economic state

<table>
<thead>
<tr>
<th>Leading indicators</th>
<th>sources</th>
<th>Coincident indicators</th>
<th>sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The change rate of each industrial stock index</td>
<td>TEJ, Datastream</td>
<td>1. Each industrial sales revenues</td>
<td>TEJ, Datastream</td>
</tr>
<tr>
<td>2. The change rate of each industrial added-orders</td>
<td>TEJ, Datastream</td>
<td>2. The change rate of each industrial production</td>
<td>TEJ, Datastream</td>
</tr>
<tr>
<td>3. For specified industry (e.g. Semiconductor, DRAM) with compiled index or goods’ price</td>
<td>SEMI, Bloomberg, Datastream</td>
<td>3. The change rate of each industrial sales volume indexes</td>
<td>TEJ</td>
</tr>
</tbody>
</table>

* The decisions of leading indicators or coincident indicators primarily depend on the business cyclical indicators selected by Council for Economic Planning and Development (in Taiwan)

** The decisions of American macroeconomic indicators are similar with Taiwan.

2. Parameters estimation of the stochastic model of solvency ratio and industrial economic state

For our stochastic solvency ratio model, we use the square root method (Chen, 1996) to estimate its parameters. Moreover, we also estimate the sensitivity ($\alpha_1$) of the lnSR relative to industrial economic state by linear regression model. The above results are illustrated as table 4.

Table 4. The Parameters’ estimation of solvency ratio model

<table>
<thead>
<tr>
<th>Industry</th>
<th>Company</th>
<th>$a$</th>
<th>$b$</th>
<th>$\sigma$</th>
<th>$\alpha_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semiconductors</td>
<td>TSMC</td>
<td>1.1051</td>
<td>3.4084</td>
<td>1.7321</td>
<td>0.0142</td>
</tr>
<tr>
<td>Semiconductors</td>
<td>UMC</td>
<td>1.5635</td>
<td>3.6302</td>
<td>1.4791</td>
<td>-0.0016</td>
</tr>
<tr>
<td>Steels</td>
<td>CSC</td>
<td>1.7491</td>
<td>2.0516</td>
<td>1.5767</td>
<td>0.0163</td>
</tr>
<tr>
<td>Plastics</td>
<td>FCFC</td>
<td>1.4379</td>
<td>1.4758</td>
<td>1.4342</td>
<td>0.0463</td>
</tr>
<tr>
<td>Plastics</td>
<td>FPC</td>
<td>0.8626</td>
<td>2.0822</td>
<td>1.3340</td>
<td>0.0260</td>
</tr>
<tr>
<td>Computers</td>
<td>Compal</td>
<td>0.8318</td>
<td>3.0624</td>
<td>1.5904</td>
<td>0.0103</td>
</tr>
<tr>
<td>Cars</td>
<td>Yulon</td>
<td>0.5268</td>
<td>2.7353</td>
<td>1.1643</td>
<td>0.0670</td>
</tr>
</tbody>
</table>
To adjust the parameters of our solvency ratio model, we have to know the industrial economic state model first. Therefore we have to estimate the parameters of the industrial economic state model. Here we select some business cyclical factors (industrial economic state) as shown in table 3 to be a proxy for industrial economic state. We employ AR(1) method (Chen, 1996) to estimate its parameters and the results are illustrated in table 5.

Table 5. Parameters’ estimation of stochastic industrial economic state model

<table>
<thead>
<tr>
<th>Industry</th>
<th>Simulation target</th>
<th>(a_\eta)</th>
<th>(b_\eta)</th>
<th>(\sigma_\eta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-conductors</td>
<td>Change rate of Electric Utilities Shipments Indexes</td>
<td>0.3146</td>
<td>0.0315</td>
<td>0.0377</td>
</tr>
<tr>
<td>Steels, Cars</td>
<td>Change rate of Total Manufacturing Shipments Indexes</td>
<td>0.4490</td>
<td>0.0106</td>
<td>0.0199</td>
</tr>
<tr>
<td>Plastics</td>
<td>Change rate of Plastic Products Shipments Indexes</td>
<td>1.0737</td>
<td>0.0013</td>
<td>0.0317</td>
</tr>
<tr>
<td>Computer &amp; its products</td>
<td>Change rate of Computers &amp; Telecom. Shipments Indexes</td>
<td>0.5570</td>
<td>0.0310</td>
<td>0.0322</td>
</tr>
</tbody>
</table>

3. Empirical results of firm’s credit rating

The empirical credit analyses results are illustrated in table 6. The third column of table 6 stands for probability of liquidity crisis of each sample firm calculated by our solvency ratio model (denoted as “model’s PLC”) during the future one year. The fourth column of table 6 represents for each firm’s theoretical long-term rating (denoted as “model’s rating”). The model’s ratings are assigned to each firm by comparing model’s PLC to the one-year default rates curve in American market\(^{31}\). Hence, the model’s ratings are equivalent to American (or global) rating. Since Taiwan market is essentially not the same as American market in terms of having different risk factors, such as political risk, country risk and so on, our model’s rating should be upgrade-adjusted to recover to firm’s local long-term rating\(^{32}\) (shown in the

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31 The cumulative default rate curve is provided by Standard and Poor’s (1981~2002).
32 In practice, a rule of thumb for the rating difference is that Taiwan local ratings are about one rating grade lower than those of Global rating. For example, in practice a twA- rating is equivalent to a global rating BBB-. From model’s perspective, Taiwan’s cumulative default rates curve should add a country rating spread to be equivalent to that of global (or American) market.
fifth column of table 6). And then we can get the firm’s theoretical short-term rating (the sixth column of table 6) by utilizing the correlation of long- and short-term ratings.  

According to empirical results illustrated in table 6, we discover that all the seven firms’ theoretical short-term ratings are the same as the actual short-term ratings and rated reasonably. Our model’s effectiveness seems preliminarily supported by the empirical evidences.

However, we primarily focus on the normal firm’s operations on our current model. It is also our model’s constraint that we cannot consider all the uncertainty (e.g. suddenly obvious changes) in our model. It could be improved by adding extra stochastic terms into our model, such as “jump diffusion model” to take care of more uncertainties.

Table 6. Empirical results of solvency ratio model

<table>
<thead>
<tr>
<th>Item</th>
<th>Company</th>
<th>Model’s PLC (One-Year)</th>
<th>Model’s rating (Long-term)</th>
<th>Model’s rating (Taiwan)</th>
<th>Model’s rating (Short-term)</th>
<th>Actual rating (short-term)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TSMC*</td>
<td>0.00%</td>
<td>AAA</td>
<td>twAAA</td>
<td>twA-1</td>
<td>twA-1</td>
</tr>
<tr>
<td>2</td>
<td>UMC*</td>
<td>0.00%</td>
<td>AAA</td>
<td>twAAA</td>
<td>twA-1</td>
<td>twA-1</td>
</tr>
<tr>
<td>3</td>
<td>CSC*</td>
<td>0.00%</td>
<td>AAA</td>
<td>twAAA</td>
<td>twA-1</td>
<td>twA-1</td>
</tr>
<tr>
<td>4</td>
<td>FCFC*</td>
<td>0.13%</td>
<td>A-</td>
<td>twAA-</td>
<td>twA-1</td>
<td>twA-1</td>
</tr>
<tr>
<td>5</td>
<td>FPC*</td>
<td>0.28%</td>
<td>BBB+</td>
<td>twA+</td>
<td>twA-1</td>
<td>twA-1</td>
</tr>
<tr>
<td>6</td>
<td>Compal*</td>
<td>0.10%</td>
<td>A-</td>
<td>twAA-</td>
<td>twA-1</td>
<td>twA-1</td>
</tr>
<tr>
<td>7</td>
<td>Yulon*</td>
<td>0.60%</td>
<td>BBB-</td>
<td>twA-</td>
<td>twA-2</td>
<td>twA-2</td>
</tr>
</tbody>
</table>

* : Model’s short-term rating is as same as the actual short-term rating given by TRC.

1. Each firm’s actual rating is acquired from TRC website.
2. Model’s PLC: The results are from 10000 times simulation of solvency ratio model for each sampled firm. And then we can calculate the one-year hazard rate through “A Solvency-based Multi-period Liquidity Crisis Prediction Model” (model’s PLC).
3. Model’s rating: Let model’s PLC correspond to American one-year default rates provided by S&P and further we can decide the credit rating for each firm.
4. Except for FCFC, the estimation periods for all other companies are 1995~2004Q2.(the estimation period of FCFC is 1998~2004Q2; this is because FCFC changes its main operating business from fibers to plastics in recent five years.)

In addition, we will also show the simulation results of multi-period solvency

33 The correlation of long- and short-term ratings is in the following: For the ratings higher than twA+, their short-term ratings will be equivalent to twA-1; for the ratings between twA- and twA, their short-term ratings will be equivalent to twA-2; for the ratings between twBBB- and twBBB+, their short-term ratings will be equivalent to twA-3 and so on. (from the TRC’s rating tables)
ratio distributions. In the following, we will take our sample firms as examples and they are illustrated as figures 7–20.

Figure 7. TSMC’s One-Year Ahead SR distr.     Figure 8. TSMC’s Multi-period SR distr.

Figure 9. UMC’s One-Year Ahead SR distr.      Figure 10. UMC’s Multi-period SR distr.

Figure 11. CSC’s One-Year Ahead SR distr.      Figure 12. CSC’s Multi-period SR distr.
Figure 13. FCFC’s One-Year Ahead SR distr.  
Figure 14. FCFC’s Multi-period SR distr.  
Figure 15. FPC’s One-Year Ahead SR distr.  
Figure 16. FPC’s Multi-period SR distr.  
Figure 17. Compal’s One-Year Ahead SR distr.  
Figure 18. Compal’s Multi-period SR distr.
V. Conclusions

The development of corporate failure prediction models grow rapidly recently and mainly on the classical statistical models and the stochastic intensity models. However, the former employs corporate historical financial data to do failure predictions and few of them propose stochastic models to estimate future liquidity measure. The latter relies on exogenous information such as credit rating rather than on a firm’s financial information such as liquidity related ratios. Besides, the developments of multi-period corporate failure prediction models are few in literatures. This study tries to fill up these literature gaps.

“Solvency Based Multi-period Liquidity Crisis Prediction Model” constructed in this study provides a systematic measuring process of corporate failure prediction. It starts from determining a firm’s future solvency ratio distributions by our “Time-dependent stochastic solvency ratio model” and then automatically performs multi-period liquidity crisis predictions by the direct criterion (when solvency ratio is less than one). Therefore, our solvency ratio model’s merits can be summarized as follows: relating to firm-liquidity; following a stochastic process; being reasonably extended to multi-period with the stochastic model of industrial economic state; providing a simple and direct criterion for liquidity crisis. The above four merits are rarely simultaneously provided by other failure prediction models.

In addition, our models can be used to do liquidity crisis predictions without knowing a firms’ credit rating. They straightly consider the firm’s future solvency ratio to do multi-period liquidity crisis analyses rather than conducting a backward
solution from firm’s credit rating to forecast a corporate failure. For both outside investors and people inside a firm, our study provides a multi-period liquidity crisis prediction model that needs only publicly available information of both corporate finance and the industrial economic state (i.e. the industrial cyclical information). We believe this “Solvency Based Multi-period Liquidity Crisis Prediction Model” has provided a new way for analyzing firm’s solvency ability.


Wilson, T., 1997a, Portfolio Credit Risk, I. RISK 10, September, 111–117.

Wilson, T., 1997b, Portfolio Credit Risk, I. RISK 10, October, 56–61.
Appendix I. The stochastic characteristics of industrial economic state

In this study, we use the change rate of each industrial seasonal-adjusted shipments indexes to be the proxies for industrial economic state factors in Taiwan market and then observe the trends (the sample period is from 1995 to 2004Q2 and the data-type is quarterly). These industrial categories include steels, cars, plastics, semiconductors, and computer& its related products. The historical trend of each industrial economic state factor is illustrated as the following figures and we discover that there exists the phenomenon of mean-reversion in all industrial categories. We therefore think industrial economic state applicable to mean-reversion stochastic model.

34 Because the shipments indexes of steels and cars can’t be acquired, we employ the manufacturing shipments indexes to substitute for.
Appendix II. The historical trend of interest rates

Figure A2-1. USA 6-month T-Bills Secondary Market

Figure A2-2: 6-month LIBOR historical trend

35 Sources: Global Financial Data, Inc
36 Sources: http://www.economagic.com/libor.htm#US
Appendix III. The actual solvency ratio distributions of firms

Through our observations for solvency ratio, we can initially suppose that it will comply with non-central positive value distributions such as lognormal, non-central chi-squared distributions and so on. However, in order to test the solvency ratio’s actual distribution, we firstly implement goodness-of-fit tests on solvency ratio and then discover the log-normal distribution for solvency ratio is preliminarily supported by statistical results. The test results will show as the following figures (figure A3-2, A3-4, A3-6, A3-8, A3-10). And solvency ratio’s “non-negative value” characteristic also matches up with lognormal distribution. In the following, we utilize Anderson-Darling statistics\(^\text{37}\) to do goodness-of-fit tests.

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\(^{37}\) Anderson-Darling tests is introduced by D’Agostino and Stephens, 1986, “Goodness-of-fit techniques”.

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Figure A3-5. CSC’s solvency ratio distribution

Figure A3-6. CSC’s goodness-of-fit test

Figure A3-7. FPC’s solvency ratio distribution

Figure A3-8. FPC’s goodness-of-fit test

Figure A3-9. Compal’s solvency ratio distribution

Figure A3-10. Compal’s goodness-of-fit test
Figure A3-11. FCFC’s solvency ratio distribution

Figure A3-12. FCFC’s goodness-of-fit test

Figure A3-13. Yulon’s solvency ratio distribution

Figure A3-14. Yulon’s goodness-of-fit test
Appendix IV. The monotonic transformation of solvency ratio

In this study, we can know that solvency ratio will comply with lognormal distribution from goodness-of-fit tests. To simplify our model designation, we take natural log on solvency ratio (later denoted as lnSR) and then lnSR will comply with normal distribution. Because taking natural log on solvency ratio belongs to monotonic transformation, it will not change solvency ratio’s essential characteristics (e.g. its mean-reverting characteristic and trend characteristics). Only one difference to original solvency ratio is that the range changes from $[0, \infty]$ to $[-\infty, \infty]$.

Figure A4-1. TSMC’s lnSR trend analysis

Figure A4-2. CSC’s lnSR trend analysis

Figure A4-3. Yulon’s lnSR trend analysis

Figure A4-4. Compal’s lnSR trend analysis

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38 We can compare figures 1–4 with figures A4-1–A4-4 and then can discover that the essential characteristics are still maintained.
Appendix V. The method to estimate parameters of time-dependent stochastic solvency ratio model

In this study, our stochastic solvency ratio model can be showed as equation (A4-1):

$$d(\ln SR_t) = a(t) \cdot [b(t) - \ln SR_{t-1}] \cdot dt + \sigma(t) \cdot dz, \quad dz = \varepsilon \sqrt{dt}, \quad \varepsilon \sim N(0,1) \quad (A4-1)$$

where,

- $d(\ln SR_t)$: lnSR’s term variation (or instantaneous changes in continuous time)
- $a(t)$: lnSR’s mean-reversion speed.
- $b(t)$: lnSR’s long-term average level
- $\sigma(t)$: standard deviation of lnSR’s term variation, namely $\sqrt{\text{Var}(d(\ln SR_t))}$.

In the estimation of stochastic solvency ratio model’s parameters ($a$, $b$, $\sigma$), we can calculate the fixed constant by using AR(1) method (Chen, 1996) firstly.

Now we want to let $b$ and $\sigma$ be time-varying so that we employ stochastic industrial economic state model to make adjustments. In the following, we let $\omega$ stand for the industrial economic state factor. Further we explore the relationship between lnSR and industrial economic state factor ($\omega$) by constructing the regression showed as equation (A4-2):

$$\ln SR_t = \alpha_0 + \alpha_1 \cdot (\omega_t) \quad (A4-2)$$

As a result, we can make time-varying adjustments on the long-term average of lnSR ($b(t)$) based on the future lnSR’s growth rate relative to $b$. According to equation (A4-2), we can further transfer the future lnSR’s growth rate to the future industrial economic state indicator’s growth rate:

$$b(t) = b \cdot (1 + \left( \frac{\ln SR(t) - \ln \overline{SR}_{\text{long-term}}}{\ln \overline{SR}_{\text{long-term}}} \right))$$

$$= b \cdot (1 + \left( \frac{\omega_t - \overline{\omega}_t}{\overline{\omega}_t} \right) \cdot \alpha_1) \quad (A4-3)$$

In equation (A4-3), $\frac{\omega_t - \overline{\omega}_t}{\overline{\omega}_t}$ stands for the growth rate of the future industrial economic state indicator relative to its long-term average ($\overline{\omega}_t$). Moreover, we have to consider the regressive coefficient $\alpha_1$ when making adjustments on the long-term
average of lnSR (b(t)) according to equation (A4-3).

In addition, we will discuss the method to make the fluctuation of lnSR (σ) be time-varying. First, we will difference on the both sides of equation (A4-2) and then take variances on the difference results. Second, we will explore the relationship between “the effect on the changes of Δ(ln SR) caused by the changes of Δ(ω) ” and “the effect on the σΔ(ln SR) caused by the changes of σΔ(ω) ”. At last, we can infer the adjustment methods for the fluctuation of lnSR(σ). In the following, we will display our inference:

Difference on both sides of equation (A4-2) as follows:

\[ \Delta(\ln SR) = \alpha \cdot \Delta(\omega) \]  
(A4-4)

Take variances on both sides of equation (A4-4) as follows:

\[ \text{Var}(\Delta(\ln SR)) = \alpha^2 \cdot \text{Var}(\Delta(\omega)) \Rightarrow \sigma_{\Delta(\ln SR)} = |\alpha| \cdot \sigma_{\Delta(\omega)} \]  
(A4-5)

So we can summarize as equation (A4-6) when \( \alpha \) is a positive constant according to equation (A4-4) and (A4-5):

\[ \alpha = \frac{\Delta(\ln SR)}{\Delta(\omega)} = \frac{\sigma_{\Delta(\ln SR)}}{\sigma_{\Delta(\omega)}} \]  
(A4-6)

And we can also get equation (A4-7) when \( \alpha \) is a negative constant:

\[ -\alpha = \frac{\Delta(\ln SR)}{\Delta(\omega)} = \frac{\sigma_{\Delta(\ln SR)}}{\sigma_{\Delta(\omega)}} \]  
(A4-7)

We can therefore conclude that the size of “effect on the changes of Δ(ln SR) caused by the changes of Δ(ω)” (called \( A \) event) will be the same with the size of “effect on the σΔ(ln SR) caused by the changes of σΔ(ω)” (called \( B \) event)
when industrial economic state changes in the future\textsuperscript{39}. But it is necessary to notice that the relationship of these two events will vary with $\alpha_i$; that is to say, the direction of $A$ event will be opposite to the direction of $B$ event when $\alpha_i$ is negative.

In equations (A4-6) and (A4-7), we can know that $\sigma_{\Delta(\ln SR_i)}$ is a function of $\sigma_{\Delta(\omega_i)}$ and $\Delta(\ln SR_i)$ is a function of $\Delta(\omega_i)$. And both two functions are estimated the same base, namely $\alpha_i$. Therefore according to the concept of \textit{varying coefficient model}, the effects on $A$ event and $B$ event will be the same by $\alpha_i$ when the industrial economic state changes in the future ($\Delta(\omega_i), \sigma_{\Delta(\omega_i)}$). As a result, we can make adjustments on the fluctuation of $\ln SR(\bar{\omega})$ by using $A$ event instead of $B$ event. In the following, we will infer the $A$ event’s effect firstly, then apply the result in $B$ event and at last we can conclude the adjustment methods of the fluctuation of $\ln SR(\bar{\omega})$:

\textbf{Inferences:}

When the industrial economic state indicator is $\omega_i$ in the future time $t$, we can get the adjustment effect of reflecting on the long-term average $\ln SR (b)$ according to equation (A4-3):

\[
\frac{b(t) - b}{b} = \alpha_i \cdot \left( \frac{\omega_i}{\bar{\omega}} - 1 \right) \quad (A4-8)
\]

When the industrial economic state factor is $\omega_{t+1}$ in the future time $t+1$, we can get the adjustment effect of reflecting on the long-term average $\ln SR (b)$ according to equation (A4-3):

\[
\frac{b(t+1) - b}{b} = \alpha_i \cdot \left( \frac{\omega_{t+1}}{\bar{\omega}} - 1 \right) \quad (A4-9)
\]

We therefore make equation (A4-9) minus equation (A4-8) and then get the influencing amount of $A$ event:

\textsuperscript{39} For the decision of solvency ratio in future time $t$, the solvency ratio at time $t-1$ has been known. We therefore can think it as a known constant when deciding the future $\ln SR$.\hfill
\[ b \cdot \alpha_1 \cdot \left( \frac{\omega_{t+1} - \omega_t}{\omega} \right) \]  \hspace{1cm} (A4-10)

As a result, the influencing size of \( A \) event will be showed as the equation (A4-11) when the base of solvency ratio is \( b \):

\[ \alpha_1 \cdot \left( \frac{\omega_{t+1} - \omega_t}{\omega} \right) \]  \hspace{1cm} (A4-11)

According to equation (A4-11), we can know that \( \ln SR \) will change by the rate of \( \alpha_1 \cdot \left( \frac{\omega_{t+1} - \omega_t}{\omega} \right) \) with the varying of industrial economic state. Moreover, we can know that \( A \) event has the effect with \( B \) event from equation (A4-6) and (A4-7). But it is necessary to notice that \( \alpha_i \) have to be taken as its absolute value.

In this study, we make regression analysis for the past \( \ln SR \) and the industrial economic state’s indictor at the same time. Further we can get the changing relationship (regressive coefficient, \( \alpha_i \)) between these two above. However, \( \alpha_i \) may be not representative when the selected proxy of industrial economic state’s indictor is not optimal (namely \( \ln SR \) may probably be explained by other variables). We will therefore try to create a new factor with high exploratory power for \( \ln SR \) by using factor analysis methods. Further we can get the better result.